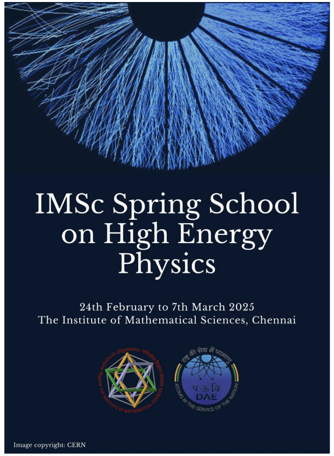
Introduction to Effective Field Theories



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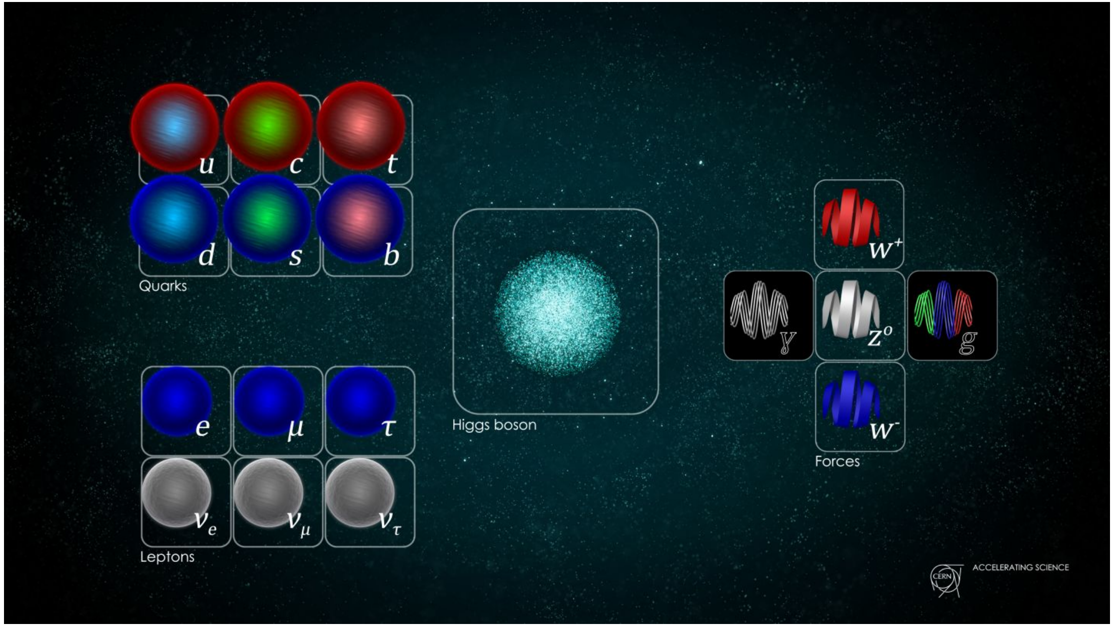
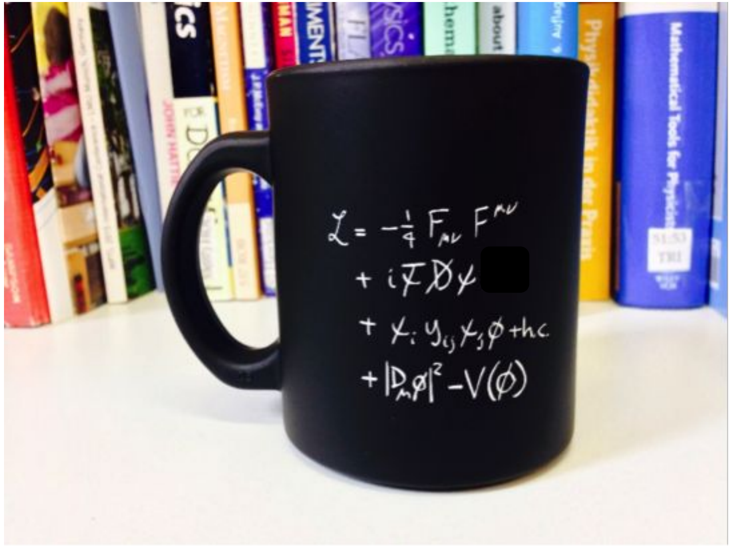
LHC EFT Working Group: Subscribe

The Standard Model of Particle Physics

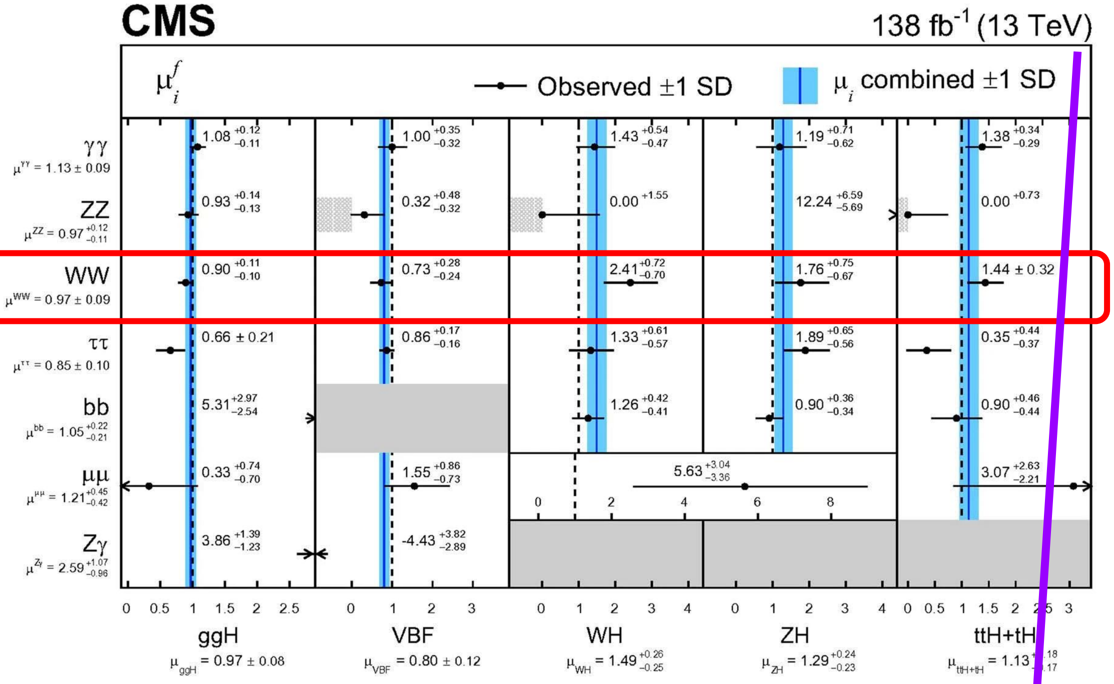


Elementary particles in the Standard Model of particle physics
Image: Daniel Dominguez/CERN



Simplified way of expressing interactions between the Standard Model particles
CERN coffee mug (corrected!)

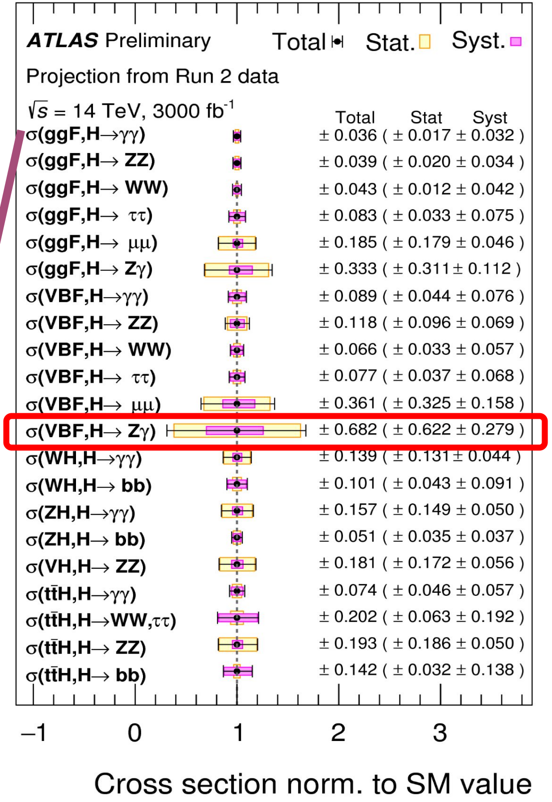
Importance of precision: the premise



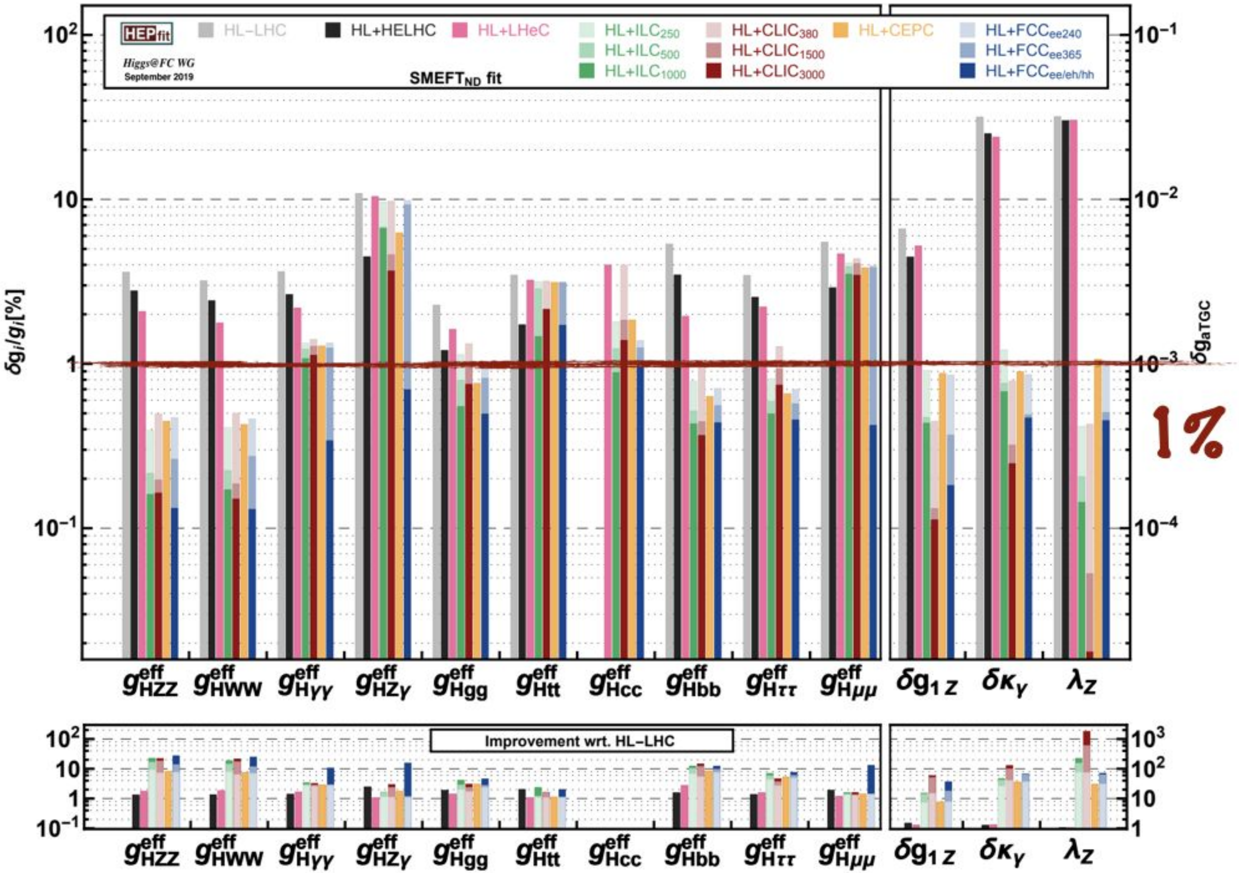
Ratios of the Higgs boson's measured interactions to other particles to its Standard Model expectations. If Standard Model predictions are exact, these numbers would eventually be 1.

From current data

Future projections from LHC, CERN

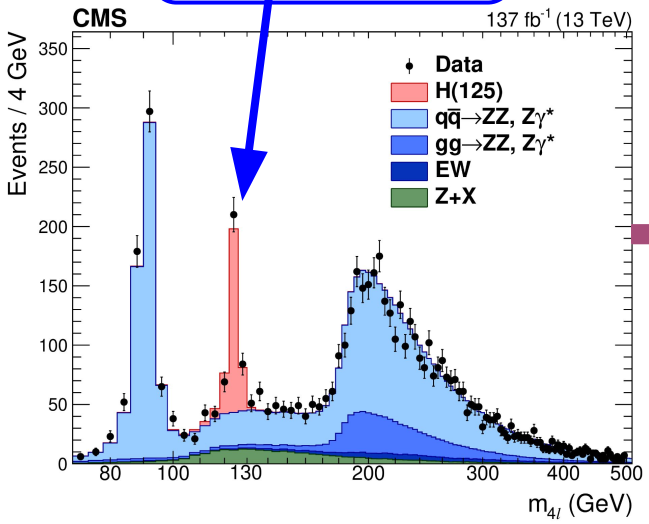


Importance of precision: the premise



Particle physics discovery: the types

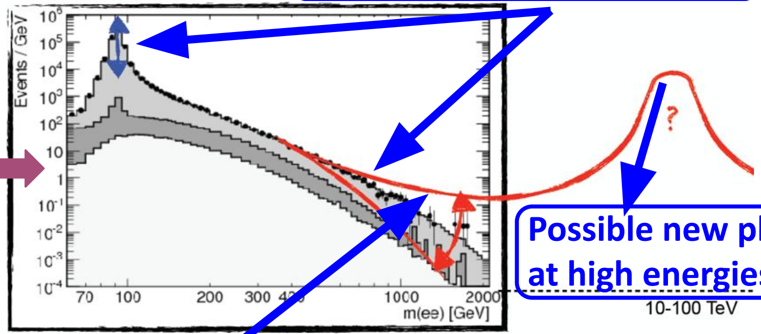
Discovery through resonance (the tested paradigm)



We are in the phase of no elementary particle resonance discoveries since the last 12+ years after the Higgs in 2012. What to do then?

New paradigm!

Discovery through precision



Possible new physics at high energies

%-level precision

Image: Francesco Riva

Possible to see hints of new physics through difference in heights, angular structure and tails of distributions without seeing the actual resonance

Introduction and Motivation (Classical Example)

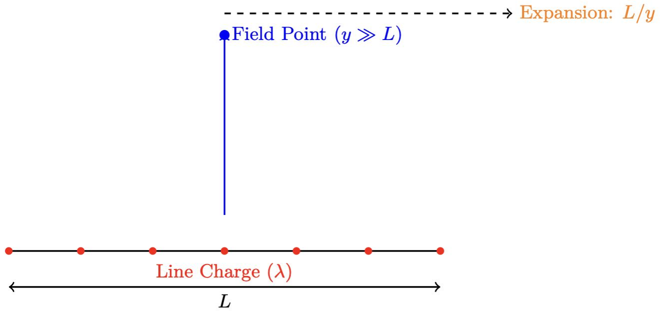
In simple terms, effective field theories (EFTs) provide a simplified description of a more fundamental theory by focusing on its low-energy (long-distance) behaviour, effectively "integrating out" the high-energy details.

- **Setup:** Consider a uniformly charged line along the x-axis, from $x = -\frac{L}{2}$ to $x = \frac{L}{2}$ with linear charge density λ. Total charge: $Q = \lambda L$. The potential at a point on the y-axis ($y \gg L$) is:

$$\phi(y) = \frac{1}{4\pi\varepsilon_0} \int_{-L/2}^{L/2} \frac{\lambda\, dx}{\sqrt{x^2 + y^2}}.$$

- **Multipole Expansion:** For $y \gg |x|$, rewrite $\sqrt{x^2+y^2} = y\sqrt{1+\frac{x^2}{y^2}}$ and expand:

$$\frac{1}{\sqrt{x^2+y^2}} \approx \frac{1}{y}\left[1 - \frac{1}{2}\frac{x^2}{y^2} + \cdots\right].$$



- **Analogy:** This expansion in powers of L/y is analogous to an EFT expansion in powers of E/Λ. Only the leading multipole (monopole, etc.) survives at large distances (low energies), just as irrelevant operators are suppressed in an EFT.

Introduction and Motivation (Classical Example)

- **Result:** After integration, the potential becomes:

$$\phi(y) = \frac{\lambda L}{4\pi\varepsilon_0\, y}\left[1 - \frac{L^2}{24\, y^2} + \cdots\right],$$

  where the leading term is the monopole and the next term (proportional to L^2/y^2) represents the first correction.

- **EFT Analogy:** In an EFT, high-energy details are encoded in a series expansion in E/Λ; here the expansion parameter is L/y.

Introduction and Motivation

What about in particle physics? The story is similar.

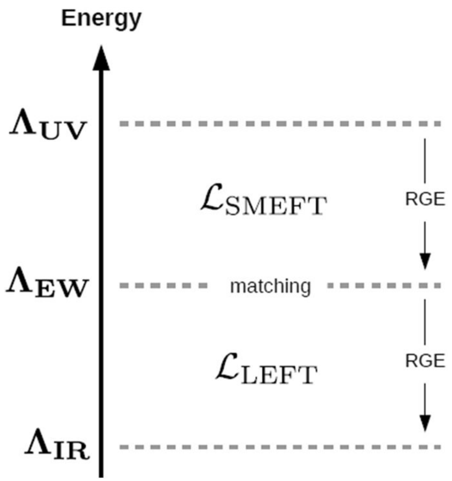
- Use energy scale $E \sim L^{-1}$ to define regimes.
- Low-energy experiments (E) don't require full high-scale $(\Lambda \gg E)$ details.
- High-scale effects are encoded in a finite set of effective parameters.
- This separation of scales enables a practical research program.
- Example: LHC collisions $(E \sim 1\,\mathrm{TeV})$ are described by an effective theory.

What are Effective Field Theories

Concept: EFT is a framework that lets us describe low-energy physics without needing full details of the high-energy dynamics.

Key Principle: Retain only the relevant low-energy degrees of freedom; the effects of heavy states are encoded in higher-dimensional operators.

Separation of Scales



In many systems, there is a clear hierarchy:

$$E \ll \Lambda,$$

where E is the energy scale of interest and Λ is the cutoff or new-physics scale.

This separation allows us to expand physical quantities in powers of $\frac{E}{\Lambda}$.

Decoupling and the Appelquist-Carazzone Theorem

The Appelquist–Carazzone theorem tells us that heavy fields contribute corrections of the form:

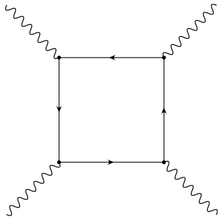
$$\Delta\mathcal{O} \sim \left(\frac{E}{\Lambda}\right)^n .$$

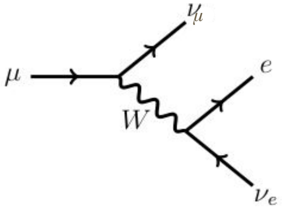
Thus, for $E \ll \Lambda$ the influence of heavy fields is suppressed.

Decoupling and Effective Field Theories

Formal EFT Decoupling:

- UV Theory: The full dynamics is given by $L_{UV}(\phi, H)$ with light fields ϕ and heavy fields H.
- Low-Energy Focus: At energies $E \ll m_H$, experiments involve only ϕ in external states; the effects of H are encoded in effective parameters.
- QED Example: Photons scatter via virtual electrons. For low-energy photons (below e^+e^- threshold), on-shell electrons don't appear.
- Muon Decay: Proceeds via a virtual W boson since $m_\mu \ll m_W$, so the details of the full SM are not required.





Integrating Out Heavy Fields: An Introduction

- **UV Theory:** The full theory is described by the Lagrangian

$$L_{UV}(\phi, H),$$

where ϕ represents light degrees of freedom and H the heavy ones.

- **Path Integral Framework:** The complete dynamics is encoded in the partition function:

$$Z_{UV}[J_\phi, J_H] = \int [\mathcal{D}\phi]\,[\mathcal{D}H]\,\exp\left[i\int d^4x\left\{L_{UV}(\phi, H) + J_\phi\,\phi + J_H\,H\right\}\right].$$

- **Effective Theory:** At low energies (when $E \ll m_H$), experiments probe only ϕ. By setting $J_H = 0$, we obtain:

$$Z_{EFT}[J_\phi] = Z_{UV}[J_\phi, 0],$$

But is this realistic?

which defines the effective theory for ϕ with heavy-field effects encoded in effective interactions.

Effective Lagrangian and Locality

- **Effective Lagrangian:** It is defined via

$$Z_{EFT}[J_\phi] = \int \mathcal{D}\phi \, \exp\left[i \int d^4x \left\{ L_{EFT}(\phi) + J_\phi \, \phi \right\} \right].$$

Effective Lagrangian and Locality

- **Local vs. Non-local Operators:**
  - A *local* operator is polynomial in fields and their derivatives, e.g.

$$\phi^2 \Box \phi^2.$$

  - A *non-local* operator involves non-polynomial functions of derivatives, e.g.

$$\phi^2 (\Box + M^2)^{-1} \phi^2.$$

Interaction occurs at a single point in spacetime.

Interactions are "smeared out" over spacetime.

- **Local Expansion & Matching:** In general,

$$L_{eff}(\phi) \neq L_{UV}(\phi, H = 0),$$

unless ϕ and H are completely decoupled. The difference

$$L_{eff}(\phi) - L_{UV}(\phi, 0)$$

is non-trivial and accounts for the effects of H exchange between the ϕs. For $M \gg E$, the heavy propagator can be expanded as

$$(\Box + M^2)^{-1} \sim \frac{1}{M^2} - \frac{\Box}{M^4} + \cdots ,$$

so that heavy-field effects appear as a series of local contact interactions.

Non-local operators can arise when heavy degrees of freedom are integrated out, capturing their propagation effects at low energies.

Some Motivations for Using EFTs

- **Simplicity:** EFTs reduce the complexity of the full theory to a finite set of effective parameters, capturing the essential low-energy dynamics.
- **Calculability:** They enable efficient multi-loop computations and resummation of large logarithms via renormalisation group techniques.
- **Model Independence:** When the underlying UV theory is unknown or too complicated, EFTs allow a systematic description of low-energy phenomena by parameterising heavy-field effects.
- **Practical Application:** For example, describing LHC collisions at $E \sim 1\,\mathrm{TeV}$ only requires the effective theory, with the details of higher-scale physics encoded in a limited number of parameters.

Infinite Interactions and Power Counting

- Even a local effective Lagrangian $L_{EFT}(\phi)$ contains an **infinite** number of interaction terms.
- **Power counting rules** organise these terms into a controlled expansion.
- For relativistic EFTs (obtained by integrating out heavy fields H), the expansion parameter is E/M_H (with E the typical experimental energy).

Coordinate and Field Rescaling and Lagrangian Transformation

- Under the rescaling

$$x_\mu \to \xi\, x'_\mu,$$

the integration measure transforms as $d^4x = \xi^4\, d^4x'$ and derivatives scale as $\partial_\mu \to \partial'_\mu/\xi$.

- **Energy Interpretation:** $\xi \to 0$ (fixed x') implies small x (short distances, high energies), while $\xi \to \infty$ corresponds to large x (long distances, low energies).

- Consider the effective action for a scalar field:

$$S_{EFT}(\phi) = \int d^4x \left[(\partial_\mu\phi)^2 - m^2\phi^2 - \kappa\,\mu\,\phi^3 - \lambda\,\phi^4 - \sum_{n+d>4} \frac{c_{n,d}}{\Lambda^{n+d-4}}\phi^{n-1}(\partial)^d\phi\right]. \tag{1}$$

- Under $x_\mu \to \xi\, x'_\mu$, it becomes

$$S_{EFT}(\phi) = \int d^4x' \left[\xi^2(\partial_\mu\phi)^2 - m^2\,\xi^4\phi^2 - \kappa\,\mu\,\xi^4\phi^3 - \lambda\,\xi^4\phi^4 - \sum_{n+d>4} \frac{c_{n,d}\,\xi^{4-d}}{\Lambda^{n+d-4}}\phi^{n-1}(\partial)^d\phi\right].$$

- To restore canonical normalisation of the kinetic term, rescale the field as

$$\phi \to \phi'\,\xi^{-1},$$

yielding

$$S_{EFT}(\phi') = \int d^4x' \left[(\partial_\mu\phi')^2 - m^2\phi'^2 - \kappa\,(\xi\mu)\,\phi'^3 - \lambda\,\phi'^4 - \sum_{n+d>4} \frac{c_{n,d}}{(\xi\,\Lambda)^{n+d-4}}\phi'^{n-1}(\partial)^d\phi'\right]. \tag{3}$$

Canonical Dimensions

- In an interaction term written as

$$\frac{c_{n,d}}{\Lambda^{n+d-4}}\,\phi^{n-1}(\partial)^d\phi,$$

  let:

  - n = number of fields,
  - d = number of derivatives.

- The canonical dimension is $D = n + d - 4$, which determines the scaling:

  - **Relevant** $(D < 0)$: Coefficients grow in the IR (e.g. ϕ^2 with $D = -2$, ϕ^3 with $D = -1$).
  - **Marginal** $(D = 0)$: Coefficients are dimensionless (e.g. ϕ^4).
  - **Irrelevant** $(D > 0)$: Coefficients are suppressed at low energies (e.g. an operator with $n = 3$, $d = 2$ has $D = 1$).

Dimensional Analysis and $\hbar$ Counting

- To derive a general selection rule (counting), temporarily reinsert the Planck constant $\hbar$ (usually set to 1).
- The action S must have dimension $\hbar^1$ since the path integrand is $e^{iS/\hbar}$.
- By convention, kinetic terms are not multiplied by any $\hbar$ factors, so each field with a quadratic kinetic term has dimension $\hbar^{1/2}$.
- Thus, an interaction term with n fields has a coefficient with dimension $\hbar^{1-\frac{n}{2}}$ (independent of derivatives).
- For example, in the UV Lagrangian:
  - $[m^2] = \hbar^0$,
  - κ has dimension $\hbar^{-1/2}$,
  - λ has dimension $\hbar^{-1}$,
  - $[c_{n,d}] = \hbar^{1-\frac{n}{2}}$.

Fermi Theory as an EFT: Overview

- **Concept:** Fermi theory illustrates EFT principles by integrating out heavy fields (e.g. W, Z, Higgs, top) of the SM below the W boson mass.
- **Degrees of Freedom:** At low energies, only light particles remain—here, we focus on muons, electrons, and their neutrinos.
- **Goal:** Derive the effective weak interaction Lagrangian for these particles.

Muon Decay in the Standard Model of Particle Physics

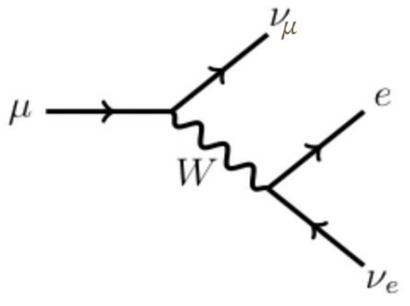
- **Process:** Consider $\mu^-(p) \to e^-(k_1)\,\bar{\nu}_e(k_2)\,\nu_\mu(k_3)$.
- **SM Interaction:** The charged current interaction is given by:

$$\mathcal{L}_{SM} = \frac{g_L}{\sqrt{2}}\Big(\bar{\nu}_\mu\,\bar{\sigma}_\rho\,\mu + \bar{\nu}_e\,\bar{\sigma}_\rho\,e\Big)W^+_\rho + \text{h.c.}$$

Here, $\bar{\sigma}^\mu$ are defined by

$$\bar{\sigma}^0 = I, \quad \bar{\sigma}^i = -\sigma^i \quad (i = 1, 2, 3),$$

where σ^i are the Pauli matrices.



Weak interaction couples to LH (RH) (anti-)fields

- **Amplitude:** The muon decay amplitude is

LH Weyl spinor

$$\mathcal{M} = \frac{g_L^2}{2}\,\bar{x}(k_3)\,\bar{\sigma}_\rho\,x(p)\,\frac{1}{q^2 - m_W^2}\,\bar{x}(k_1)\,\bar{\sigma}_\rho\,y(k_2), \quad q = p - k_3.$$

RH Weyl spinor

- Because $q^2 \le m_\mu^2 \ll m_W^2$, one approximates:

$$\mathcal{M} \simeq -\frac{g_L^2}{2m_W^2}\Big[\bar{x}(k_3)\,\bar{\sigma}_\rho\,x(p)\Big]\Big[\bar{x}(k_1)\,\bar{\sigma}_\rho\,y(k_2)\Big]\Big[1 + \mathcal{O}(q^2/m_W^2)\Big].$$

Matching to the Fermi EFT

- **Effective Lagrangian:** At low energies, the W boson is integrated out, leading to a 4-fermion interaction:

$$\mathcal{L}_{EFT} \supset \frac{c}{\Lambda^2}(\bar{\nu}_\mu \bar{\sigma}_\rho \mu)(\bar{e}\bar{\sigma}_\rho \nu_e) + \text{h.c.}$$

- **Matching:** Reproducing the SM amplitude in the limit $q^2 \ll m_W^2$ requires

$$\Lambda = m_W, \quad c = -\frac{g_L^2}{2}.$$

- **Validity:** This EFT describes all charged current processes (like muon decay) at energies $E \ll m_W$. At higher energies, the EFT fails to reproduce the full SM behavior.

We will discuss the validity of EFTS in the next lecture.

Example: Toy UV Theory and EFT Lagrangian

UV Lagrangian:

Consider a light real scalar field ϕ (mass m_L) and a heavy field H (mass M). The UV Lagrangian is:

$$\mathcal{L}_{UV} = \frac{1}{2}\left[(\partial_\mu \phi)^2 - m_L^2 \phi^2 + (\partial_\mu H)^2 - M^2 H^2\right] - \frac{\lambda_0}{4!}\phi^4 - \frac{\lambda_1}{2} M\, \phi^2 H - \frac{\lambda_2}{4}\phi^2 H^2.$$

Notes:

- Heavy scale M is factored out in the trilinear term.
- A $\mathbb{Z}_2$ symmetry $\phi \to -\phi$ is imposed, so odd powers of ϕ do not appear.
- H^3 and H^4 interactions are neglected (they can be generated by loops).

EFT Lagrangian (for $E \ll M$):

After integrating out H, the EFT Lagrangian is assumed to be

$$\mathcal{L}_{EFT} = \frac{1}{2}\left[(\partial_\mu \phi)^2 - m^2 \phi^2\right] - C_4 \frac{\phi^4}{4!} - \frac{C_6}{M^2}\frac{\phi^6}{6!} + \mathcal{O}(M^{-4}).$$

Interaction terms are organised as an expansion in inverse powers of M, with each operator $\mathcal{O}_d$ of canonical dimension d. Only operators with an even number of ϕs appear by virtue of the $\mathbb{Z}_2$ symmetry.

Example: Toy UV Theory and EFT Lagrangian (Concept of Basis)

Additional Operators at $\mathcal{O}(M^{-2})$:

One could also write operators such as

$$\hat{\mathcal{O}}_6 \equiv (\Box\phi)^2, \quad \tilde{O}_6 \equiv \phi^3\Box\phi, \quad \tilde{O}'_6 \equiv \phi^2\Box\phi^2, \quad \tilde{O}''_6 \equiv \phi^2\,\partial_\mu\phi\,\partial^\mu\phi.$$

Redundancy via Integration by Parts:

- One can show that

$$\phi^2(\partial_\mu\phi)^2 = -\frac{1}{3}\phi^3\Box\phi, \quad \phi^2\Box\phi^2 = \frac{4}{3}\phi^3\Box\phi.$$

H.W.: Show these explicitly!

- Thus, $\tilde{O}'_6$ and $\tilde{O}''_6$ are not independent.

Field Redefinitions and Equivalence:

- Using the classical equations of motion (EOM), one may eliminate $\hat{\mathcal{O}}_6$ and $\tilde{O}_6$ in favour of the operator already present in the EFT.
- Shifting the Lagrangian by terms proportional to the EOM does not affect the S-matrix (the *equivalence theorem*).
- For example, going from the *unbox basis* to the *box basis* corresponds to a field redefinition:

$$\phi \to \phi\Big(1 - \frac{C_6}{120\,C_4\,M^2}\phi^2\Big).$$

H.W.: Show this!

Example: Toy UV Theory and EFT Lagrangian

EOM and Operator Relation:

The classical equation of motion for ϕ (ignoring $\mathcal{O}(M^{-2})$ corrections) is

$$\Box\phi + m^2\phi + \frac{C_4}{6}\phi^2 = \mathcal{O}(M^{-2}).$$

Using the EOM, one can show that

$$\frac{1}{M^2}\phi^3\Box\phi = -\frac{m^2}{M^2}\phi^4 - \frac{C_4}{6M^2}\phi^6 + \mathcal{O}(M^{-4}).$$

Thus, the operator $\tilde{O}_6 \equiv \phi^3\Box\phi$ has the same effect on on-shell amplitudes as a particular combination of the ϕ^4 and ϕ^6 interactions already present.

Alternate EFT Lagrangians: One may also write the EFT as

$$\mathcal{L}_{EFT} = \frac{1}{2}\left[(\partial_\mu\phi)^2 - m^2\phi^2\right] - \tilde{C}_4\frac{\phi^4}{4!} - \frac{\tilde{C}_6}{4!M^2}\phi^3\Box\phi + \mathcal{O}(M^{-4}).$$

These two forms – the *unbox basis* and the *box basis* – are equivalent on-shell, with the coefficients related by:

$$\tilde{C}_4 = C_4 - \frac{m^2}{5M^2}\frac{C_6}{C_4}, \qquad \tilde{C}_6 = -\frac{C_6}{5C_4}.$$

Translation between bases

Homework Task: Explain why field redefinitions, such as the one above, do not affect physical S-matrix elements (hint: review the equivalence theorem).

well-behaved field redefinitions

Example: Toy UV Theory and EFT Lagrangian (Tree-Level Matching)

At tree level, requiring the same propagator in the UV theory and the EFT gives

$$m^2 = m_L^2.$$

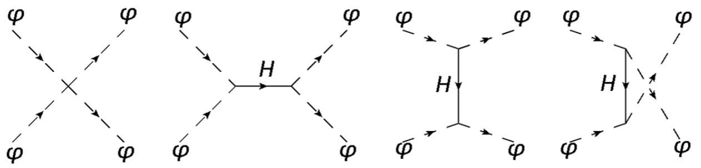
$$\mathcal{L}_{UV} = \frac{1}{2}\left[(\partial_\mu\phi)^2 - m_L^2\phi^2 + (\partial_\mu H)^2 - M^2H^2\right] - \frac{\lambda_0}{4!}\phi^4 - \frac{\lambda_1}{2}M\,\phi^2 H - \frac{\lambda_2}{4}\phi^2H^2$$

$$\mathcal{L}_{EFT} = \frac{1}{2}\left[(\partial_\mu\phi)^2 - m^2\phi^2\right] - C_4\frac{\phi^4}{4!} - \frac{C_6}{M^2}\frac{\phi^6}{6!} + \mathcal{O}(M^{-4})$$

2-to-2 Scattering in the UV Theory:

The process $\phi\phi \to \phi\phi$ receives contributions from:

- The contact ϕ^4 interaction.
- s-, t-, and u-channel exchange of the heavy field H.



The resulting amplitude is

$$\mathcal{M}_4^{UV} = -\lambda_0 - \lambda_1^2 M^2\left[\frac{1}{s-M^2} + \frac{1}{t-M^2} + \frac{1}{u-M^2}\right].$$

For $s,t,u \ll M^2$, expanding gives

$$\mathcal{M}_4^{UV} \simeq -\lambda_0 + 3\lambda_1^2 + \frac{\lambda_1^2}{M^2}(s+t+u) + \mathcal{O}(M^{-4}).$$

Since $s+t+u = 4m_L^2$, we have

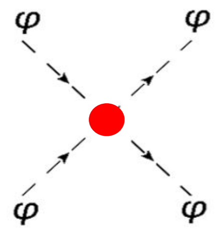
$$\mathcal{M}_4^{UV} \simeq -\lambda_0 + 3\lambda_1^2 + \frac{4m_L^2\lambda_1^2}{M^2} + \mathcal{O}(M^{-4}).$$

Example: Toy UV Theory and EFT Lagrangian (Tree-Level Matching)

EFT Amplitude:

In the EFT (unbox basis), only the contact diagram contributes:

$$\mathcal{M}_4^{EFT} = -C_4.$$



Matching $\mathcal{M}_4^{EFT} = \mathcal{M}_4^{UV} + \mathcal{O}(M^{-2})$ then yields:

Matching Summary:

The tree-level matching conditions up to $\mathcal{O}(M^{-2})$ are:

Matching tools: CoDEx, Matchete, Matchmakereft

$$m^2 = m_L^2, \qquad C_4 = \lambda_0 - 3\lambda_1^2 - 4\lambda_1^2 \frac{m_L^2}{M^2},$$

and (from a lengthy 6-point matching)

$$C_6 = 45\lambda_1^2\lambda_2 - 20\lambda_0\lambda_1^2 + 60\lambda_1^4.$$

H.W.: What about the box basis? Try!

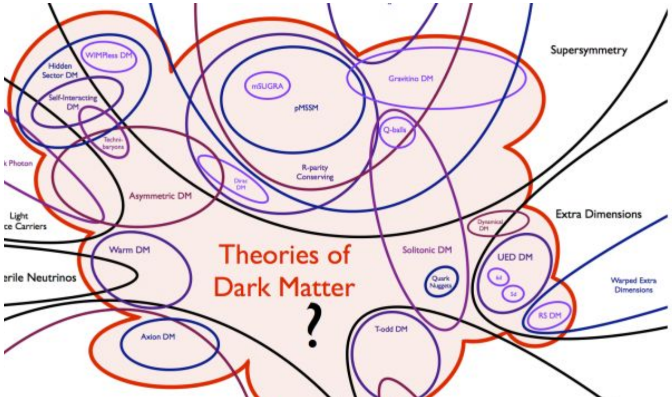
H.W.: Try this out!

What about matching at one-loop?

Limitations of the Standard Model of Particle Physics

- **The Standard Model (SM):** A theory of quarks and leptons interacting via strong, weak, and electromagnetic forces. It is valid up to very high energies and has passed countless experimental tests.
- **Limitations:** Despite its success, the SM does not explain dark matter, neutrino masses, matter–antimatter asymmetry, or cosmic inflation. Theoretical issues (e.g. strong CP problem, flavour hierarchies, unification) further suggest that the SM is incomplete.

Model theistic versus Model agnostic (EFT) approaches



No direct hints towards new physics explaining the various observations which require physics beyond the Standard Model.

No consensus. Every model comes with additional baggage which needs to be discovered.

Is new physics hiding somewhere that we are obviously missing?

Is the reach just above the present experimental reach?

Are the interactions with Standard Model particles extremely feeble?

Are the theoretical and experimental precisions not good enough?

Image: Tim Tait

Imprints of new physics could show up as tiny deviations in standard measurements ⟶ Hint towards new physics?

Theory precision is thus crucial to minimise uncertainties.

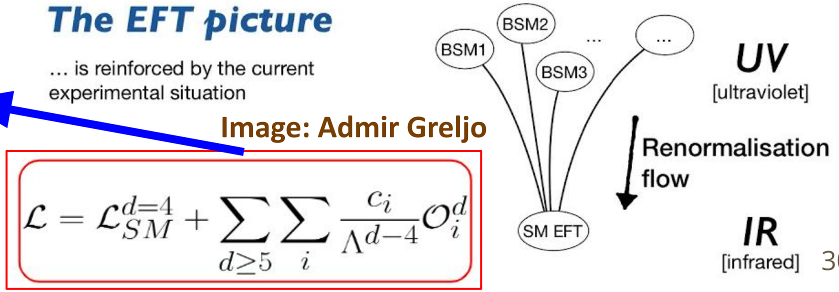


Image: Admir Greljo

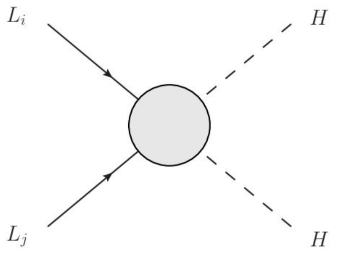
EFT Approach and Motivation for SMEFT

- **Beyond the SM:** New particles and interactions are strongly motivated but—so far—no direct or indirect collider signals have been observed.
- **Effective Field Theory (EFT):** If new particles are much heavier than the weak scale, they can be integrated out. Their effects are then captured by adding higher-dimensional operators to the SM.
- **SMEFT Framework:** SMEFT extends the SM by including all gauge-invariant operators built from SM fields (with the same SU(3)×SU(2)×U(1) symmetry), organised in a series expansion:

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \frac{1}{\Lambda}\sum_i c_i^{(5)}\mathcal{O}_i^{(5)} + \frac{1}{\Lambda^2}\sum_i c_i^{(6)}\mathcal{O}_i^{(6)} + \frac{1}{\Lambda^3}\sum_i c_i^{(7)}\mathcal{O}_i^{(7)} + \cdots .$$

- The expansion is valid when $v \ll \Lambda$, with Λ representing the mass scale of the new particles.

Dimension-5 (Weinberg) Operators and Neutrino Masses



- **Dimension-5 Operator:** The unique $D = 5$ operator in SMEFT is

$$[\mathcal{O}_5]_{IJ} = (\epsilon_{ij} H^i L_I^j)(\epsilon_{ij} H^i L_J^j),$$

where $I, J = 1, 2, 3$ are flavour indices and ϵ_{ij} is the antisymmetric tensor.

- **Lepton Number Violation:** This operator violates lepton number (and $B - L$), leading to Majorana masses.

Homework exercise!

- **After EWSB:** When the Higgs gets a VEV v, the operator generates a neutrino mass term:

$$\frac{1}{\Lambda} [c_5]_{IJ} [\mathcal{O}_5]_{IJ} \to \frac{v^2}{2\Lambda} [c_5]_{IJ} \, \nu_I \nu_J .$$

- Given neutrino masses $<$ eV and at least one $\gtrsim 0.06$ eV, we deduce $\Lambda / c_5 > 10^{15}$ GeV. This implies that dimension-5 effects are tiny (except in neutrino oscillations), while dimension-6 operators provide the leading corrections at the weak scale.

What if the neutrinos are Dirac particles? Will SMEFT suffice?

Overview of EFT Applications and Approaches

EFT as a Framework:

- EFT techniques are widely applied in high-energy physics.
- They provide a systematic expansion of the Lagrangian by organising operators according to symmetry and energy scale.

UV Completion vs. EFT:

- When UV completion is known: (e.g. Fermi, χPT, Einstein-Hilbert)
  - EFT Wilson coefficients may be calculable (Fermi, EH) or not (χPT).
  - Calculable coefficients may appear at tree level (Fermi) or only at loop level (EH).
- When UV completion is not known: (e.g. SMEFT)
  - The degrees of freedom may be fundamental (Fermi, EH), emergent (χPT), or unknown (SMEFT).

Overview of EFT Applications and Approaches

Benefits of EFT:

- Systematic expansion based on symmetries leads to increased calculability and simplification.
- Even with limitations in each case, organising the Lagrangian helps extract meaningful low-energy predictions.

Additional Applications:

- HQET: For composite particles with one heavy quark (e.g. B mesons).
- SCET: For QCD collisions producing energetic jets.
- NRQED: For precision calculations of atomic spectra.

Motivation and the Two New Physics Scales

- **Motivation:** Although the SM is highly successful, it does not explain neutrino masses, dark matter, or the matter–antimatter asymmetry. New physics is therefore expected at scales $\Lambda \gg v$ (with $v = 246$ GeV).
- **Two New Physics Scales:**
  - Λ_L is the scale for odd-dimensional operators (e.g. dimension-5), which violate $B - L$. Experimental constraints from neutrino masses suggest $\Lambda_L \sim 10^{15}$ GeV.
  - Λ is the scale for even-dimensional operators (e.g. dimension-6). This scale may be much lower, possibly in the few-TeV range, making these effects accessible at current or future experiments.
- The assumed hierarchy

$$v \ll \Lambda \quad \text{and} \quad \Lambda^2 \ll v\,\Lambda_L,$$

ensures that the EFT expansion converges quickly and that the leading new physics contributions at low energies come from dimension-6 operators.

SMEFT Lagrangian and New Physics Scales

The SMEFT Lagrangian is written as:

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \frac{1}{\Lambda_L} \sum_i c_i^{(5)} \mathcal{O}_i^{(5)} + \frac{1}{\Lambda^2} \sum_i c_i^{(6)} \mathcal{O}_i^{(6)} + \frac{1}{\Lambda_L^3} \sum_i c_i^{(7)} \mathcal{O}_i^{(7)} + \frac{1}{\Lambda^4} \sum_i c_i^{(8)} \mathcal{O}_i^{(8)} + \cdots$$

Key Points:

- $\mathcal{L}_{\mathrm{SM}}$ is the Standard Model Lagrangian.
- Each operator $\mathcal{O}_i^{(D)}$ is a gauge-invariant combination of SM fields with canonical dimension D.
- The Wilson coefficients $c_i^{(D)}$ are dimensionless.
- Λ_L and Λ are interpreted as the characteristic mass scales of the UV completion; the former is associated with operators that violate $B - L$ (and are hence highly suppressed), while the latter governs the leading new-physics effects.

SMEFT @ D=6

Motivation:

- The importance of dimension-6 operators for characterising low-energy effects of heavy particles has been recognised long ago.
- More recently, advantages of using a complete and non-redundant set of operators have been emphasised.
- Seemingly different higher-dimensional operators may yield identical S-matrix elements if they are related by:
    - Equations of motion (EOM),
    - Integration by parts (IBP),
    - Field redefinitions, or
    - Fierz transformations.
- Removing redundant operators simplifies the EFT description and yields an unambiguous map from observables to EFT Wilson coefficients.
- There exist infinitely many equivalent bases; common examples for D=6 include the **Warsaw basis** and the **SILH basis**.

Construction of a Basis

- Starting from all distinct D=6 operators that can be constructed from SM fields, many are redundant since they are equivalent to linear combinations of others.
- For example, a complete basis for one generation was constructed only a few years ago and later extended to three generations; the resulting Warsaw basis has 2499 independent parameters.
- The redundant operators can be removed (using IBP, EOM, etc.), and any complete basis (e.g. Warsaw, SILH) yields equivalent physical predictions.
- More systematic methods (e.g. Hilbert series techniques) can be used to construct such a basis.

The Warsaw Basis

Bosonic CP-even		Bosonic CP-odd	
O_H	$(H^\dagger H)^3$		
$O_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$		
O_{HD}	$\left\|H^\dagger D_\mu H\right\|^2$		
O_{HG}	$H^\dagger H\, G^a_{\mu\nu} G^a_{\mu\nu}$	$O_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}^a_{\mu\nu} G^a_{\mu\nu}$
O_{HW}	$H^\dagger H\, W^i_{\mu\nu} W^i_{\mu\nu}$	$O_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}^i_{\mu\nu} W^i_{\mu\nu}$
O_{HB}	$H^\dagger H\, B_{\mu\nu} B_{\mu\nu}$	$O_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu} B_{\mu\nu}$
O_{HWB}	$H^\dagger \sigma^i H\, W^i_{\mu\nu} B_{\mu\nu}$	$O_{H\widetilde{W}B}$	$H^\dagger \sigma^i H\, \widetilde{W}^i_{\mu\nu} B_{\mu\nu}$
O_W	$\epsilon^{ijk} W^i_{\mu\nu} W^j_{\nu\rho} W^k_{\rho\mu}$	$O_{\widetilde{W}}$	$\epsilon^{ijk} \widetilde{W}^i_{\mu\nu} W^j_{\nu\rho} W^k_{\rho\mu}$
O_G	$f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$	$O_{\widetilde{G}}$	$f^{abc} \widetilde{G}^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$

Bosonic operators

$(\bar{R}R)(\bar{R}R)$	
O_{ee}	$\eta(e^c\sigma_\mu\bar{e}^c)(e^c\sigma_\mu\bar{e}^c)$
O_{uu}	$\eta(u^c\sigma_\mu\bar{u}^c)(u^c\sigma_\mu\bar{u}^c)$
O_{dd}	$\eta(d^c\sigma_\mu\bar{d}^c)(d^c\sigma_\mu\bar{d}^c)$
O_{eu}	$(e^c\sigma_\mu\bar{e}^c)(u^c\sigma_\mu\bar{u}^c)$
O_{ed}	$(e^c\sigma_\mu\bar{e}^c)(d^c\sigma_\mu\bar{d}^c)$
O_{ud}	$(u^c\sigma_\mu\bar{u}^c)(d^c\sigma_\mu\bar{d}^c)$
$O^{(8)}_{ud}$	$(u^c\sigma_\mu T^a\bar{u}^c)(d^c\sigma_\mu T^a\bar{d}^c)$

$(\bar{L}L)(\bar{R}R)$	
$O_{\ell e}$	$(\bar{\ell}\bar{\sigma}_\mu\ell)(e^c\sigma_\mu\bar{e}^c)$
$O_{\ell u}$	$(\bar{\ell}\bar{\sigma}_\mu\ell)(u^c\sigma_\mu\bar{u}^c)$
$O_{\ell d}$	$(\bar{\ell}\bar{\sigma}_\mu\ell)(d^c\sigma_\mu\bar{d}^c)$
O_{eq}	$(e^c\sigma_\mu\bar{e}^c)(\bar{q}\bar{\sigma}_\mu q)$
O_{qu}	$(\bar{q}\bar{\sigma}_\mu q)(u^c\sigma_\mu\bar{u}^c)$
$O^{(8)}_{qu}$	$(\bar{q}\bar{\sigma}_\mu T^a q)(u^c\sigma_\mu T^a\bar{u}^c)$
O_{qd}	$(\bar{q}\bar{\sigma}_\mu q)(d^c\sigma_\mu\bar{d}^c)$
$O^{(8)}_{qd}$	$(\bar{q}\bar{\sigma}_\mu T^a q)(d^c\sigma_\mu T^a\bar{d}^c)$

$(\bar{L}L)(\bar{L}L)$	
$O_{\ell\ell}$	$\eta(\bar{\ell}\bar{\sigma}_\mu\ell)(\bar{\ell}\bar{\sigma}_\mu\ell)$
O_{qq}	$\eta(\bar{q}\bar{\sigma}_\mu q)(\bar{q}\bar{\sigma}_\mu q)$
O'_{qq}	$\eta(\bar{q}\bar{\sigma}_\mu\sigma^i q)(\bar{q}\bar{\sigma}_\mu\sigma^i q)$
$O_{\ell q}$	$(\bar{\ell}\bar{\sigma}_\mu\ell)(\bar{q}\bar{\sigma}_\mu q)$
$O'_{\ell q}$	$(\bar{\ell}\bar{\sigma}_\mu\sigma^i\ell)(\bar{q}\bar{\sigma}_\mu\sigma^i q)$

$(\bar{L}R)(\bar{L}R)$	
O_{quqd}	$(u^c q^j)\epsilon_{jk}(d^c q^k)$
$O^{(8)}_{quqd}$	$(u^c T^a q^j)\epsilon_{jk}(d^c T^a q^k)$
$O_{\ell equ}$	$(\bar{\ell}^j\bar{e}^c)\epsilon_{jk}(\bar{q}^k\bar{u}^c)$
$O^{(3)}_{\ell equ}$	$(\bar{\ell}^j\bar{\sigma}_{\mu\nu}\bar{e}^c)\epsilon_{jk}(\bar{q}^k\bar{\sigma}^{\mu\nu}u^c)$
$O_{\ell edq}$	$(\bar{\ell}\bar{e}^c)(d^c q)$

Four-Fermi Operators

Yukawa	
$[O^\dagger_{eH}]_{IJ}$	$H^\dagger H e^c_I H^\dagger \ell_J$
$[O^\dagger_{uH}]_{IJ}$	$H^\dagger H u^c_I \widetilde{H}^\dagger q_J$
$[O^\dagger_{dH}]_{IJ}$	$H^\dagger H d^c_I H^\dagger q_J$

Vertex	
$[O_{H\ell}]_{IJ}$	$i\bar{\ell}_I\bar{\sigma}_\mu\ell_J H^\dagger \overleftrightarrow{D}_\mu H$
$[O^{(3)}_{H\ell}]_{IJ}$	$i\bar{\ell}_I\sigma^i\bar{\sigma}_\mu\ell_J H^\dagger\sigma^i \overleftrightarrow{D}_\mu H$
$[O_{He}]_{IJ}$	$ie^c_I\sigma_\mu\bar{e}^c_J H^\dagger \overleftrightarrow{D}_\mu H$
$[O_{Hq}]_{IJ}$	$i\bar{q}_I\bar{\sigma}_\mu q_J H^\dagger \overleftrightarrow{D}_\mu H$
$[O^{(3)}_{Hq}]_{IJ}$	$i\bar{q}_I\sigma^i\bar{\sigma}_\mu q_J H^\dagger\sigma^i \overleftrightarrow{D}_\mu H$
$[O_{Hu}]_{IJ}$	$iu^c_I\sigma_\mu\bar{u}^c_J H^\dagger \overleftrightarrow{D}_\mu H$
$[O_{Hd}]_{IJ}$	$id^c_I\sigma_\mu\bar{d}^c_J H^\dagger \overleftrightarrow{D}_\mu H$
$[O_{Hud}]_{IJ}$	$iu^c_I\sigma_\mu\bar{d}^c_J \tilde{H}^\dagger D_\mu H$

Dipole	
$[O^\dagger_{eW}]_{IJ}$	$e^c_I\sigma_{\mu\nu}H^\dagger\sigma^i\ell_J W^i_{\mu\nu}$
$[O^\dagger_{eB}]_{IJ}$	$e^c_I\sigma_{\mu\nu}H^\dagger\ell_J B_{\mu\nu}$
$[O^\dagger_{uG}]_{IJ}$	$u^c_I\sigma_{\mu\nu}T^a\widetilde{H}^\dagger q_J\, G^a_{\mu\nu}$
$[O^\dagger_{uW}]_{IJ}$	$u^c_I\sigma_{\mu\nu}\widetilde{H}^\dagger\sigma^i q_J\, W^i_{\mu\nu}$
$[O^\dagger_{uB}]_{IJ}$	$u^c_I\sigma_{\mu\nu}\widetilde{H}^\dagger q_J\, B_{\mu\nu}$
$[O^\dagger_{dG}]_{IJ}$	$d^c_I\sigma_{\mu\nu}T^a H^\dagger q_J\, G^a_{\mu\nu}$
$[O^\dagger_{dW}]_{IJ}$	$d^c_I\sigma_{\mu\nu}\bar{H}^\dagger\sigma^i q_J\, W^i_{\mu\nu}$
$[O^\dagger_{dB}]_{IJ}$	$d^c_I\sigma_{\mu\nu}H^\dagger q_J\, B_{\mu\nu}$

Two-fermion operators

The Warsaw Basis

Bosonic CP-even	
O_H	$(H^\dagger H)^3$
$O_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
O_{HD}	$\left\|H^\dagger D_\mu H\right\|^2$
O_{HG}	$H^\dagger H\, G^a_{\mu\nu}G^a_{\mu\nu}$
O_{HW}	$H^\dagger H\, W^i_{\mu\nu}W^i_{\mu\nu}$

Bosonic CP-odd	
$O_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}^a_{\mu\nu}G^a_{\mu\nu}$
$O_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}^i_{\mu\nu}W^i_{\mu\nu}$

$(\bar{R}R)(\bar{R}R)$	
O_{ee}	$\eta(e^c\sigma_\mu\bar{e}^c)(e^c\sigma_\mu\bar{e}^c)$
O_{uu}	$\eta(u^c\sigma_\mu\bar{u}^c)(u^c\sigma_\mu\bar{u}^c)$
O_{dd}	$\eta(d^c\sigma_\mu\bar{d}^c)(d^c\sigma_\mu\bar{d}^c)$
O_{eu}	$(e^c\sigma_\mu\bar{e}^c)(u^c\sigma_\mu\bar{u}^c)$
O_{ed}	$(e^c\sigma_\mu\bar{e}^c)(d^c\sigma_\mu\bar{d}^c)$
O_{ud}	$(u^c\sigma_\mu\bar{u}^c)(d^c\sigma_\mu\bar{d}^c)$
$O_{ud}^{(8)}$	$(u^c\sigma_\mu T^a\bar{u}^c)(d^c\sigma_\mu T^a\bar{d}^c)$

$(\bar{L}L)(\bar{R}R)$	
$O_{\ell e}$	$(\bar{\ell}\bar{\sigma}_\mu\ell)(e^c\sigma_\mu\bar{e}^c)$
$O_{\ell u}$	$(\bar{\ell}\bar{\sigma}_\mu\ell)(u^c\sigma_\mu\bar{u}^c)$
$O_{\ell d}$	$(\bar{\ell}\bar{\sigma}_\mu\ell)(d^c\sigma_\mu\bar{d}^c)$
O_{eq}	$(e^c\sigma_\mu\bar{e}^c)(\bar{q}\bar{\sigma}_\mu q)$
O_{qu}	$(\bar{q}\bar{\sigma}_\mu q)(u^c\sigma_\mu\bar{u}^c)$
$O_{qu}^{(8)}$	$(\bar{q}\bar{\sigma}_\mu T^a q)(u^c\sigma_\mu T^a\bar{u}^c)$
O_{qd}	$(\bar{q}\bar{\sigma}_\mu q)(d^c\sigma_\mu\bar{d}^c)$

Yukawa	
$[O^\dagger_{eH}]_{IJ}$	$H^\dagger H e^c_I H^\dagger \ell_J$
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$[O^{(3)}_{H\ell}]_{IJ}$	$i\bar{\ell}_I\sigma^i\bar{\sigma}_\mu\ell_J H^\dagger\sigma^i\overleftrightarrow{D}_\mu H$

Dipole	
$[O^\dagger_{eW}]_{IJ}$	$e^c_I\sigma_{\mu\nu}H^\dagger\sigma^i\ell_J W^i_{\mu\nu}$
$[O^\dagger_{eB}]_{IJ}$	$e^c_I\sigma_{\mu\nu}H^\dagger\ell_J B_{\mu\nu}$

Two-fermion $D = 6$ operators: Flavour indices are denoted by I, J. For complex operators (O_H^{ud}, Yukawa, and dipole terms), the complex conjugate is implicitly included.

Four-fermion $D = 6$ operators: Flavour indices are suppressed for clarity. The factor $\eta = 1/2$ when all indices are identical (e.g., $[O_{ee}]_{1111}$), otherwise $\eta = 1$. Complex operators include their conjugates in the Lagrangian, with complex Wilson coefficients.

Construction of a Basis

Example Operator:

$$\mathcal{O}'_{HD} = (H^\dagger H)\, D_\mu H^\dagger D^\mu H.$$

$$\partial_\nu B_{\nu\mu} = -\frac{i g_Y}{2} H^\dagger \overleftrightarrow{D}_\mu H - g_Y j^Y_\mu,$$
$$(\partial_\nu W^i_{\nu\mu} + \epsilon^{ijk} g_L W^j_\nu W^k_{\nu\mu}) = D_\nu W^i_{\nu\mu} = -\frac{i}{2} g_L H^\dagger \sigma^i \overleftrightarrow{D}_\mu H - g_L j^i_\mu,$$
$$(\partial_\nu G^a_{\nu\mu} + f^{abc} g_s G^b_\nu G^c_{\nu\mu}) = D_\nu G^a_{\nu\mu} = -g_s j^a_\mu,$$
$$\Box H = \mu_H^2 H - 2\lambda (H^\dagger H) - j_H,$$

Step 1: Integration by Parts (IBP)

By integrating by parts, we rewrite

$$\mathcal{O}'_{HD} = \frac{1}{2} H^\dagger H \Box (H^\dagger H) - \left[H^\dagger H \left(H^\dagger D_\mu D^\mu H + D_\mu D^\mu H^\dagger\, H \right) \right].$$

$$j^Y_\mu = \sum_{f \in \nu,e,u,d} Y_f \bar{f} \bar{\sigma}_\mu f + \sum_{f \in e,u,d} Y_{\bar{f}^c} f^c \sigma_\mu \bar{f}^c,$$
$$j^i_\mu = \bar{q} \bar{\sigma}_\mu \frac{\sigma^i}{2} q + \bar{\ell} \bar{\sigma}_\mu \frac{\sigma^i}{2} \ell,$$
$$j^a_\mu = \bar{q} \bar{\sigma}_\mu T^a q + u^c \sigma_\mu T^a \bar{u}^c + d^c \sigma_\mu T^a \bar{d}^c,$$

Step 2: Apply the Higgs EOM

Using the leading-order Higgs equation of motion (neglecting $\mathcal{O}(M^{-2})$ corrections),

$$\Box (H^\dagger H) \to -\mu_H^2 (H^\dagger H) + 2\lambda\, (H^\dagger H)^2 + \ldots,$$

$$H^\dagger \overleftrightarrow{D}_\mu H \equiv H^\dagger D_\mu H - D_\mu H^\dagger H,$$
$$j_H \equiv -\bar{u}^c y_u^\dagger \tilde{q} + d^c y_d q + e^c y_e \ell, \qquad \tilde{q}_i \equiv \epsilon_{ij} \bar{q}_j$$

the operator becomes

$$\mathcal{O}'_{HD} = -\mu_H^2 (H^\dagger H)^2 + \frac{1}{2} (H^\dagger H) \Box (H^\dagger H) + 2\lambda\, (H^\dagger H)^3 + \frac{1}{2} (H^\dagger H) \left[-\bar{u}^c\, y_u^\dagger\, \tilde{q} + d^c\, y_d\, q + c^c\, y_e\, \ell + \text{h.c.} \right].$$

Conclusion:

- All operators on the right-hand side are in the Warsaw basis.
- Thus, $\mathcal{O}'_{HD}$ is redundant—it can be written as a specific linear combination of Warsaw basis ops.

Motivation and Example - Heavy Neutral Vector Boson

Motivation:

- We use dimension-6 operators as a prop to parametrise the effects of heavy BSM particles on weak-scale observables.
- Suppose one day it is demonstrated that a linear combination of higher-dimensional operators must be present in the SM EFT Lagrangian to account for all experimental results. What will this tell us about the new physics?
- The best way to answer is through examples that relate dimension-6 operators to the couplings and masses in BSM models.

Example 1 (Fermi-like):

Consider a heavy neutral vector boson V_μ with mass M_V coupled to SM fermionic currents:

$$\mathcal{L}_{UV} \supset V_\mu \Big(g_{Xf,L}\, \bar{f}\, \bar{\sigma}^\mu f + g_{Xf,R}\, f_c\, \sigma^\mu \bar{f}_c \Big).$$

For energies well below M_V the momentum dependence in the V propagator can be ignored, effectively leading to a contact interaction:

$$\mathcal{L}_{EFT} \supset -\frac{1}{2M_V^2} \Big(g_{Vf,L}\, \bar{f}\, \bar{\sigma}^\mu f + g_{Vf,R}\, f_c\, \sigma^\mu \bar{f}_c \Big)^2.$$

Motivation and Example - Heavy Neutral Vector Boson

Implications for New Physics:

- If experimental data require the presence of certain dimension-6 operators in the SM EFT, this provides indirect evidence for heavy BSM particles.
- The Wilson coefficients, being proportional to the ratio of BSM couplings and masses, can be used to set an upper limit on the new physics mass scale.
- For instance, in Example 1, the matching condition

$$\frac{c_{f_1 f_2}}{\Lambda^2} = -\frac{g_{Vf_1} g_{Xf_2}}{M_V^2}$$

Experiments only probe c/Λ^2

Upper-limit on Λ comes from the fact that the couplings can't be larger than 4π!

Some Clarifications

(i) Basis Independence:

A complete, non-redundant operator basis at a fixed mass dimension forms a vector space whose dimension is invariant under an invertible change of basis. In other words, while the form of operators may differ (e.g. Warsaw vs SILH), the number of independent operators remains the same.

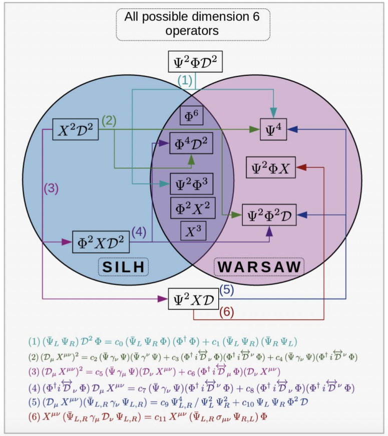
(ii) Hilbert Series Method and SMEFT Counting:

The Hilbert series is a generating function

Arrows depict relations among the classes based on the equations of motion (EOMs) of various elds.

$$H(t) = \sum_{n=0}^{\infty} a_n\, t^n,$$

where the formal variable t tracks the weight (e.g. mass dimension) and a_n counts the number of independent, gauge-invariant operators of that weight. This method utilises the plethystic exponential and Molien–Weyl integrals to systematically account for all invariants and redundancies. In SMEFT, such techniques have been used to count operators—for instance, in the Warsaw basis for dimension-6 operators with three generations, one finds 2499 independent parameters.



Banerjee, et al.

More details on Hilbert series in the backup slides

Some Clarifications

(iii) Partition Function and Field Redefinitions:

The partition function

$$Z = \int \mathcal{D}\phi \, e^{iS[\phi]},$$

remains invariant (up to an overall factor) under local, invertible field redefinitions. Thus, physical observables (the S-matrix) remain unchanged even if the Lagrangian appears different.

(iv) Equations of Motion in EFT:

In matching EFTs to UV theories, the classical equations of motion (derived via the Euler–Lagrange equation) are typically obtained from the leading (renormalisable) Lagrangian. Higher-dimensional operators are subleading in the $1/\Lambda$ expansion and are usually omitted when deriving the EOM used to remove redundant operators.

From Operators to Observables

- To study the phenomenological effects of higher-dimensional operators in the SM EFT, it is often convenient to work with the mass eigenstates (after electroweak symmetry breaking) rather than with the full $SU(3) \times SU(2) \times U(1)$ invariant formulation.
- Higher-dimensional operators lead to deviations from the SM in two main ways:
  1. **Modified Couplings** – corrections to the strength of SM-like interactions.
  2. **New Vertices** – additional interaction terms that do not exist in the SM Lagrangian.

From Operators to Observables

- Consider the operator

$$\mathcal{O}_{He} = i\, e_c\, \sigma_\mu\, \bar{e}_c \left(H^\dagger D_\mu H - D_\mu H^\dagger H \right).$$

- Inserting the Higgs vacuum expectation value (VEV) leads to a coupling of the Z boson to right-handed electrons:

$$\frac{c_{He}}{\Lambda^2}\, \mathcal{O}_{He} \;\to\; -\frac{c_{He}\sqrt{g_L^2 + g_Y^2}\, v^2}{2\Lambda^2}\, Z_\mu\, e_c\, \sigma^\mu\, \bar{e}_c\,.$$

- In the SM, the Z-boson coupling to a fermion is given by

$$g_{Zf} = \sqrt{g_L^2 + g_Y^2}\left(T_3 - s_\theta^2\, Q_f \right),$$

so for the right-handed electron (with $T_3 = 0$):

$$g_{Ze} = \sqrt{g_L^2 + g_Y^2}\, s_\theta^2\,.$$

From Operators to Observables

- The effect of $\mathcal{O}_{He}$ is to shift the coupling by

$$\Delta g_{Ze} = -\frac{c_{He}\sqrt{g_L^2 + g_Y^2}\, v^2}{2\Lambda^2}.$$

- The same operator $\mathcal{O}_{He}$ also generates a vertex that is absent in the SM:

$$\frac{c_{He}}{\Lambda^2}\,\mathcal{O}_{He} \;\to\; -\frac{c_{He}\sqrt{g_L^2 + g_Y^2}\, v}{2\Lambda^2}\, h\, Z_\mu\, e_c\, \sigma^\mu\, \bar{e}_c.$$

Leads to correlations between various observables!

- This new vertex (involving the Higgs boson h, the Z boson, and right-handed electrons) affects amplitudes for Higgs processes. For example, it modifies the decay width and differential distributions in the Higgs decay to 4 leptons, which is studied at the LHC.

From Operators to Observables

- The effective Lagrangian for the Higgs boson h can be written as

$$\mathcal{L} \supset \frac{1}{2}(\partial_\mu h)^2 - \frac{m_h^2}{2} h^2 - \frac{m_h^2}{2v}\left(1 + \delta_1 \frac{v^2}{\Lambda^2}\right) h^3 - \delta_2 \frac{v}{\Lambda^2}\, h\, \partial_\mu h\, \partial^\mu h + \ldots$$

- Here, the δ_1-term modifies the SM triple Higgs coupling, and the δ_2-term is a new two-derivative interaction.
- By redefining the Higgs field as

$$h \to h + \delta_2 \frac{v}{2\Lambda^2} h^2,$$

the Lagrangian becomes

$$\mathcal{L} \supset \frac{1}{2}(\partial_\mu h)^2 - \frac{m_h^2}{2} h^2 - \frac{m_h^2}{2v}\left(1 + (\delta_1 + \delta_2) \frac{v^2}{\Lambda^2}\right) h^3 + \ldots$$

- By the equivalence theorem, this redefinition does not change physical observables.

From Operators to Observables

- In the SM the electroweak parameters g_L, g_Y and v are determined from three precisely measured observables:
  1. The Fermi constant: $\sqrt{2}G_F = \frac{1}{v^2}$.
  2. The electromagnetic fine structure constant: $\alpha = \frac{g_L^2\, g_Y^2}{4\pi\,(g_L^2+g_Y^2)}$.
  3. The Z-boson mass: $m_Z^2 = \frac{(g_L^2+g_Y^2)v^2}{4}$.
- Higher-dimensional operators can shift these relations. For example, the operator

$$\frac{c_{HD}}{\Lambda^2}\,|H^\dagger D_\mu H|^2$$

contributes after electroweak symmetry breaking as

Check these explicitly!

$$\frac{c_{HD}}{\Lambda^2}\,|H^\dagger D_\mu H|^2 \to \frac{c_{HD}\,v^2}{2\Lambda^2}\,\frac{(g_L^2+g_Y^2)v^2}{8}\,Z_\mu Z^\mu.$$

- Additionally, one obtains a shift in the W boson mass:

$$\frac{\delta m_W^2}{2m_W^2} = -\frac{c_{HD}\,g_L^2\,v^2}{4(g_L^2-g_Y^2)\,\Lambda^2}.$$

From Operators to Observables

- The Z boson mass is measured at LEP with high precision:

$$m_Z^{\text{exp}} = 91.1876 \pm 0.0021\,\text{GeV}.$$

- Since m_Z is used to extract g_L, g_Y and v, the contribution from $\mathcal{O}_{HD}$ complicates this determination.
- Assuming $\mathcal{O}_{HD}$ is the only higher-dimensional operator present, the constraint on its Wilson coefficient is:

Check this!

$$\frac{c_{HD}}{\Lambda^2} = \frac{-1.2 \pm 0.9}{(10\,\text{TeV})^2}.$$

A detailed example in the backup slides!

- This indicates that electroweak precision measurements can probe weakly coupled new physics at scales ~ 10 TeV, and strongly coupled new physics at scales up to ~ 100 TeV.

Toy UV Theory: One-Loop Matching of 2-Point Function

$$\mathcal{L}_{UV} = \frac{1}{2}\left[(\partial_\mu \phi)^2 - m_L^2 \phi^2 + (\partial_\mu H)^2 - M^2 H^2\right] - \frac{\lambda_0}{4!}\phi^4 - \frac{\lambda_1}{2} M\, \phi^2 H - \frac{\lambda_2}{4}\phi^2 H^2$$

$$\mathcal{L}_{EFT} = \frac{1}{2}\left[(\partial_\mu \phi)^2 - m^2 \phi^2\right] - C_4 \frac{\phi^4}{4!} - \frac{C_6}{M^2}\frac{\phi^6}{6!} + \mathcal{O}(M^{-4})$$

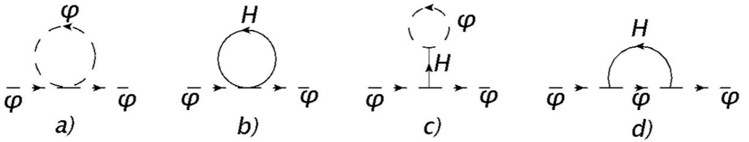
- **Tree-Level:**

1PI

$$\Pi_0^{EFT} = p^2 - m^2, \quad \Pi_0^{UV} = p^2 - m_L^2.$$

- **Loop Corrections:** In dimensional regularisation the EFT one-loop correction reads

$$\delta\Pi^{EFT} = C_4 \frac{m^2}{32\pi^2}\left[\frac{1}{\bar{\epsilon}} + \log\left(\frac{\mu^2}{m^2}\right) + 1\right]$$



with

$$\frac{1}{\bar{\epsilon}} = \frac{1}{\epsilon} + \gamma_E + \log(4\pi).$$

- **Basis Dependence:** Two bases are considered:
  - Unbox basis: No wave-function renormalisation.
  - Box basis: Includes off-shell momentum (p^2) dependence and non-trivial wave-function renormalisation.

Toy UV Theory: One-Loop Matching of 2-Point Function

- $\overline{\text{MS}}$ **Prescription:** Simply subtract the $1/\bar{\epsilon}$ pole to render amplitudes finite.
- **Renormalisation Scale:** The scale μ is introduced by dimensional regularisation and the Lagrangian mass parameter becomes μ-dependent.
- **Physical Mass:** Defined as the pole of $\Pi(p^2)$,

$$m^2_{\text{phys}} = m^2 - C_4 \frac{m^2}{32\pi^2}\left[\log\left(\frac{\mu^2}{m^2}\right) + 1\right].$$

- **RG Equation:** To keep physical observables μ-independent, the mass must satisfy

$$\frac{dm^2}{d\log\mu} = \frac{C_4\, m^2}{16\pi^2}.$$

Toy UV Theory: One-Loop Matching of 2-Point Function

- **UV Propagator Correction:** For the light scalar in the UV theory, the one-loop corrected physical mass is

$$C_4 = \lambda_0 - 3\lambda_1^2 - 4\lambda_1^2 \frac{m_L^2}{M^2}$$

$$m_{\text{phys}}^2 = m_L^2 - \left(\lambda_0 - 3\lambda_1^2 - 4\lambda_1^2 \frac{m_L^2}{M^2}\right) \frac{m_L^2}{32\pi^2}\left[\ln\left(\frac{\mu^2}{m^2}\right) + 1\right]$$
$$- \frac{1}{32\pi^2} \ln\left(\frac{\mu^2}{m^2}\right)\left[M^2(\lambda_2^2 + 2\lambda_1^2) + 2\lambda_1^2 m_L^2 + 4\lambda_1^2 \frac{m_L^4}{M^2}\right]$$
$$- \frac{1}{32\pi^2}\left[M^2(\lambda_2^2 + 2\lambda_1^2) + 3\lambda_1^2 m_L^2 + \frac{22}{3}\lambda_1^2 \frac{m_L^4}{M^2}\right].$$

- **Key Point:** The UV theory has different mass parameters and couplings, leading to extra contributions compared to the EFT.

Toy UV Theory: One-Loop Matching of 2-Point Function

- **Matching Condition:** By equating the physical masses computed in the EFT and UV theories, one obtains:

$$m^2(\mu) = m_L^2(\mu) - \frac{1}{32\pi^2} \ln\left(\frac{\mu^2}{m^2}\right) \left[M^2(\lambda_2^2 + 2\lambda_1^2) + 2\lambda_1^2 m_L^2 + 4\lambda_1^2 \frac{m_L^4}{M^2}\right]$$
$$- \frac{1}{32\pi^2} \left[M^2(\lambda_2^2 + 2\lambda_1^2) + 3\lambda_1^2 m_L^2 + \frac{22}{3}\lambda_1^2 \frac{m_L^4}{M^2}\right].$$

- **Choice of Matching Scale:** Setting $\mu \sim M$ cancels large logarithms, thus preserving the validity of the perturbative expansion.

Toy UV Theory: One-Loop Matching of 2-Point Function

- **Dimensional Regularisation:** Regularises loop integrals by working in $d = 4 - \epsilon$ dimensions.
- **$\overline{\text{MS}}$ Scheme:** Simplifies renormalisation by subtracting the universal $1/\bar{\epsilon}$ term.
- **EFT vs. UV:** EFT loop corrections are matched to the UV theory to define scale-dependent mass parameters consistently.
- **Naturalness and Matching:** The matching condition reveals how UV parameters determine the EFT mass, with the natural scale being $m^2 \sim M^2/(16\pi^2)$. Careful matching (with $\mu \sim M$) avoids large logarithms and ensures robust perturbative calculations.

Toy UV Theory: RG Equations in the EFT (Unbox Basis)

- To ensure that physical observables (e.g. the physical mass and S-matrix elements) remain independent of the renormalisation scale μ, the EFT parameters run with μ.
- In the unbox basis the one-loop RG equations are:

$$\frac{d\, m^2}{d \ln \mu} = \frac{m^2\, C_4}{16\pi^2},$$

Comes from one-loop matching of 2-point function.

$$\frac{d\, C_4}{d \ln \mu} = \frac{1}{16\pi^2}\left[3\, C_4{}^2 + \frac{m^2}{M^2}\, C_6\right].$$

Comes from one-loop matching of 4-point function.

- The $\mathcal{O}(M^0)$ term reproduces the standard ϕ^4 theory, while the $\mathcal{O}(M^{-2})$ term shows the effect of the dimension-6 operator.
- In general, at one loop the Wilson coefficients of higher-dimensional operators affect the running of lower-dimensional ones when explicit mass scales are present.

Toy UV Theory: RG Equations in the EFT (Unbox Basis)

- Solving the first RG equation at one-loop yields

$$m^2(\mu) = m^2(M)\left(\frac{\mu}{M}\right)^{\frac{C_4}{16\pi^2}} .$$

- For small logarithms, i.e. when $\frac{C_4}{16\pi^2}\ln(\mu/M) \ll 1$, this can be approximated by

$$m^2(\mu) \approx m^2(M)\left[1 + \frac{C_4}{16\pi^2}\ln\left(\frac{\mu}{M}\right)\right].$$

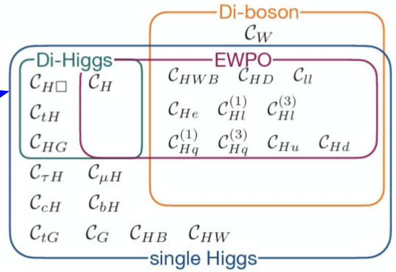
- This modification of the naive scaling $m^2 \sim M^2$ is due to the anomalous dimension $C_4/(16\pi^2)$.

$a^\epsilon = e^{\epsilon \log a} \approx 1 + \epsilon \log a$

For $C_4 \ln \frac{\mu}{M} \ll 16\pi^2$

- The RG evolution re-sums large logarithms from the UV theory into the EFT parameters, ensuring consistency between the EFT and the full theory even when $C_4 \ln(\mu/M)$ becomes sizable.

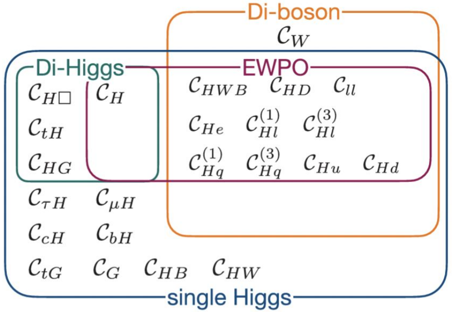
SMEFT-UV matching: the dataset

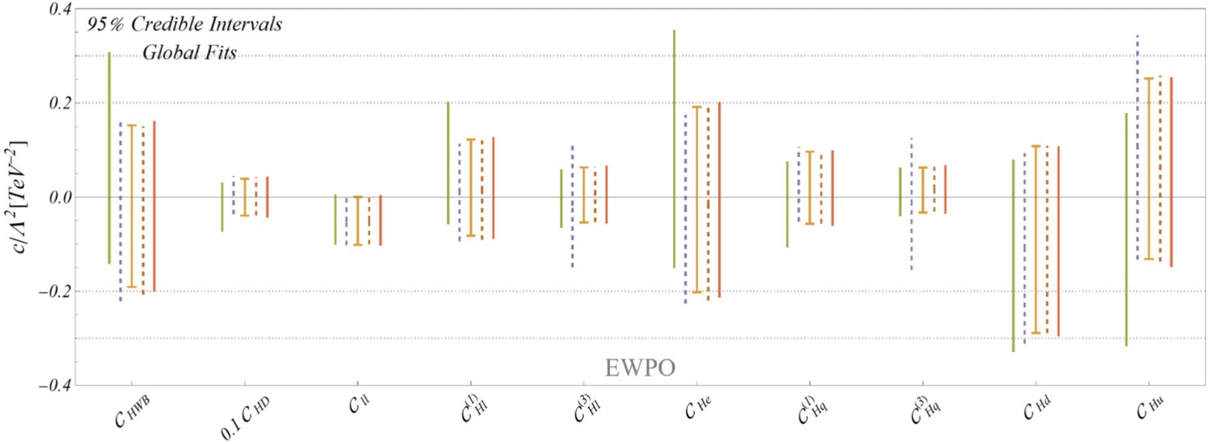


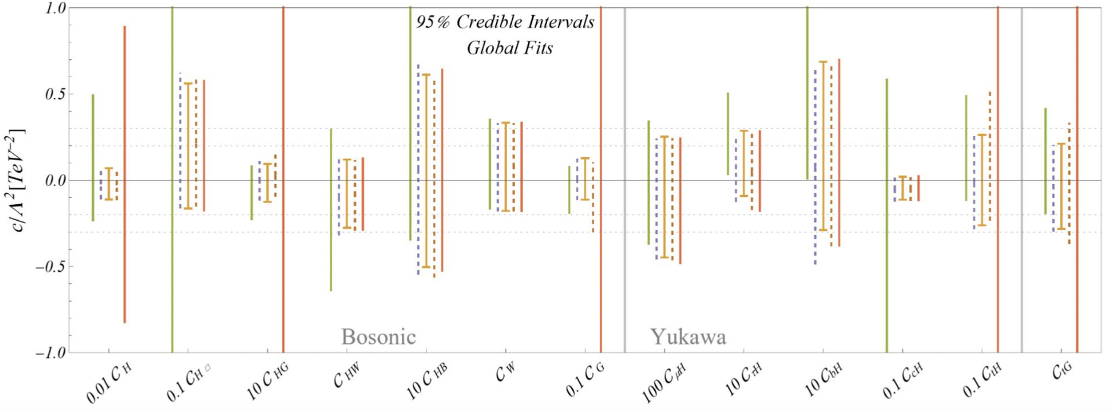
The sectors

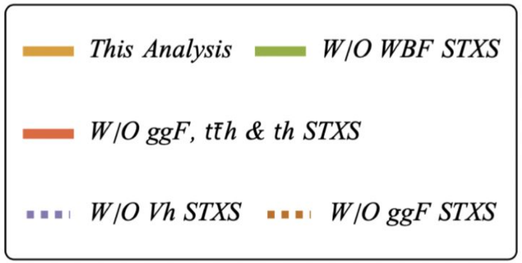
Observables		no. of measurements	References	2020
Electroweak Precision Observables (EWPO) Γ_Z, σ^0_{had}, R^0_l, A_l, A_l(SLD), A^l_{FB}, $\sin^2\theta^l_{\text{eff}}$(Tev), R^0_c, A_c, A^c_{FB}, R^0_b, A_b, A^b_{FB}, m_W, Γ_W		15	tab. 1 of ref. [168] correlations in ref. [1]	✓ ✓
LEP-2 WW data		74	tabs. 12-15 of ref. [2]	✓
Higgs Data				
7 and 8 TeV Run-I data	ATLAS & CMS combination	20	tab. 8 of ref. [3]	✓
	ATLAS & CMS combination $\mu(h \to \mu\mu)$	1	tab. 13 of ref. [3]	✓
	ATLAS $\mu(h \to Z\gamma)$	1	fig. 1 of ref. [4]	✓
13 TeV ATLAS Run-II data	$\mu(h \to Z\gamma)$ at 139 fb^{-1}	1	[5]	✓
	$\mu(h \to \mu\mu)$ at 139 fb^{-1}	1	[6]	✓
	$\mu(h \to \tau\tau)$ at 139 fb^{-1}	4	fig. 14 of ref. [7]	
	$\mu(h \to bb)$ in VBF and ttH at 139 fb^{-1}	1+1	[8, 9]	
	STXS Higgs combination	25	figs. 20/21 of ref. [169]	✓
	STXS $h \to \gamma\gamma/ZZ/b\bar{b}$ at 139 fb^{-1}	42	figs. 1 and 2 of ref. [10]	
	STXS $h \to WW$ in ggF, VBF at 139 fb^{-1}	11	figs. 12 and 14 of ref. [11]	
13 TeV CMS Run-II data	CMS combination at up to 137 fb^{-1}	23	tab. 4 of ref. [12]	✓
	$\mu(h \to b\bar{b})$ in Vh at 35.9/41.5 fb^{-1}	2	entries from tab. 4 of ref. [12]	
	$\mu(h \to WW)$ in ggF at 137 fb^{-1}	1	[13]	
	$\mu(h \to \mu\mu)$ at 137 fb^{-1}	4	fig. 11 of ref. [14]	
	$\mu(h \to \tau\tau/WW)$ in $t\bar{t}h$ at 137 fb^{-1}	3	fig. 14 of ref. [15]	
	STXS $h \to WW$ at 137 fb^{-1} in Vh	4	tab. 9 of ref. [16]	
	STXS $h \to \tau\tau$ at 137 fb^{-1}	11	figs. 11/12 of ref. [17]	
	STXS $h \to \gamma\gamma$ at 137 fb^{-1}	27	tab. 13 and fig. 21 of ref. [18]	
	STXS $h \to ZZ$ at 137 fb^{-1}	18	tab. 6 and fig. 15 of ref. [19]	
ATLAS WZ 13 TeV m_T^{WZ} at 36.1 fb^{-1}		6 bins	fig. 4(c) of ref. [20]	✓
ATLAS Zjj 13 TeV $\Delta\phi_{jj}$ at 139 fb^{-1}		12 bins	fig. 7(d) of ref. [21]	✓
ATLAS WW 13 TeV $p_T^{\ell 1}$ at 36.1 fb^{-1}		7 bins	bins 8-14 of fig. 7(a) of ref. [22]	✓
Di-Higgs signal strengths ATLAS & CMS 13 TeV data $\mu^{b\bar{b}b\bar{b}}_{HH}$, $\mu^{b\bar{b}\tau\bar{\tau}}_{HH}$, $\mu^{b\bar{b}\gamma\gamma}_{HH}$		6	[23–28]	

Fitting the gauge-Higgs sector: The fits









Top down + Bottom up: 2HDM Lagrangian (example)

The 2HDM Lagrangian ($\mathcal{H}_2 \equiv (1_C, 2_L, -\frac{1}{2}\big|_Y)$):

$$\mathcal{L}_{\mathcal{H}_2} = \mathcal{L}_{\rm SM}^{d\leq 4} + |\mathcal{D}_\mu \mathcal{H}_2|^2 - m_{\mathcal{H}_2}^2 |\mathcal{H}_2|^2 - \frac{\lambda_{\mathcal{H}_2}}{4}|\mathcal{H}_2|^4 - (\eta_H|\widetilde{H}|^2 + \eta_{\mathcal{H}_2}|\mathcal{H}_2|^2)(\widetilde{H}^\dagger \mathcal{H}_2 + \mathcal{H}_2^\dagger \widetilde{H})$$
$$- \lambda_{\mathcal{H}_2,1}|\widetilde{H}|^2|\mathcal{H}_2|^2 - \lambda_{\mathcal{H}_2,2}|\widetilde{H}^\dagger \mathcal{H}_2|^2 - \lambda_{\mathcal{H}_2,3}\left[(\widetilde{H}^\dagger \mathcal{H}_2)^2 + (\mathcal{H}_2^\dagger \widetilde{H})^2\right]$$
$$- \left\{Y_{\mathcal{H}_2}^{(e)} \bar{l}_L \widetilde{\mathcal{H}}_2 e_R + Y_{\mathcal{H}_2}^{(u)} \bar{q}_L \mathcal{H}_2 u_R + Y_{\mathcal{H}_2}^{(d)} \bar{q}_L \widetilde{\mathcal{H}}_2 d_R + \text{h.c.}\right\}.$$

2HDM contains an extra isospin-doublet scalar ($\mathcal{H}_2$) which is a colour-singlet with hypercharge $Y = -\frac{1}{2}$.

Top down + Bottom up: 2HDM couplings + SMEFT WCs

37 operators generated at scale Λ!

[arXiv:2111.05876: Anisha, Bakshi, SB, Biekötter, Chakrabortty, Patra, Spannowsky, 2021]

Functions of SM parameters only except the mass scale

Functions of 2HDM parameters

Do not affect current set of observables

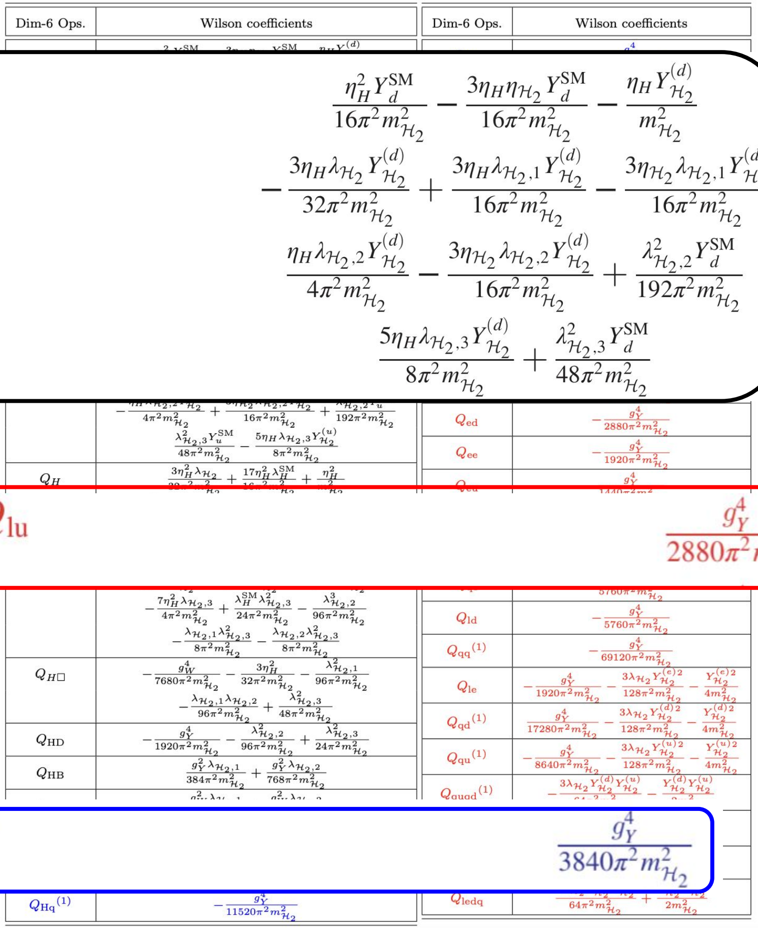
Dim-6 Ops.	Wilson coefficients
Q_{dH}	$\frac{\eta_H^2 Y_d^{\mathrm{SM}}}{16\pi^2 m_{\mathcal{H}_2}^2} - \frac{3\eta_H \eta_{\mathcal{H}_2} Y_d^{\mathrm{SM}}}{16\pi^2 m_{\mathcal{H}_2}^2} - \frac{\eta_H Y_{\mathcal{H}_2}^{(d)}}{m_{\mathcal{H}_2}^2} - \frac{3\eta_H \lambda_{\mathcal{H}_2} Y_{\mathcal{H}_2}^{(d)}}{32\pi^2 m_{\mathcal{H}_2}^2} + \frac{3\eta_H \lambda_{\mathcal{H}_2,1} Y_{\mathcal{H}_2}^{(d)}}{16\pi^2 m_{\mathcal{H}_2}^2} - \frac{3\eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,1} Y_{\mathcal{H}_2}^{(d)}}{16\pi^2 m_{\mathcal{H}_2}^2} \frac{\eta_H \lambda_{\mathcal{H}_2,2} Y_{\mathcal{H}_2}^{(d)}}{4\pi^2 m_{\mathcal{H}_2}^2} - \frac{3\eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,2} Y_{\mathcal{H}_2}^{(d)}}{16\pi^2 m_{\mathcal{H}_2}^2} + \frac{\lambda_{\mathcal{H}_2,2}^2 Y_d^{\mathrm{SM}}}{192\pi^2 m_{\mathcal{H}_2}^2} \frac{5\eta_H \lambda_{\mathcal{H}_2,3} Y_{\mathcal{H}_2}^{(d)}}{8\pi^2 m_{\mathcal{H}_2}^2} + \frac{\lambda_{\mathcal{H}_2,3}^2 Y_d^{\mathrm{SM}}}{48\pi^2 m_{\mathcal{H}_2}^2}$
Q_{eH}	$\frac{\eta_H^2 Y_e^{\mathrm{SM}}}{16\pi^2 m_{\mathcal{H}_2}^2} - \frac{3\eta_H \eta_{\mathcal{H}_2} Y_e^{\mathrm{SM}}}{16\pi^2 m_{\mathcal{H}_2}^2} - \frac{\eta_H Y_{\mathcal{H}_2}^{(e)}}{m_{\mathcal{H}_2}^2} - \frac{3\eta_H \lambda_{\mathcal{H}_2} Y_{\mathcal{H}_2}^{(e)}}{32\pi^2 m_{\mathcal{H}_2}^2} + \frac{3\eta_H \lambda_{\mathcal{H}_2,1} Y_{\mathcal{H}_2}^{(e)}}{16\pi^2 m_{\mathcal{H}_2}^2} - \frac{3\eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,1} Y_{\mathcal{H}_2}^{(e)}}{16\pi^2 m_{\mathcal{H}_2}^2} \frac{\eta_H \lambda_{\mathcal{H}_2,2} Y_{\mathcal{H}_2}^{(e)}}{4\pi^2 m_{\mathcal{H}_2}^2} - \frac{3\eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,2} Y_{\mathcal{H}_2}^{(e)}}{16\pi^2 m_{\mathcal{H}_2}^2} + \frac{\lambda_{\mathcal{H}_2,2}^2 Y_e^{\mathrm{SM}}}{192\pi^2 m_{\mathcal{H}_2}^2} \frac{5\eta_H \lambda_{\mathcal{H}_2,3} Y_{\mathcal{H}_2}^{(e)}}{8\pi^2 m_{\mathcal{H}_2}^2} + \frac{\lambda_{\mathcal{H}_2,3}^2 Y_e^{\mathrm{SM}}}{48\pi^2 m_{\mathcal{H}_2}^2}$
Q_{uH}	$\frac{\eta_H^2 Y_u^{\mathrm{SM}}}{16\pi^2 m_{\mathcal{H}_2}^2} + \frac{3\eta_H \lambda_{\mathcal{H}_2} Y_{\mathcal{H}_2}^{(u)}}{32\pi^2 m_{\mathcal{H}_2}^2} + \frac{\eta_H Y_{\mathcal{H}_2}^{(u)}}{m_{\mathcal{H}_2}^2} - \frac{3\eta_H \eta_{\mathcal{H}_2} Y_u^{\mathrm{SM}}}{16\pi^2 m_{\mathcal{H}_2}^2} - \frac{3\eta_H \lambda_{\mathcal{H}_2,1} Y_{\mathcal{H}_2}^{(u)}}{16\pi^2 m_{\mathcal{H}_2}^2} + \frac{3\eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,1} Y_{\mathcal{H}_2}^{(u)}}{16\pi^2 m_{\mathcal{H}_2}^2} - \frac{\eta_H \lambda_{\mathcal{H}_2,2} Y_{\mathcal{H}_2}^{(u)}}{4\pi^2 m_{\mathcal{H}_2}^2} + \frac{3\eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,2} Y_{\mathcal{H}_2}^{(u)}}{16\pi^2 m_{\mathcal{H}_2}^2} + \frac{\lambda_{\mathcal{H}_2,2}^2 Y_u^{\mathrm{SM}}}{192\pi^2 m_{\mathcal{H}_2}^2} \frac{\lambda_{\mathcal{H}_2,3}^2 Y_u^{\mathrm{SM}}}{48\pi^2 m_{\mathcal{H}_2}^2} - \frac{5\eta_H \lambda_{\mathcal{H}_2,3} Y_{\mathcal{H}_2}^{(u)}}{8\pi^2 m_{\mathcal{H}_2}^2}$
Q_H	$\frac{3\eta_H^2 \lambda_{\mathcal{H}_2}}{32\pi^2 m_{\mathcal{H}_2}^2} + \frac{17\eta_H^2 \lambda_H^{\mathrm{SM}}}{16\pi^2 m_{\mathcal{H}_2}^2} + \frac{\eta_H^2}{m_{\mathcal{H}_2}^2} - \frac{3\eta_H^2 \lambda_{\mathcal{H}_2,1}}{4\pi^2 m_{\mathcal{H}_2}^2} - \frac{3\eta_H \eta_{\mathcal{H}_2} \lambda_H^{\mathrm{SM}}}{8\pi^2 m_{\mathcal{H}_2}^2} + \frac{3\eta_H \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,1}}{8\pi^2 m_{\mathcal{H}_2}^2} - \frac{13\eta_H^2 \lambda_{\mathcal{H}_2,2}}{16\pi^2 m_{\mathcal{H}_2}^2} + \frac{3\eta_H \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,2}}{8\pi^2 m_{\mathcal{H}_2}^2} - \frac{\lambda_{\mathcal{H}_2,1}^3}{48\pi^2 m_{\mathcal{H}_2}^2} \frac{\lambda_H^{\mathrm{SM}} \lambda_{\mathcal{H}_2,2}^2}{96\pi^2 m_{\mathcal{H}_2}^2} - \frac{\lambda_{\mathcal{H}_2,1}^2 \lambda_{\mathcal{H}_2,2}}{32\pi^2 m_{\mathcal{H}_2}^2} - \frac{\lambda_{\mathcal{H}_2,1} \lambda_{\mathcal{H}_2,2}^2}{32\pi^2 m_{\mathcal{H}_2}^2} - \frac{7\eta_H^2 \lambda_{\mathcal{H}_2,3}}{4\pi^2 m_{\mathcal{H}_2}^2} + \frac{\lambda_H^{\mathrm{SM}} \lambda_{\mathcal{H}_2,3}^2}{24\pi^2 m_{\mathcal{H}_2}^2} - \frac{\lambda_{\mathcal{H}_2,2}^3}{96\pi^2 m_{\mathcal{H}_2}^2} - \frac{\lambda_{\mathcal{H}_2,1} \lambda_{\mathcal{H}_2,3}^2}{8\pi^2 m_{\mathcal{H}_2}^2} - \frac{\lambda_{\mathcal{H}_2,2} \lambda_{\mathcal{H}_2,3}^2}{8\pi^2 m_{\mathcal{H}_2}^2}$
$Q_{H\Box}$	$-\frac{g_W^4}{7680\pi^2 m_{\mathcal{H}_2}^2} - \frac{3\eta_H^2}{32\pi^2 m_{\mathcal{H}_2}^2} - \frac{\lambda_{\mathcal{H}_2,1}^2}{96\pi^2 m_{\mathcal{H}_2}^2} - \frac{\lambda_{\mathcal{H}_2,1} \lambda_{\mathcal{H}_2,2}}{96\pi^2 m_{\mathcal{H}_2}^2} + \frac{\lambda_{\mathcal{H}_2,3}^2}{48\pi^2 m_{\mathcal{H}_2}^2}$
Q_{HD}	$-\frac{g_Y^4}{1920\pi^2 m_{\mathcal{H}_2}^2} - \frac{\lambda_{\mathcal{H}_2,2}^2}{96\pi^2 m_{\mathcal{H}_2}^2} + \frac{\lambda_{\mathcal{H}_2,3}^2}{24\pi^2 m_{\mathcal{H}_2}^2}$
Q_{HB}	$\frac{g_Y^2 \lambda_{\mathcal{H}_2,1}}{384\pi^2 m_{\mathcal{H}_2}^2} + \frac{g_Y^2 \lambda_{\mathcal{H}_2,2}}{768\pi^2 m_{\mathcal{H}_2}^2}$
Q_{HW}	$\frac{g_W^2 \lambda_{\mathcal{H}_2,1}}{384\pi^2 m_{\mathcal{H}_2}^2} + \frac{g_W^2 \lambda_{\mathcal{H}_2,2}}{768\pi^2 m_{\mathcal{H}_2}^2}$
Q_{HWB}	$\frac{g_W g_Y \lambda_{\mathcal{H}_2,2}}{384\pi^2 m_{\mathcal{H}_2}^2}$
$Q_{\mathrm{Hl}}{}^{(1)}$	$\frac{g_Y^4}{3840\pi^2 m_{\mathcal{H}_2}^2}$
$Q_{\mathrm{Hq}}{}^{(1)}$	$-\frac{g_Y^4}{11520\pi^2 m_{\mathcal{H}_2}^2}$

Dim-6 Ops.	Wilson coefficients
Q_{Hd}	$\frac{g_Y^4}{5760\pi^2 m_{\mathcal{H}_2}^2}$
Q_{He}	$\frac{g_Y^4}{1920\pi^2 m_{\mathcal{H}_2}^2}$
Q_{Hu}	$-\frac{g_Y^4}{2880\pi^2 m_{\mathcal{H}_2}^2}$
$Q_{\mathrm{Hl}}{}^{(3)}$	$-\frac{g_W^4}{1920\pi^2 m_{\mathcal{H}_2}^2}$
$Q_{\mathrm{Hq}}{}^{(3)}$	$-\frac{g_W^4}{1920\pi^2 m_{\mathcal{H}_2}^2}$
Q_W	$\frac{g_W^3}{5760\pi^2 m_{\mathcal{H}_2}^2}$
Q_{ll}	$-\frac{g_W^4}{7680\pi^2 m_{\mathcal{H}_2}^2} - \frac{g_Y^4}{7680\pi^2 m_{\mathcal{H}_2}^2}$
$Q_{\mathrm{ud}}{}^{(1)}$	$\frac{g_Y^4}{4320\pi^2 m_{\mathcal{H}_2}^2}$
$Q_{\mathrm{lq}}{}^{(3)}$	$-\frac{g_W^4}{3840\pi^2 m_{\mathcal{H}_2}^2}$
$Q_{\mathrm{qq}}{}^{(3)}$	$-\frac{g_W^4}{7680\pi^2 m_{\mathcal{H}_2}^2}$
Q_{dd}	$-\frac{g_Y^4}{17280\pi^2 m_{\mathcal{H}_2}^2}$
Q_{ed}	$-\frac{g_Y^4}{2880\pi^2 m_{\mathcal{H}_2}^2}$
Q_{ee}	$-\frac{g_Y^4}{1920\pi^2 m_{\mathcal{H}_2}^2}$
Q_{eu}	$\frac{g_Y^4}{1440\pi^2 m_{\mathcal{H}_2}^2}$
Q_{uu}	$-\frac{g_Y^4}{4320\pi^2 m_{\mathcal{H}_2}^2}$
Q_{lu}	$\frac{g_Y^4}{2880\pi^2 m_{\mathcal{H}_2}^2}$
Q_{qe}	$\frac{g_Y^4}{5760\pi^2 m_{\mathcal{H}_2}^2}$
Q_{ld}	$-\frac{g_Y^4}{5760\pi^2 m_{\mathcal{H}_2}^2}$
$Q_{\mathrm{qq}}{}^{(1)}$	$-\frac{g_Y^4}{69120\pi^2 m_{\mathcal{H}_2}^2}$
Q_{le}	$-\frac{g_Y^4}{1920\pi^2 m_{\mathcal{H}_2}^2} - \frac{3\lambda_{\mathcal{H}_2} Y_{\mathcal{H}_2}^{(e)2}}{128\pi^2 m_{\mathcal{H}_2}^2} - \frac{Y_{\mathcal{H}_2}^{(e)2}}{4m_{\mathcal{H}_2}^2}$
$Q_{\mathrm{qd}}{}^{(1)}$	$\frac{g_Y^4}{17280\pi^2 m_{\mathcal{H}_2}^2} - \frac{3\lambda_{\mathcal{H}_2} Y_{\mathcal{H}_2}^{(d)2}}{128\pi^2 m_{\mathcal{H}_2}^2} - \frac{Y_{\mathcal{H}_2}^{(d)2}}{4m_{\mathcal{H}_2}^2}$
$Q_{\mathrm{qu}}{}^{(1)}$	$-\frac{g_Y^4}{8640\pi^2 m_{\mathcal{H}_2}^2} - \frac{3\lambda_{\mathcal{H}_2} Y_{\mathcal{H}_2}^{(u)2}}{128\pi^2 m_{\mathcal{H}_2}^2} - \frac{Y_{\mathcal{H}_2}^{(u)2}}{4m_{\mathcal{H}_2}^2}$
$Q_{\mathrm{quqd}}{}^{(1)}$	$-\frac{3\lambda_{\mathcal{H}_2} Y_{\mathcal{H}_2}^{(d)} Y_{\mathcal{H}_2}^{(u)}}{64\pi^2 m_{\mathcal{H}_2}^2} - \frac{Y_{\mathcal{H}_2}^{(d)} Y_{\mathcal{H}_2}^{(u)}}{2m_{\mathcal{H}_2}^2}$
$Q_{\mathrm{lequ}}{}^{(1)}$	$\frac{3\lambda_{\mathcal{H}_2} Y_{\mathcal{H}_2}^{(e)} Y_{\mathcal{H}_2}^{(u)}}{64\pi^2 m_{\mathcal{H}_2}^2} + \frac{Y_{\mathcal{H}_2}^{(e)} Y_{\mathcal{H}_2}^{(u)}}{2m_{\mathcal{H}_2}^2}$
$Q_{\mathrm{lq}}{}^{(1)}$	$\frac{g_Y^4}{11520\pi^2 m_{\mathcal{H}_2}^2}$
Q_{ledq}	$\frac{3\lambda_{\mathcal{H}_2} Y_{\mathcal{H}_2}^{(d)} Y_{\mathcal{H}_2}^{(e)}}{64\pi^2 m_{\mathcal{H}_2}^2} + \frac{Y_{\mathcal{H}_2}^{(d)} Y_{\mathcal{H}_2}^{(e)}}{2m_{\mathcal{H}_2}^2}$

Top down + Bottom up: 2HDM couplings + SMEFT WCs

37 operators generated at scale Λ!

[arXiv:2111.05876: Anisha, Bakshi, SB, Biekötter, Chakrabortty, Patra, Spannowsky, 2021]



$Q_{\rm dH}$

$$\frac{\eta_H^2 Y_d^{\rm SM}}{16\pi^2 m_{\mathcal{H}_2}^2} - \frac{3\eta_H \eta_{\mathcal{H}_2} Y_d^{\rm SM}}{16\pi^2 m_{\mathcal{H}_2}^2} - \frac{\eta_H Y_{\mathcal{H}_2}^{(d)}}{m_{\mathcal{H}_2}^2} - \frac{3\eta_H \lambda_{\mathcal{H}_2} Y_{\mathcal{H}_2}^{(d)}}{32\pi^2 m_{\mathcal{H}_2}^2} + \frac{3\eta_H \lambda_{\mathcal{H}_2,1} Y_{\mathcal{H}_2}^{(d)}}{16\pi^2 m_{\mathcal{H}_2}^2} - \frac{3\eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,1} Y_{\mathcal{H}_2}^{(d)}}{16\pi^2 m_{\mathcal{H}_2}^2}$$
$$\frac{\eta_H \lambda_{\mathcal{H}_2,2} Y_{\mathcal{H}_2}^{(d)}}{4\pi^2 m_{\mathcal{H}_2}^2} - \frac{3\eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,2} Y_{\mathcal{H}_2}^{(d)}}{16\pi^2 m_{\mathcal{H}_2}^2} + \frac{\lambda_{\mathcal{H}_2,2}^2 Y_d^{\rm SM}}{192\pi^2 m_{\mathcal{H}_2}^2}$$
$$\frac{5\eta_H \lambda_{\mathcal{H}_2,3} Y_{\mathcal{H}_2}^{(d)}}{8\pi^2 m_{\mathcal{H}_2}^2} + \frac{\lambda_{\mathcal{H}_2,3}^2 Y_d^{\rm SM}}{48\pi^2 m_{\mathcal{H}_2}^2}$$

Functions of 2HDM parameters

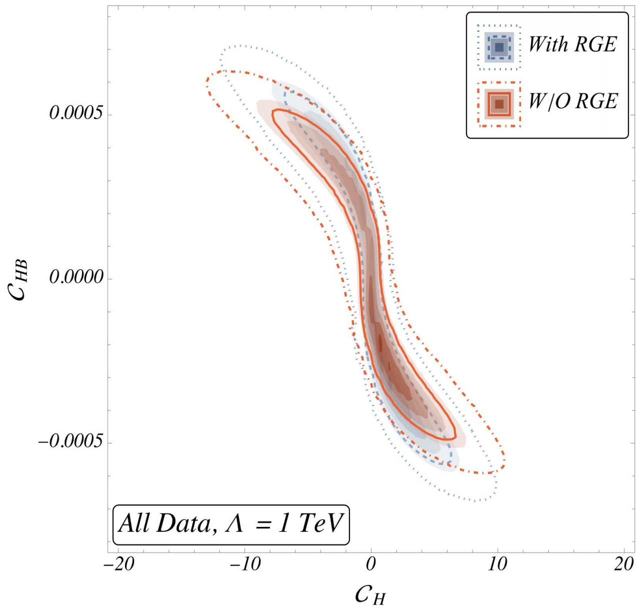
$Q_{\rm lu}$ $$\frac{g_Y^4}{2880\pi^2 m_{\mathcal{H}_2}^2}$$

Do not affect current set of observables

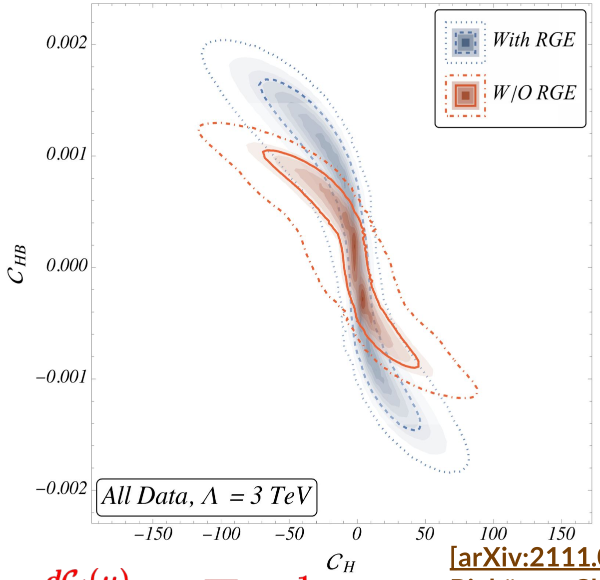
Functions of SM parameters only except the mass scale

$Q_{\rm Hl}{}^{(1)}$ $$\frac{g_Y^4}{3840\pi^2 m_{\mathcal{H}_2}^2}$$

Theory uncertainties in EFT analyses: RGE effects + operator mixing



Usually, the running of the SMEFT operators ignored which emerge at **Λ**. But, the measurements are at different scales.



For 2HDM, 51 operators generated of which 14 are from RGE!

Increase in mass scale relaxes the parameter bounds!

At LO

$$\mathcal{C}_i(M_Z) = \mathcal{C}_i(\Lambda) + \sum_j \frac{1}{16\pi^2}\gamma_{ij}\mathcal{C}_j(\Lambda)\log[\frac{M_Z}{\Lambda}]$$

$$\frac{d\mathcal{C}_i(\mu)}{d\log(\mu)} = \sum_j \frac{1}{16\pi^2}\gamma_{ij}\mathcal{C}_j$$

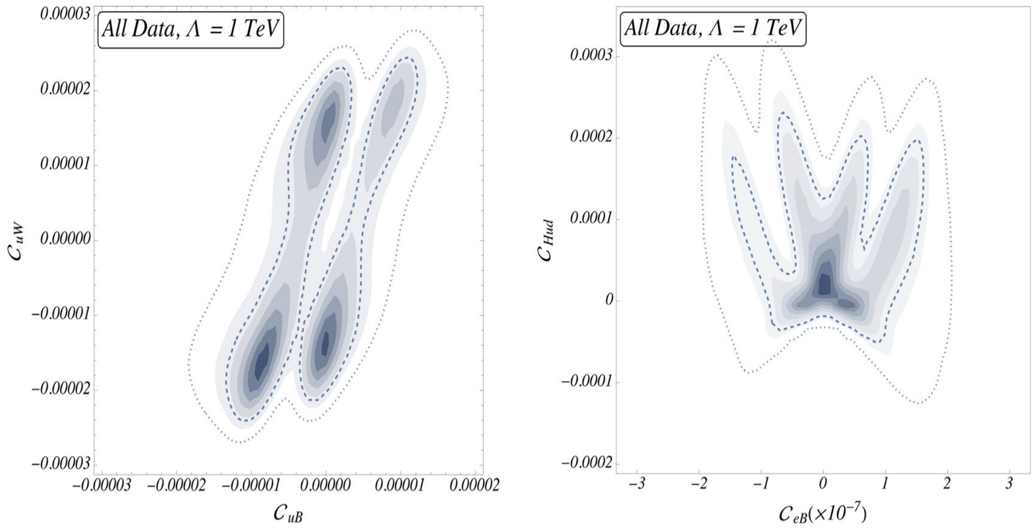
[arXiv:2111.05876: Anisha, Bakshi, SB, Biekötter, Chakrabortty, Patra, Spannowsky, 2021]

Theory uncertainties in EFT analyses: RGE effects + operator mixing

Usually, the running of the SMEFT operators ignored which emerge at Λ. But, the measurements are at different scales.

For 2HDM, 51 operators generated (top-down matching) of which 14 are from RGE! Examples (all suppressed by $16\pi^2$):

$\mathcal{O}_{uB}, \mathcal{O}_{uW}, \mathcal{O}_{dB}, \mathcal{O}_{dW}, \mathcal{O}_{eB}, \mathcal{O}_{eW}, \mathcal{O}_{Hud}$



(a) $\mathcal{C}_{uB}$ - $\mathcal{C}_{uW}$ (b) $\mathcal{C}_{eB}$ - $\mathcal{C}_{Hud}$

[arXiv:2111.05876: Anisha, Bakshi, SB, Biekötter, Chakrabortty, Patra, Spannowsky, 2021]

Higgs Effective Field Theory (HEFT)

HEFT is the most general parametrisation of low-energy physics with only SM DOFs!!!

HEFT ⊃ SMEFT ⊃ SM **Is there any scenario where only HEFT can describe low-energy effects of BSM?**

1. Low-energy interactions only follow $U(1)_{em}$
2. The interactions can't tell us more about the properties of the microscopic theory
3. New non-decoupling strong dynamics → spontaneous EW symmetry breaking → Higgs-like scalar
4. SM not recovered when all BSM masses taken to infinity
5. Non-analyticity in Lagrangians can't be removed by field redefinitions → arises when **new states integrated out acquire mass from EWSB** → **violates decoupling** See Falkowski, Rattazzi

Unlike in the SMEFT, h is considered a gauge singlet and the Goldstone bosons, ω^a as an $SU(2)_L$ triplet. HEFT treats these separately → Goldstones embedded in Unitary matrix, U.

Part of the Lagrangian:

$$\mathcal{L}_{\text{HEFT}} \supset \frac{v^2}{4}\mathcal{F}(h)Tr\{D_\mu U^\dagger D^\mu U\} + \frac{1}{2}(\partial_\mu h)^2 - V(h) - \frac{v}{\sqrt{2}}(\bar{u}^i_L \bar{d}^i_L)\mathcal{F}(h)\begin{pmatrix} y^u_{ij} u^j_R \\ y^d_{ij} d^j_R \end{pmatrix} + \text{h.c.}$$

$$\mathcal{F}(h) = 1 + 2a\frac{h}{v} + b\frac{h^2}{v^2} + \ldots$$

$$V(h) = \frac{1}{2}m_h^2 v^2 (1 + d_3\frac{h}{v} + \frac{d_4}{4}\frac{h^2}{v^2}) + \ldots$$

$$D_\mu U = \partial_\mu U + igW^a_\mu \frac{\sigma^a}{2}U - ig' U \frac{\sigma^3}{2} B_\mu$$

See geometric interpretation of HEFT with Higgs and Goldstone bosons as coordinates of Riemannian manifold

Motivated by Cohen et al.

SMEFT versus HEFT

SMEFT

1. Most general set of local operators invariant under $SU(3)_C \times SU(2)_L \times U(1)_Y$
2. Operators suppressed by powers of new-physics scale, Λ
3. Low energy states modelled using fields transforming linearly under aforementioned symmetries
4. Observed Higgs, h, is a component of an electroweak doublet scalar, H
5. More restrictive symmetry structure → less number of parameters which are correlated

HEFT

1. Manifest gauge symmetry is $SU(3)_C \times U(1)_{em}$
2. Operators suppressed by electroweak breaking scale, v
3. The $SU(2)_L \times U(1)_Y$ symmetry is non-linearly realised using a multiplet of Goldstone bosons
4. No relation between h and the Goldstone bosons
5. Less restrictive symmetry structure → more number of uncorrelated parameters

Backup Slides

Scaling of Wilson Coefficients and Selection Rules

Wilson Coefficient Estimates:

Assuming a single new-physics coupling g_* and one mass scale in the UV, power counting yields:

$$\mathcal{O}_H = |H|^6 : \quad c_H \sim g_*^4,$$
$$\mathcal{O}_{eH} = |H|^2\, \bar{\ell}\, H\, e_c : \quad c_{eH} \sim g_*^3,$$
$$\mathcal{O}_{H\Box} = |H|^2\, \Box\, |H|^2 : \quad c_{H\Box} \sim g_*^2,$$
$$\mathcal{O}_W = \epsilon_{ijk}\, W^i_{\mu\nu}\, W^{j,\nu\rho}\, W^{k,\mu}_{\rho} : \quad c_W \sim g_* \quad \text{(naïvely)}.$$

In general, the Wilson coefficients are free parameters with only experimental constraints.

Selection Rules and Corrections:

- In a strongly coupled UV theory ($g_* \gg 1$), naïve scaling for $\mathcal{O}_H$ would be very large. However, the same UV dynamics also contributes to the SM quartic Higgs interaction $\lambda|H|^4$ with $\lambda \sim g_*^2$. Since experimentally $\lambda \sim 0.1$, a protection mechanism (e.g. an approximate shift symmetry) is required, leading to:

$$c_H \sim \lambda\, g_*^2 \quad \Rightarrow \quad c_H \lesssim 10.$$

Scaling of Wilson Coefficients and Selection Rules

Selection Rules and Corrections:

- Chirality-violating operators (e.g. $\mathcal{O}_{eH}$) must be accompanied by the corresponding Yukawa coupling:

$$c_{eH} \sim y_e \, g_*^2 .$$

- For the gauge boson operator, if the $SU(2)_L$ bosons are fundamental, amplitudes with n external W bosons carry n powers of the gauge coupling g_L. Therefore, $\mathcal{O}_W$ is not generated at tree level and scales as:

$$c_W \sim \frac{g_L^3}{16\pi^2},$$

typically $c_W \lesssim 10^{-3}$.

Loop Momenta in Dimensional Regularisation

- **Integration Measure:** The loop integration is performed in $d = 4 - \epsilon$ dimensions:

$$\int \frac{d^d k}{(2\pi)^d}, \quad d = 4 - \epsilon.$$

- **No Hard Cut-Off:** Dimensional regularisation integrates over the entire momentum range,

$$\Lambda_{\text{DR}} = \infty,$$

so even momenta $k > \Lambda_{\text{EFT}}$ contribute.

High-Energy Contributions and UV Matching

- **Analytic High-Energy Contributions:** Contributions from large loop momenta yield analytic terms in external momenta:

$$f(p) = \sum_n a_n \left(\frac{p}{\mu}\right)^n,$$

which can be absorbed into EFT counterterms.

- **UV Matching:** Finite loop corrections have the form

$$\delta\Pi^{\text{EFT}} \propto \left[\frac{1}{\bar{\epsilon}} + \ln\left(\frac{\mu^2}{m^2}\right) + 1\right],$$

where the high-energy effects are encoded and matched with the UV theory.

The MSbar Scheme and its Advantages

- **Subtraction Prescription:** The $\overline{\text{MS}}$ scheme subtracts not only the $1/\epsilon$ pole but also universal constants:

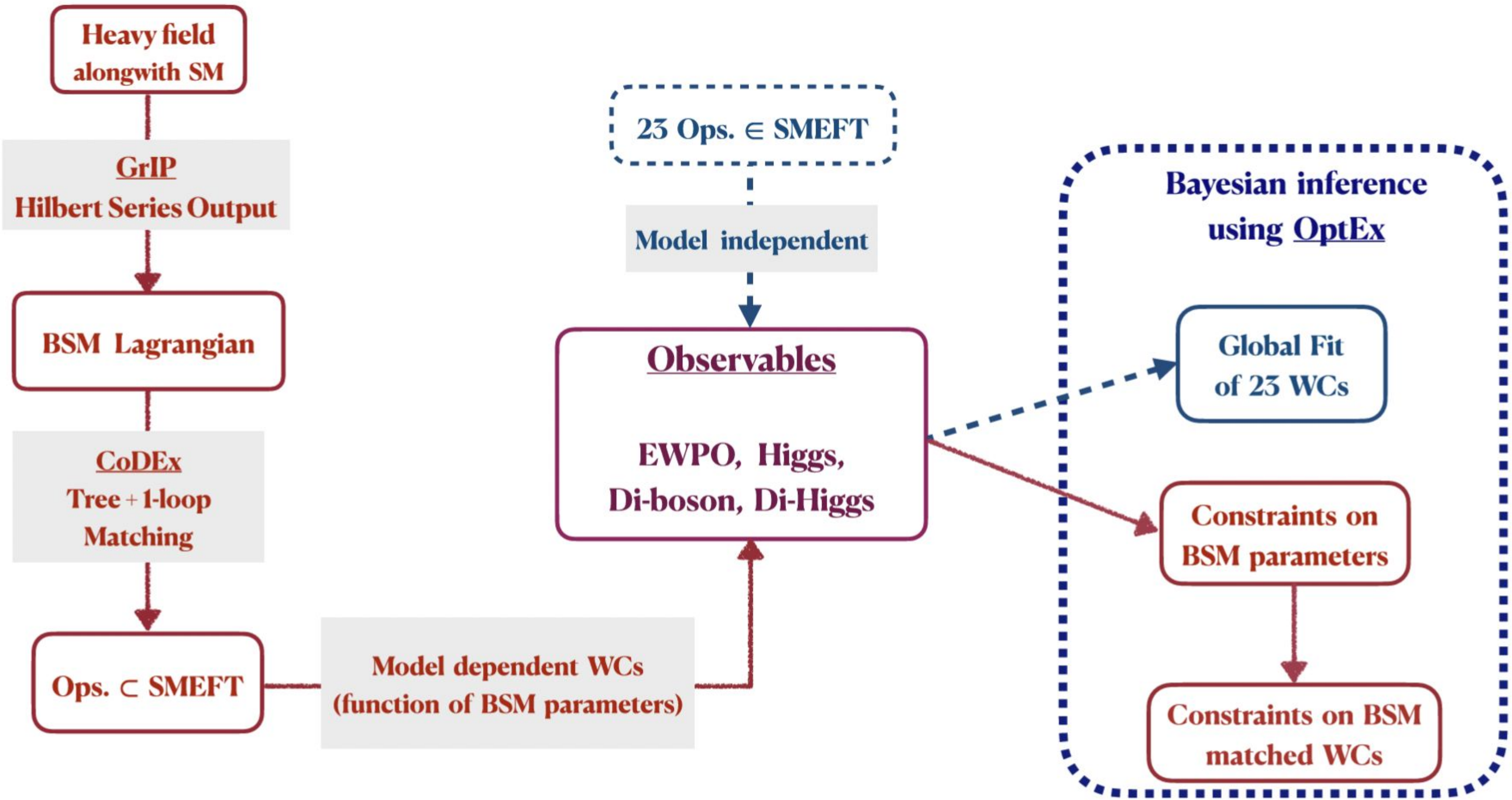
$$\frac{1}{\bar{\epsilon}} = \frac{1}{\epsilon} + \gamma_E + \ln(4\pi).$$

- **Renormalisation Outcome:** After subtraction, the finite result becomes

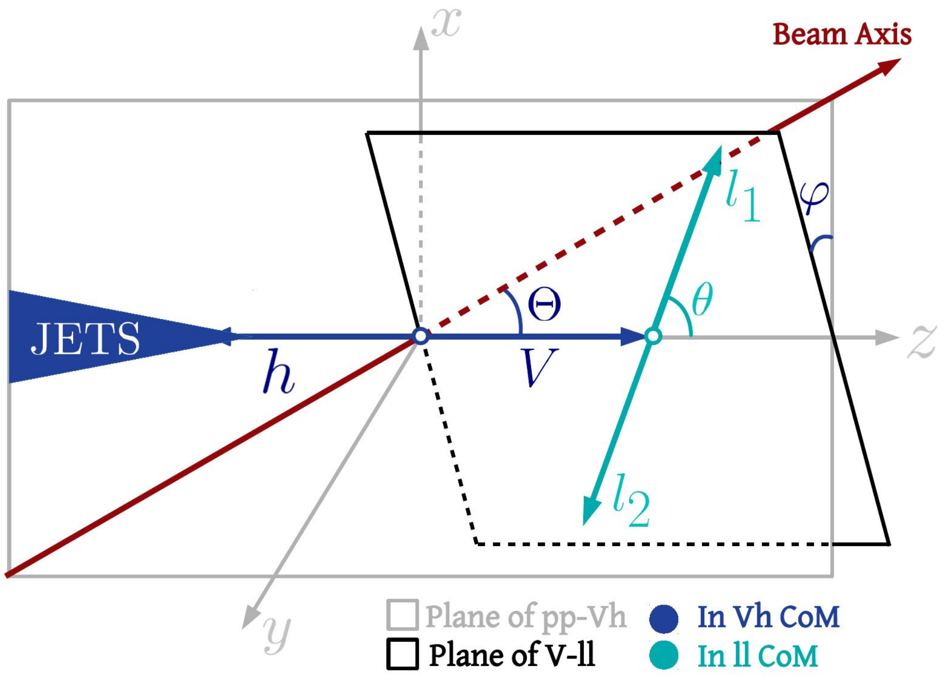
$$\delta\Pi^{\text{EFT}} \propto \ln\left(\frac{\mu^2}{m^2}\right) + 1.$$

- **Advantages:** This approach preserves gauge invariance, simplifies calculations, and naturally introduces the renormalisation scale μ to absorb high-energy effects via matching.

SMEFT-UV matching: flowchart



Vh production at *pp* colliders

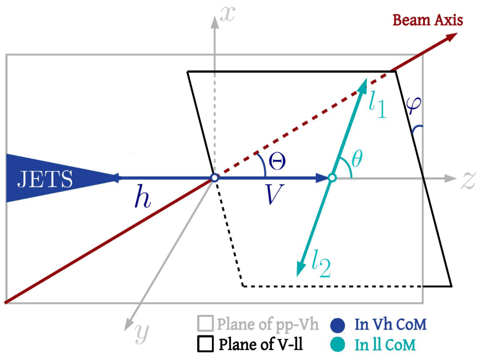


Vh production at *pp* colliders

$$
\begin{aligned}
f_{LL} &= S_\Theta^2 S_\theta^2, \\
f_{TT}^1 &= C_\Theta C_\theta, \\
f_{TT}^2 &= (1 + C_\Theta^2)(1 + C_\theta^2), \\
f_{LT}^1 &= C_\varphi S_\Theta S_\theta, \\
f_{LT}^2 &= C_\varphi S_\Theta S_\theta C_\Theta C_\theta, \\
\tilde{f}_{LT}^1 &= S_\varphi S_\Theta S_\theta, \\
\tilde{f}_{LT}^2 &= S_\varphi S_\Theta S_\theta C_\Theta C_\theta, \\
f_{TT'} &= C_{2\varphi} S_\Theta^2 S_\theta^2, \\
\tilde{f}_{TT'} &= S_{2\varphi} S_\Theta^2 S_\theta^2,
\end{aligned}
$$

Many angular distributions testable at the LHC

- φ, Θ and $\{x, y, z\}$ in *Vh* CoM frame (z identified as direction of V-boson; y identified as normal to the plane of V and beam axis; x defined to complete the right-handed set), θ in V CoM frame
- Q: How much differential information can one extract from this process?
- For three body phase space, $3 \times 3 - 4 = 5$ kinematic variables completely define final state
- Barring boost factor, the variables are $\sqrt{s}, \Theta, \theta, \varphi$



Vh production at *pp* colliders

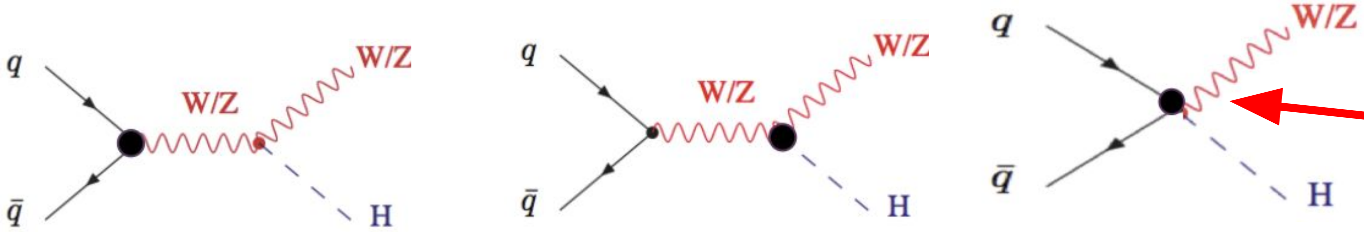
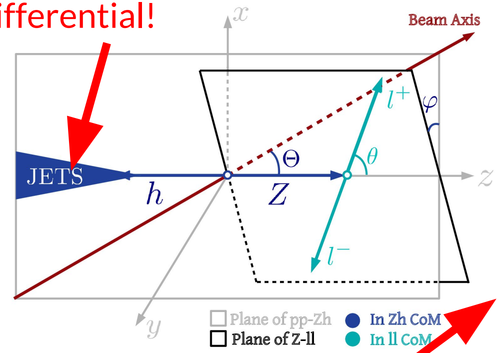


Diagram not there in SM

Can be made more differential!



$$
\begin{aligned}
f_{LL} &= S_\Theta^2 S_\theta^2,\\
f_{TT}^1 &= C_\Theta C_\theta,\\
f_{TT}^2 &= (1+C_\Theta^2)(1+C_\theta^2)\\
f_{LT}^1 &= C_\varphi S_\Theta S_\theta,\\
f_{LT}^2 &= C_\varphi S_\Theta S_\theta C_\Theta C_\theta,\\
\tilde{f}_{LT}^1 &= S_\varphi S_\Theta S_\theta,\\
\tilde{f}_{LT}^2 &= S_\varphi S_\Theta S_\theta C_\Theta C_\theta,\\
f_{TT'} &= C_{2\varphi} S_\Theta^2 S_\theta^2,\\
\tilde{f}_{TT'} &= S_{2\varphi} S_\Theta^2 S_\theta^2\,,
\end{aligned}
$$

Possible to probe multiple angular observables

$\mathcal{O}_{H\Box} = (H^\dagger H)\Box(H^\dagger H)$	$\mathcal{O}_{HL}^{(3)} = iH^\dagger\sigma^a \overleftrightarrow{D}_\mu H \bar{L}\sigma^a\gamma^\mu L$
$\mathcal{O}_{HD} = (H^\dagger D_\mu H)^*(H^\dagger D_\mu H)$	$\mathcal{O}_{HB} = \lvert H\rvert^2 B_{\mu\nu}B^{\mu\nu}$
$\mathcal{O}_{Hu} = iH^\dagger \overleftrightarrow{D}_\mu H \bar{u}_R\gamma^\mu u_R$	$\mathcal{O}_{HWB} = H^\dagger\sigma^a H W^a_{\mu\nu}B^{\mu\nu}$
$\mathcal{O}_{Hd} = iH^\dagger \overleftrightarrow{D}_\mu H \bar{d}_R\gamma^\mu d_R$	$\mathcal{O}_{HW} = \lvert H\rvert^2 W_{\mu\nu}W^{\mu\nu}$
$\mathcal{O}_{He} = iH^\dagger \overleftrightarrow{D}_\mu H \bar{e}_R\gamma^\mu e_R$	$\mathcal{O}_{H\tilde{B}} = \lvert H\rvert^2 B_{\mu\nu}\tilde{B}^{\mu\nu}$
$\mathcal{O}_{HQ}^{(1)} = iH^\dagger \overleftrightarrow{D}_\mu H \bar{Q}\gamma^\mu Q$	$\mathcal{O}_{H\tilde{W}B} = H^\dagger\sigma^a H W^a_{\mu\nu}\tilde{B}^{\mu\nu}$
$\mathcal{O}_{HQ}^{(3)} = iH^\dagger\sigma^a \overleftrightarrow{D}_\mu H \bar{Q}\sigma^a\gamma^\mu Q$	$\mathcal{O}_{H\tilde{W}} = \lvert H\rvert^2 W^a_{\mu\nu}\tilde{W}^{a\mu\nu}$
$\mathcal{O}_{HL}^{(1)} = iH^\dagger \overleftrightarrow{D}_\mu H \bar{L}\gamma^\mu L$	$\mathcal{O}_{y_b} = y_b\lvert H\rvert^2(\bar{Q}Hb_R + h.c).$

CP-odd operators

D6 operators in Warsaw basis contributing to anomalous hVV*/hVff couplings

Zh and *Wh* production at the LHC

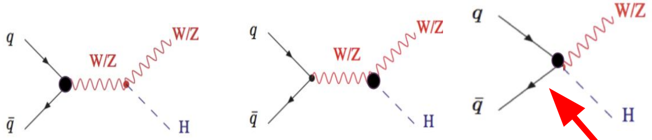


Diagram not in the SM

SM scaling κ-framework

$$
\begin{aligned}
\Delta\mathcal{L}_6 \supset\ & \delta\hat{g}^h_{WW}\,\frac{2m_W^2}{v}\,hW^{+\mu}W^-_\mu + \boxed{\delta\hat{g}^h_{ZZ}\,\frac{2m_Z^2}{v}\,h\frac{Z^\mu Z_\mu}{2}} + \delta g^W_Q\,(W^+_\mu \bar{u}_L\gamma^\mu d_L + h.c.) \\
+\ & \delta g^W_L\,(W^+_\mu \bar{\nu}_L\gamma^\mu e_L + h.c.) + g^h_{WL}\,\frac{h}{v}(W^+_\mu \bar{\nu}_L\gamma^\mu e_L + h.c.) \\
+\ & g^h_{WQ}\,\frac{h}{v}(W^+_\mu \bar{u}_L\gamma^\mu d_L + h.c.) + \sum_f \delta g^Z_f Z_\mu \bar{f}\gamma^\mu f + \boxed{\sum_f g^h_{Zf}\,\frac{h}{v}Z_\mu \bar{f}\gamma^\mu f} \\
+\ & \kappa_{WW}\,\frac{h}{v}W^{+\mu\nu}W^-_{\mu\nu} + \tilde{\kappa}_{WW}\,\frac{h}{v}W^{+\mu\nu}\tilde{W}^-_{\mu\nu} + \boxed{\kappa_{ZZ}\,\frac{h}{2v}Z^{\mu\nu}Z_{\mu\nu}} \\
+\ & \boxed{\tilde{\kappa}_{ZZ}\,\frac{h}{2v}Z^{\mu\nu}\tilde{Z}_{\mu\nu}} + \kappa_{Z\gamma}\,\frac{h}{v}A^{\mu\nu}Z_{\mu\nu} + \tilde{\kappa}_{Z\gamma}\,\frac{h}{v}A^{\mu\nu}\tilde{Z}_{\mu\nu} + \delta\hat{g}^h_{bb}\,\frac{\sqrt{2}m_b}{v}\,hb\bar{b} \\
+\ & \kappa_{\gamma\gamma}\,\frac{h}{v}A^{\mu\nu}A_{\mu\nu}
\end{aligned}
$$

4 directions relevant for the high-energy primaries (to follow)

Contact interaction; no propagator; Energy growth

CP-even new Lorentz structure (angular deformation)

CP-odd new Lorentz structure (angular deformation)

Deformations written in broken phase after symmetry breaking

Mapping on to the Warsaw basis

$$\delta g_f^W = \frac{g}{\sqrt{2}} \frac{v^2}{\Lambda^2} c_{HF}^{(3)} + \frac{\delta m_Z^2}{m_Z^2} \frac{\sqrt{2} g c_{\theta_W}^2}{4 s_{\theta_W}^2}, \quad \text{where} \quad \frac{\delta m_Z^2}{m_Z^2} = \frac{v^2}{\Lambda^2}(2 t_{\theta_W} c_{WB} + \frac{c_{HD}}{2})$$

$$g_{Wf}^h = \sqrt{2} g \frac{v^2}{\Lambda^2} c_{HF}^{(3)}, \quad \delta \hat{g}_{WW}^h = \frac{v^2}{\Lambda^2}\left(c_{H\Box} - \frac{c_{HD}}{4}\right)$$

$$\kappa_{WW} = \frac{2v^2}{\Lambda^2} c_{HW}, \quad \tilde{\kappa}_{WW} = \frac{2v^2}{\Lambda^2} c_{H\tilde{W}}$$

$$\delta g_f^Z = -\frac{g' Y_f}{c_{\theta_W}} c_{WB} \frac{v^2}{\Lambda^2} - \frac{g}{c_{\theta_W}} \frac{v^2}{\Lambda^2}(|T_3^f| c_{HF}^{(1)} - T_3^f c_{HF}^{(3)} + (1/2 - |T_3^f|) c_{Hf}) c_{\theta_W}$$
$$+ \frac{\delta m_Z^2}{m_Z^2} \frac{g}{2 c_{\theta_W} s_{\theta_W}^2} (T_3 c_{\theta_W}^2 + Y_f s_{\theta_W}^2)$$

$$\delta \hat{g}_{ZZ}^h = \frac{v^2}{\Lambda^2}\left(c_{H\Box} + \frac{c_{HD}}{4}\right), \quad g_{Zf}^h = -\frac{2g}{c_{\theta_W}} \frac{v^2}{\Lambda^2}(|T_3^f| c_{HF}^{(1)} - T_3^f c_{HF}^{(3)} + (1/2 - |T_3^f|) c_{Hf})$$

$$\kappa_{ZZ} = \frac{2v^2}{\Lambda^2}(c_{\theta_W}^2 c_{HW} + s_{\theta_W}^2 c_{HB} + s_{\theta_W} c_{\theta_W} c_{HWB})$$

$$\tilde{\kappa}_{ZZ} = \frac{2v^2}{\Lambda^2}(c_{\theta_W}^2 c_{H\tilde{W}} + s_{\theta_W}^2 c_{H\tilde{B}} + s_{\theta_W} c_{\theta_W} c_{H\tilde{W}B})$$

$$\delta \hat{g}_{b\bar{b}}^h = -\frac{v^2}{\Lambda^2} \frac{v}{\sqrt{2} m_b} c_{y_b} + \frac{v^2}{\Lambda^2}(c_{H\Box} - \frac{c_{HD}}{4})$$

***VH*: Relations to the Warsaw Basis**

Check out Rosetta: an operator basis translator for SMEFT

High-energy primaries

1. The four channels, *viz.*, Zh, $W^{\pm}h$, $W^{+}W^{-}$ and $W^{\pm}Z$ can be expressed (at high energies) respectively as $G^{0}h$, $G^{\pm}h$, $G^{+}G^{-}$ and $G^{\pm}G^{0}$ and the Higgs field can be written as

$$\begin{pmatrix} G^{+} \\ \frac{h+iG^{0}}{2} \end{pmatrix}$$

2. These four final states are **intrinsically connected by gauge symmetry** even though they are very different from a collider physics point of view

3. With the **Goldstone boson equivalence theorem,** it is possible to compute amplitudes for various components of the Higgs in the unbroken phase

4. **Full *SU(2)* theory is manifest** [Franceschini, Panico,Pomarol, Riva, Wulzer, 2017]

High-energy primaries

Amplitude	High-energy primaries
$\bar{u}_L d_L \to W_L Z_L, W_L h$	$\sqrt{2} a_q^{(3)}$
$\bar{u}_L u_L \to W_L W_L$ $\bar{d}_L d_L \to Z_L h$	$a_q^{(1)} + a_q^{(3)}$
$\bar{d}_L d_L \to W_L W_L$ $\bar{u}_L u_L \to Z_L h$	$a_q^{(1)} - a_q^{(3)}$
$\bar{f}_R f_R \to W_L W_L, Z_L h$	a_f

Amplitude	High-energy primaries
$\bar{u}_L d_L \to W_L Z_L, W_L h$	$\frac{g^h_{Z d_L d_L} - g^h_{Z u_L u_L}}{\sqrt{2}}$
$\bar{u}_L u_L \to W_L W_L$ $\bar{d}_L d_L \to Z_L h$	$g^h_{Z d_L d_L}$
$\bar{d}_L d_L \to W_L W_L$ $\bar{u}_L u_L \to Z_L h$	$g^h_{Z u_L u_L}$
$\bar{f}_R f_R \to W_L W_L, Z_L h$	$g^h_{Z f_R f_R}$

Vh and *VV* channels are entwined by symmetry and they constrain the same set of observables at High energies but may have different directions [Franceschini, Panico,Pomarol, Riva, Wulzer, 2017, SB, Gupta, Seth, Reiness, Spannowsky, 2020]

High-energy primaries

Amplitude	High-energy primaries	Low-energy primaries
$\bar{u}_L d_L \to W_L Z_L, W_L h$	$\sqrt{2} a_q^{(3)}$	$\sqrt{2}\frac{g^2}{m_W^2}\left[c_{\theta_W}(\delta g_{uL}^Z - \delta g_{dL}^Z)/g - c_{\theta_W}^2 \delta g_1^Z\right]$
$\bar{u}_L u_L \to W_L W_L$ $\bar{d}_L d_L \to Z_L h$	$a_q^{(1)} + a_q^{(3)}$	$-\frac{2g^2}{m_W^2}\left[Y_L t_{\theta_W}^2 \delta\kappa_\gamma + T_Z^{u_L}\delta g_1^Z + c_{\theta_W}\delta g_{dL}^Z/g\right]$
$\bar{d}_L d_L \to W_L W_L$ $\bar{u}_L u_L \to Z_L h$	$a_q^{(1)} - a_q^{(3)}$	$-\frac{2g^2}{m_W^2}\left[Y_L t_{\theta_W}^2 \delta\kappa_\gamma + T_Z^{d_L}\delta g_1^Z + c_{\theta_W}\delta g_{uL}^Z/g\right]$
$\bar{f}_R f_R \to W_L W_L, Z_L h$	a_f	$-\frac{2g^2}{m_W^2}\left[Y_{f_R} t_{\theta_W}^2 \delta\kappa_\gamma + T_Z^{f_R}\delta g_1^Z + c_{\theta_W}\delta g_{fR}^Z/g\right]$

Vh and *VV* channels are entwined by symmetry and they constrain the same set of observables at High energies but may have different directions [Franceschini, Panico,Pomarol, Riva, Wulzer, 2017, SB, Gupta, Reiness, Seth, Spannowsky, 2020]

High-energy primaries

SILH basis	Warsaw basis
$\mathcal{O}_W = \frac{ig}{2}(H^\dagger\sigma^a \overleftrightarrow{D}^\mu H)D^\nu W^a_{\mu\nu}$	$\mathcal{O}^{(3)}_L = (\bar{Q}_L\sigma^a\gamma^\mu Q_L)(iH^\dagger\sigma^a \overleftrightarrow{D}_\mu H)$
$\mathcal{O}_B = \frac{ig'}{2}(H^\dagger \overleftrightarrow{D}^\mu H)\partial^\nu B^a_{\mu\nu}$	$\mathcal{O}_L = (\bar{Q}_L\gamma^\mu Q_L)(iH^\dagger \overleftrightarrow{D}_\mu H)$
$\mathcal{O}_{HW} = ig(D^\mu H)^\dagger\sigma^a(D^\nu H)W^a_{\mu\nu}$	$\mathcal{O}^u_R = (\bar{u}_R\gamma^\mu u_R)(iH^\dagger \overleftrightarrow{D}_\mu H)$
$\mathcal{O}_{HB} = ig(D^\mu H)^\dagger(D^\nu H)B_{\mu\nu}$	$\mathcal{O}^d_R = (\bar{d}_R\gamma^\mu d_R)(iH^\dagger \overleftrightarrow{D}_\mu H)$
$\mathcal{O}_{2W} = -\frac{1}{2}(D^\mu W^a_{\mu\nu})^2$	
$\mathcal{O}_{2B} = -\frac{1}{2}(\partial^\mu B_{\mu\nu})^2$	

Dimension-6 operators contributing to the high energy longitudinal diboson production channels in the SILH and Warsaw bases [Franceschini, Panico,Pomarol, Riva, Wulzer, 2017]

$$a_u = 4\frac{c^u_R}{\Lambda^2}, a_d = 4\frac{c^d_R}{\Lambda^2}, a^{(1)}_q = 4\frac{c^{(1)}_L}{\Lambda^2}, \text{ and } a^{(3)}_q = 4\frac{c^{(3)}_L}{\Lambda^2}$$

Relating the high-energy primaries with the Warsaw basis operators

We are dealing with four channels and there are only four independent couplings at play at high energies.

Zh production (Helicity amplitude)

- For a $2 \to 2$ process $f(\sigma)\bar{f}(-\sigma) \to Zh$, the helicity amplitudes are given by

$$\mathcal{M}_\sigma^{\lambda=\pm} = \sigma \frac{1+\sigma\lambda\cos\Theta}{\sqrt{2}} G_V \frac{m_V}{\boxed{\sqrt{\hat{s}}}} \left[\boxed{1} + \left(\frac{g_{Vf}^h}{g_f^V} + \hat{\kappa}_{VV} - i\lambda\hat{\tilde{\kappa}}_{VV} \right) \frac{\hat{s}}{2m_V^2} \right]$$

$$\mathcal{M}_\sigma^{\lambda=0} = -\frac{\sin\Theta}{2} G_V \left[\boxed{1} + \delta\hat{g}_{VV}^h + 2\hat{\kappa}_{VV} + \delta g_f^Z + \frac{g_{Vf}^h}{g_f^V} \left(-\frac{1}{2} + \frac{\hat{s}}{2m_V^2} \right) \right]$$

$$\begin{aligned} \hat{\kappa}_{WW} &= \kappa_{WW} \\ \hat{\kappa}_{ZZ} &= \kappa_{ZZ} + \frac{Q_f e}{g_f^Z} \kappa_{Z\gamma}, \\ \hat{\tilde{\kappa}}_{ZZ} &= \tilde{\kappa}_{ZZ} + \frac{Q_f e}{g_f^Z} \tilde{\kappa}_{Z\gamma} \end{aligned}$$

- $\lambda = \pm 1$ and $\sigma = \pm 1$ are, respectively, the helicities of the Z-boson and initial-state fermions, $g_f^Z = g(T_3^f - Q_f s_{\theta_W}^2)/c_{\theta_W}$
- Leading SM is longitudinal ($\lambda = 0$), Leading effect of $\kappa_{WW}, \kappa_{ZZ}, \tilde{\kappa}_{ZZ}$ is in the transverse-longitudinal (LT) interference, LT term vanishes if we aren't careful

Angular observables: *Zh* and *Wh* production at the LHC

$$\epsilon_{LR} = \frac{(g^V_{l_R})^2 - (g^V_{l_L})^2}{(g^V_{l_R})^2 + (g^V_{l_L})^2} \qquad \mathcal{G} = g g^Z_f \sqrt{(g^Z_{l_L})^2 + (g^Z_{l_R})^2} / (\cos\theta_W \Gamma_Z) \qquad \gamma = \sqrt{\hat{s}}/(2m_V)$$

$$\begin{aligned}\sum_{L,R} |\mathcal{A}(\hat{s},\Theta,\theta,\varphi)|^2 &= a_{LL}\sin^2\Theta\sin^2\theta + a^1_{TT}\cos\Theta\cos\theta \\ &+ a^2_{TT}(1+\cos^2\Theta)(1+\cos^2\theta) + \cos\varphi\sin\Theta\sin\theta \\ &\times (a^1_{LT} + a^2_{LT}\cos\theta\cos\Theta) + \sin\varphi\sin\Theta\sin\theta \\ &\times (\tilde{a}^1_{LT} + \tilde{a}^2_{LT}\cos\theta\cos\Theta) + a_{TT'}\cos 2\varphi\sin^2\Theta\sin^2\theta \\ &+ \tilde{a}_{TT'}\sin 2\varphi\sin^2\Theta\sin^2\theta\end{aligned}$$

a_{LL}	$\frac{\mathcal{G}^2}{4}\left[1 + 2\delta\hat{g}^h_{VV} + 4\hat{\kappa}_{VV} + 2\delta g^Z_f + \frac{g^h_{Vf}}{g^V_f}(-1+4\gamma^2)\right]$
a^1_{TT}	$\frac{\mathcal{G}^2\sigma\epsilon_{RL}}{2\gamma^2}\left[1 + 4\left(\frac{g^h_{Vf}}{g^V_f} + \hat{\kappa}_{VV}\right)\gamma^2\right]$
a^2_{TT}	$\frac{\mathcal{G}^2}{8\gamma^2}\left[1 + 4\left(\frac{g^h_{Vf}}{g^V_f} + \hat{\kappa}_{VV}\right)\gamma^2\right]$
a^1_{LT}	$-\frac{\mathcal{G}^2\sigma\epsilon_{RL}}{2\gamma}\left[1 + 2\left(\frac{2g^h_{Vf}}{g^V_f} + \hat{\kappa}_{VV}\right)\gamma^2\right]$
a^2_{LT}	$-\frac{\mathcal{G}^2}{2\gamma}\left[1 + 2\left(\frac{2g^h_{Vf}}{g^V_f} + \hat{\kappa}_{VV}\right)\gamma^2\right]$
$\tilde{a}^1_{LT}$	$-\mathcal{G}^2\sigma\epsilon_{RL}\hat{\tilde{\kappa}}_{VV}\gamma$
$\tilde{a}^2_{LT}$	$-\mathcal{G}^2\hat{\tilde{\kappa}}_{VV}\gamma$
$a_{TT'}$	$\frac{\mathcal{G}^2}{8\gamma^2}\left[1 + 4\left(\frac{g^h_{Vf}}{g^V_f} + \kappa\hat{V}V\right)\gamma^2\right]$
$\tilde{a}_{TT'}$	$\frac{\mathcal{G}^2}{2}\hat{\tilde{\kappa}}_{VV}$

Suppressed moments

Zh and *Wh* production at the LHC

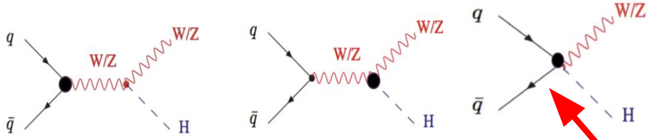


Diagram not in the SM

SM scaling κ-framework

$$
\begin{aligned}
\Delta\mathcal{L}_6 \supset\ & \delta\hat{g}^h_{WW}\frac{2m_W^2}{v}hW^{+\mu}W^-_\mu + \boxed{\delta\hat{g}^h_{ZZ}\frac{2m_Z^2}{v}h\frac{Z^\mu Z_\mu}{2}} + \delta g^W_Q\,(W^+_\mu\bar{u}_L\gamma^\mu d_L + h.c.)\\
& + \delta g^W_L\,(W^+_\mu\bar{\nu}_L\gamma^\mu e_L + h.c.) + g^h_{WL}\,\frac{h}{v}(W^+_\mu\bar{\nu}_L\gamma^\mu e_L + h.c.)\\
& + g^h_{WQ}\,\frac{h}{v}(W^+_\mu\bar{u}_L\gamma^\mu d_L + h.c.) + \sum_f \delta g^Z_f Z_\mu\bar{f}\gamma^\mu f + \boxed{\sum_f g^h_{Zf}\,\frac{h}{v}Z_\mu\bar{f}\gamma^\mu f}\\
& + \kappa_{WW}\,\frac{h}{v}W^{+\mu\nu}W^-_{\mu\nu} + \tilde{\kappa}_{WW}\,\frac{h}{v}W^{+\mu\nu}\tilde{W}^-_{\mu\nu} + \boxed{\kappa_{ZZ}\,\frac{h}{2v}Z^{\mu\nu}Z_{\mu\nu}}\\
& + \boxed{\tilde{\kappa}_{ZZ}\,\frac{h}{2v}Z^{\mu\nu}\tilde{Z}_{\mu\nu}} + \kappa_{Z\gamma}\,\frac{h}{v}A^{\mu\nu}Z_{\mu\nu} + \tilde{\kappa}_{Z\gamma}\,\frac{h}{v}A^{\mu\nu}\tilde{Z}_{\mu\nu} + \delta\hat{g}^h_{bb}\,\frac{\sqrt{2}m_b}{v}hb\bar{b}\\
& + \kappa_{\gamma\gamma}\,\frac{h}{v}A^{\mu\nu}A_{\mu\nu}
\end{aligned}
$$

4 directions relevant for the high-energy primaries (to follow)

Contact interaction; no propagator; Energy growth

CP-even new Lorentz structure (angular deformation)

CP-odd new Lorentz structure (angular deformation)

Deformations written in broken phase after symmetry breaking

EFT validity

- We estimate the scale of new physics for a measured g^h_{Zf}
- Example: Heavy $SU(2)_L$ triplet (singlet) vector W'^a (Z') couples to SM fermion current $\bar{f}\sigma^a\gamma_\mu f$ ($\bar{f}\gamma_\mu f$) with g_f and to the Higgs current $iH^\dagger\sigma^a \overleftrightarrow{D}_\mu H$ ($iH^\dagger \overleftrightarrow{D}_\mu H$) with g_H

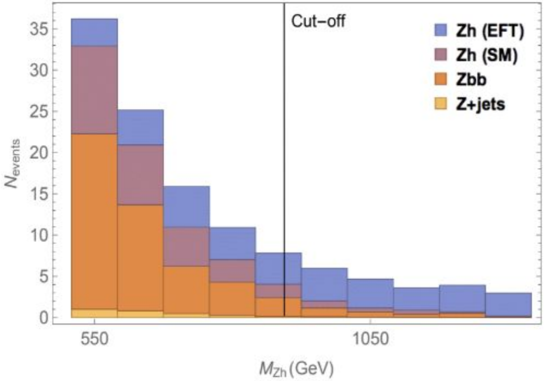
$$g^h_{Zu_L,d_L} \sim \frac{g_H g^2 v^2}{2\Lambda^2},$$

$$g^h_{Zf} \sim \frac{g_H g g_f v^2}{\Lambda^2} \qquad g^h_{Zu_R,d_R} \sim \frac{g_H g g' Y_{u_R,d_R} v^2}{\Lambda^2}$$

- $\Lambda \rightarrow$ mass scale of vector and thus cut-off for low energy EFT
- Assumed g_f to be a combination of $g_B = g' Y_f$ and $g_W = g/2$ for universal case

Higgs-Strahlung at the LHC (hZZ*/ hZff) (at high energies: contact interaction)

- We study the impact of constraining TGC couplings at higher energies
- We study the channel $pp \to Zh \to \ell^+\ell^- b\bar{b}$
- The backgrounds are SM $pp \to Zh$, $Zb\bar{b}$, $t\bar{t}$ and the fake $pp \to Zjj$ ($j \to b$ fake rate taken as 2%)
- Major background $Zb\bar{b}$ (b-tagging efficiency taken to be 70%)
- Boosted substructure analysis with fat-jets of $R = 1.2$ used



Cuts	Zbb	Zh (SM)
At least 1 fat jet with 2 B-mesons with $p_T > 15$ GeV	0.23	0.41
2 OSSF isolated leptons	0.41	0.50
80 GeV $< M_{\ell\ell} <$ 100 GeV, $p_{T,\ell\ell} > 160$ GeV, $\Delta R_{\ell\ell} > 0.2$	0.83	0.89
At least 1 fat jet with 2 B-meson tracks with $p_T > 110$ GeV	0.96	0.98
2 Mass drop subjets and ≥ 2 filtered subjets	0.88	0.92
2 b-tagged subjets	0.38	0.41
115 GeV $< m_h <$ 135 GeV	0.15	0.51
$\Delta R(b_i, \ell_j) > 0.4$, $\not{E}_T < 30$ GeV, $\lvert y_h \rvert < 2.5$, $p_{T,h/Z} > 200$ GeV	0.47	0.69

Event generator (hard-process simulation, resonance decay, PDF sampling, etc) → parton showering (ISR, FSR, etc) → hadronisation (baryon, meson production, etc) → underlying events (MPI) → detector simulation (smearing, efficiencies, energy deposition, etc) → event reconstruction (particle identification, jet clustering, MET, etc)

Differential in energy: constraining the contact terms

Exclusion from *WZ* [Franceschini et al, 2017]

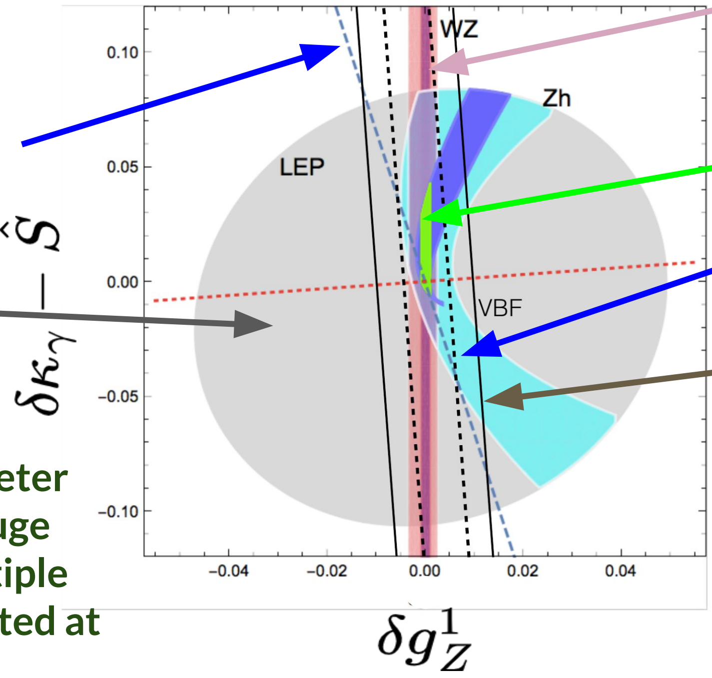
Accidental cancellation of interference terms

Zh + *WZ* combined

Exclusion from *Zh*

LEP exclusion region

WBF analysis in diphoton channel



A common parameter space in triple-gauge couplings for multiple channels - correlated at high energies

$\sigma_{Zh}{}^{SM}/\sigma_{Zbb}$ without cuts ~ 4.6/165

With regular cut-based analysis ~ 0.26

With BDT optimisation ~ 0.50

Differential in energy: constraining the contact terms

Single parameter fits from Zh

	Our 100 TeV Projection	Our 14 TeV projection	LEP Bound
$\delta g^Z_{u_L}$	$\pm 0.0003\ (\pm 0.0001)$	$\pm 0.002\ (\pm 0.0007)$	-0.0026 ± 0.0032
$\delta g^Z_{d_L}$	$\pm 0.0003\ (\pm 0.0001)$	$\pm 0.003\ (\pm 0.001)$	0.0023 ± 0.002
$\delta g^Z_{u_R}$	$\pm 0.0005\ (\pm 0.0002)$	$\pm 0.005\ (\pm 0.001)$	-0.0036 ± 0.0070
$\delta g^Z_{d_R}$	$\pm 0.0015\ (\pm 0.0006)$	$\pm 0.016\ (\pm 0.005)$	0.016 ± 0.0104
δg^Z_1	$\pm 0.0005\ (\pm 0.0002)$	$\pm 0.005\ (\pm 0.001)$	$-0.009^{+0.043}_{-0.042}$
$\delta \kappa_\gamma$	$\pm 0.0035\ (\pm 0.0015)$	$\pm 0.032\ (\pm 0.009)$	$-0.016^{+0.085}_{-0.096}$
$\hat{S}$	$\pm 0.0035\ (\pm 0.0015)$	$\pm 0.032\ (\pm 0.009)$	0.0004 ± 0.0007
W	$\pm 0.0004\ (\pm 0.0002)$	$\pm 0.003\ (\pm 0.001)$	-0.0003 ± 0.0006
Y	$\pm 0.0035\ (\pm 0.0015)$	$\pm 0.032\ (\pm 0.009)$	0.0000 ± 0.0006

Directions from *VBF*, *Zh*, *Wh*, and *WZ*

$$|(-0.04\ c^1_Q + 1.4\ c^{(3)}_Q + 0.1\ c_{uR} - 0.03\ c_{dR})\xi| < 0.003 \qquad [VBF]$$

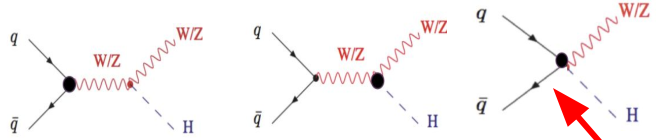
$$|(-0.18\ c^1_Q + 1.3\ c^{(3)}_Q + 0.3\ c_{uR} - 0.1\ c_{dR})\xi| < 0.0005 \qquad [Zh]$$

$$|c^{(3)}_Q \xi| < 0.0004 \qquad [Wh]$$

$$-0.0004 < c^{(3)}_Q \xi < 0.0003 \qquad [WZ]$$

What about the W^+W^- direction?

Zh and *Wh* production at the LHC



SM scaling
κ-framework

Diagram not in the SM

$$\begin{aligned}\Delta\mathcal{L}_6 \supset\ & \delta\hat{g}^h_{WW}\frac{2m_W^2}{v} h W^{+\mu}W^-_\mu + \delta\hat{g}^h_{ZZ}\frac{2m_Z^2}{v} h\frac{Z^\mu Z_\mu}{2} + \delta g^W_Q \left(W^+_\mu \bar{u}_L\gamma^\mu d_L + h.c.\right) \\ & + \delta g^W_L\left(W^+_\mu\bar{\nu}_L\gamma^\mu e_L + h.c.\right) + g^h_{WL}\frac{h}{v}\left(W^+_\mu\bar{\nu}_L\gamma^\mu e_L+h.c.\right) \\ & + g^h_{WQ}\frac{h}{v}\left(W^+_\mu\bar{u}_L\gamma^\mu d_L+h.c.\right) + \sum_f \delta g^Z_f Z_\mu\bar{f}\gamma^\mu f + \sum_f g^h_{Zf}\frac{h}{v}Z_\mu\bar{f}\gamma^\mu f \\ & + \kappa_{WW}\frac{h}{v}W^{+\mu\nu}W^-_{\mu\nu} + \tilde{\kappa}_{WW}\frac{h}{v}W^{+\mu\nu}\tilde{W}^-_{\mu\nu} + \kappa_{ZZ}\frac{h}{2v}Z^{\mu\nu}Z_{\mu\nu} \\ & + \tilde{\kappa}_{ZZ}\frac{h}{2v}Z^{\mu\nu}\tilde{Z}_{\mu\nu} + \kappa_{Z\gamma}\frac{h}{v}A^{\mu\nu}Z_{\mu\nu} + \tilde{\kappa}_{Z\gamma}\frac{h}{v}A^{\mu\nu}\tilde{Z}_{\mu\nu} + \delta\hat{g}^h_{bb}\frac{\sqrt{2}m_b}{v}hb\bar{b} \\ & + \kappa_{\gamma\gamma}\frac{h}{v}A^{\mu\nu}A_{\mu\nu}\end{aligned}$$

4 directions relevant for the high-energy primaries (to follow)

Contact interaction; no propagator; Energy growth

CP-even new Lorentz structure (angular deformation)

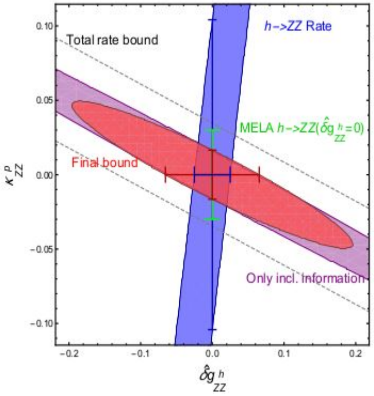
CP-odd new Lorentz structure (angular deformation)

Deformations written in broken phase after symmetry breaking

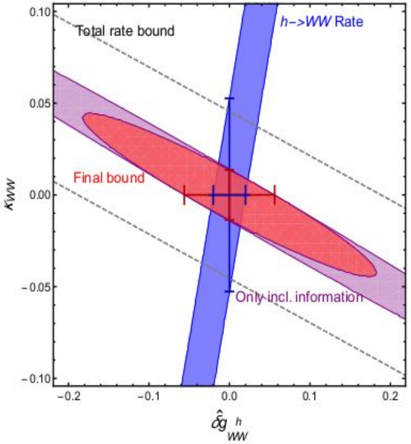
Differential in angles: constraining the angular terms

- Method of moments used to constrain the other couplings
- We obtain **percent level bounds on κ_{ZZ} and in the $(\kappa_{ZZ}, \delta\hat{g}^h_{ZZ})$ plane**
- Competitive and complementary bounds to previous analyses
- Independent bound on the *CP*-odd coupling, $|\tilde{\kappa}^p_{ZZ}| < 0.03$

CP-odd couplings!



- We obtain **percent level bounds on κ_{WW} and in the $(\kappa_{WW}, \delta\hat{g}^h_{WW})$ plane**
- Competitive and complementary bounds to previous analyses
- Independent bound on the *CP*-odd coupling, $|\tilde{\kappa}^p_{WW}| < 0.04$

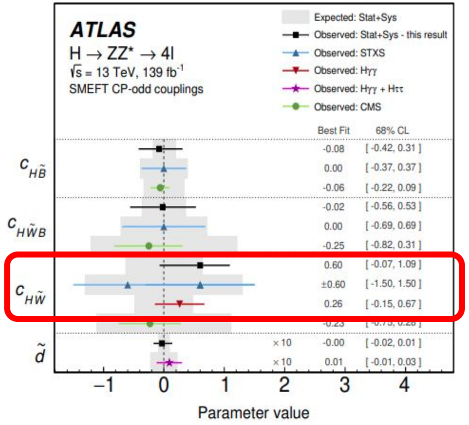


$$\tilde{\kappa}_{WW} = \frac{2v^2}{\Lambda^2} c_{H\tilde{W}}$$

$$\tilde{\kappa}_{ZZ} = \frac{2v^2}{\Lambda^2}\left(\cos^2\theta_W c_{H\tilde{W}} + \sin^2\theta_W c_{H\tilde{B}} + sin\theta_W cos\theta_W c_{H\tilde{W}B}\right)$$

Assuming Λ = 1 Te V, $c_{H\tilde{W}} < 0.33$ at 68% C.L. at HL-LHC!

We consider all operators simultaneously!
ATLAS considers one at a time



Some Clarifications

Hilbert Series: Mathematical Details

- The Hilbert series is defined as

$$H(t) = \sum_{n=0}^{\infty} a_n t^n,$$

where a_n is the number of independent invariants of degree n. Here, the variable t is a formal parameter that tracks the weight (e.g. the mass dimension or another grading) of an operator.

- For fields ϕ_i with assigned weights w_i, the *single-letter partition function* is:

$$f(t) = \sum_{i} t^{w_i}.$$

For instance, if a field has mass dimension 1, its contribution is t^1.

Some Clarifications

Plethystic Exponentials:

- Given a single-letter partition function $f(t)$ for a set of fields (with each field's contribution weighted by its mass dimension or other quantum number), the plethystic exponential is defined as:

$$\mathrm{PE}[f(t)] = \exp\left(\sum_{k=1}^{\infty} \frac{1}{k} f(t^k)\right).$$

- This function generates the full set of multi-field operators (or monomials) by summing over all symmetric products of the fields.

Molien–Weyl Integrals:

- To count only the gauge-invariant combinations, one projects the full generating function onto the invariants by integrating over the gauge group.

Some Clarifications

- The Molien–Weyl formula is:

$$H(t) = \int_G d\mu(g)\,\mathrm{PE}[f(t;g)],$$

  where $d\mu(g)$ is the invariant Haar measure on the gauge group G and $f(t;g)$ includes the dependence on the group elements (via characters of the representations).
- This integral effectively sums over all group transformations, leaving only the combinations that are invariant under the gauge symmetry.

Together, the plethystic exponential and the Molien–Weyl integral provide a systematic and powerful method for counting and classifying the independent operators in an EFT.

Some Clarifications

Partition Function:

- The partition function is defined by:

$$Z = \int \mathcal{D}\phi \, e^{iS[\phi]}.$$

- Under a local, invertible field redefinition,

$$\phi(x) \to \phi'(x) = F[\phi(x)],$$

  the measure transforms as:

$$\mathcal{D}\phi = J[F] \, \mathcal{D}\phi',$$

  where $J[F]$ is the Jacobian determinant.

- In many regularisation schemes (e.g. dimensional regularisation) $J[F]$ is trivial (or its effect can be absorbed), ensuring that physical observables (like the S-matrix) remain invariant.

Equivalence Theorem:

This theorem guarantees that local, invertible field redefinitions do not affect on-shell S-matrix elements, so different operator bases related by such redefinitions yield the same physical predictions.

Some Clarifications

Regularisation and Dimensional Regularisation:

Loop integrals in EFT are divergent. *Dimensional regularisation* sets $d = 4 - \epsilon$ so that divergences appear as poles in ϵ. For example,

$$\int \frac{d^d p}{(2\pi)^d} \frac{1}{(p^2 - m^2)^2} \sim \frac{i}{(4\pi)^2} \left(\frac{1}{\epsilon} + \cdots \right).$$

Renormalisation Group Equations (RGEs):

After renormalisation (typically in the $\overline{\text{MS}}$ scheme), the Wilson coefficients $c_i(\mu)$ become scale-dependent, obeying

$$\mu \frac{d}{d\mu} c_i(\mu) = \gamma_{ij} \, c_j(\mu),$$

ensuring that physical observables remain μ-independent.

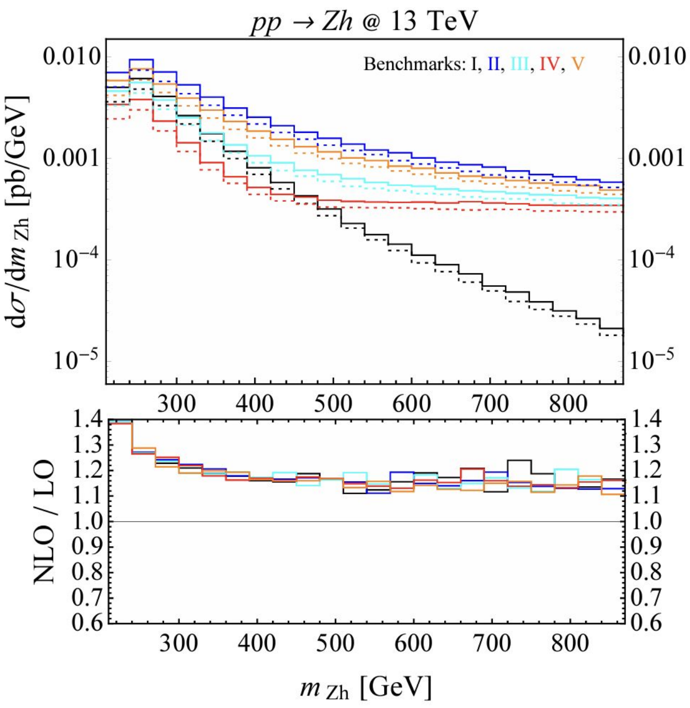
Matching Procedure:

Matching the EFT to the UV theory involves equating on-shell S-matrix elements (or 1PI functions) order by order in $1/\Lambda$ and in the loop expansion, thereby fixing the EFT Wilson coefficients in terms of the UV parameters.

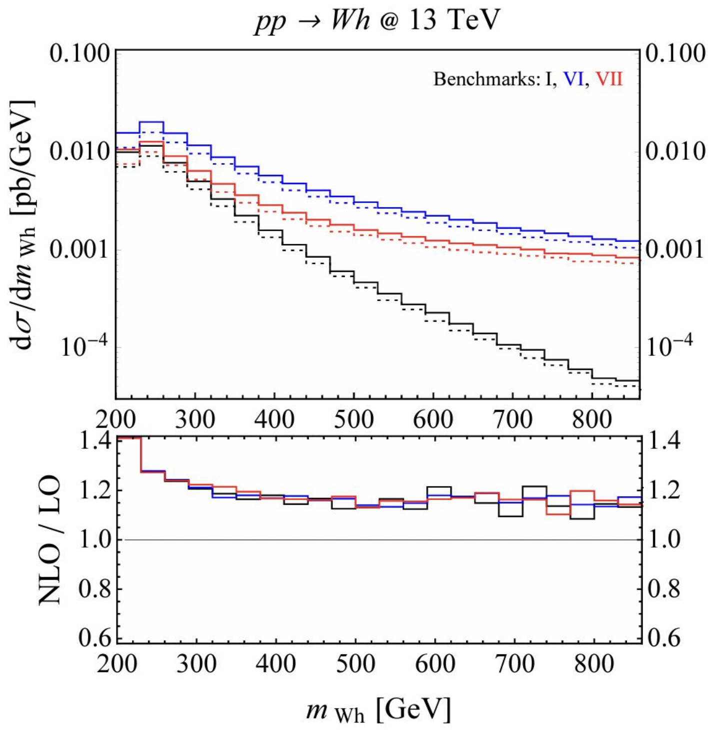
Theory uncertainties in EFT analyses

Type of Uncertainty	Source	Example
Truncation	Missing higher-order operators	Ignoring $\mathcal{O}(1/\Lambda^4)$ terms when fitting SMEFT parameters at the LHC
Matching	Dependence on the unknown UV theory	Different UV completions (e.g., integrating out a heavy scalar vs. a heavy vector boson) yield different EFT coefficients
Renormalisation Scale	Missing higher-loop effects in SMEFT Wilson coefficient running	Scale dependence in next-to-leading-order (NLO) SMEFT fits due to missing next-to-next-to-leading-order (NNLO) corrections
Operator Mixing	Running & basis dependence	Warsaw basis vs. Higgs basis in SMEFT leading to different constraints on Wilson coefficients
Non-Perturbative	Strong interaction effects	Hadronic form factors in lattice QCD calculations affecting flavour physics EFTs (e.g., $B \to K\ell^+\ell^-$ anomalies)
Flavour Assumptions	Assuming universality or Minimal Flavour Violation (MFV)	Assuming MFV in SMEFT may underestimate new physics contributions to rare $b \to s\ell^+\ell^-$ transitions in LHCb anomalies
EFT Validity	Energy scale exceeding Λ, making EFT expansion unreliable	High-p_T regions at the LHC may invalidate a dimension-6 SMEFT truncation
Parametric	Uncertainty in Standard Model (SM) input parameters	Uncertainties in m_W, α_s, CKM matrix elements, and the top quark mass affecting SMEFT global fits

Theory uncertainties in EFT analyses: NLO effects (QCD)



Automated in MG5_aMC@NLO through NLOCT!



Greljo et al., 2017

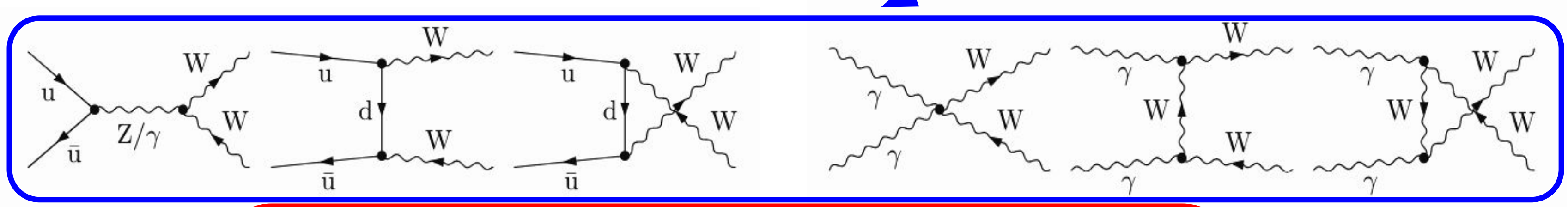
The W^+W^- channel

$$
\begin{aligned}
\Delta\mathcal{L}_{\rm BSM} = & \,\delta g^Z_{uL}\left[Z^\mu \bar{u}_L\gamma_\mu u_L + \frac{\cos\theta_W}{\sqrt{2}}(W^{+\mu}\bar{u}_L\gamma_\mu d_L + \text{h.c.}) + \ldots\right] + \delta g^Z_{uR}\left[Z^\mu \bar{u}_R\gamma_\mu u_R\right] \\
& + \delta g^Z_{dL}\left[Z^\mu \bar{d}_L\gamma_\mu d_L - \frac{\cos\theta_W}{\sqrt{2}}(W^{+\mu}\bar{u}_L\gamma_\mu d_L + \text{h.c.}) + \ldots\right] + \delta g^Z_{dR}\left[Z^\mu \bar{d}_R\gamma_\mu d_R\right] \\
& + ig\cos\theta_W\delta g^Z_1\left[Z^\mu(W^{+\nu}W^-_{\mu\nu} - \text{h.c.}) + Z^{\mu\nu}W^+_\mu W^-_\nu + \ldots\right] \\
& + ie\delta\kappa_\gamma[(A_{\mu\nu} - \tan\theta_W Z_{\mu\nu})W^{+\mu}W^{-\nu} + \ldots],
\end{aligned}
$$

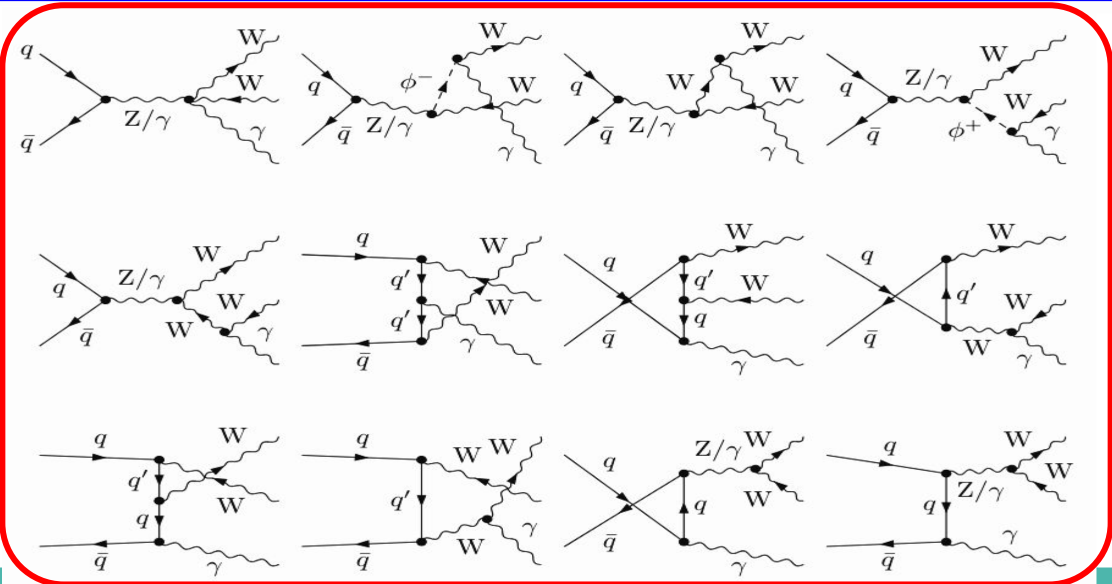
with $Z_{\mu\nu} \equiv \hat{Z}_{\mu\nu} - iW^+_{[\mu}W^-_{\nu]}$, $A_{\mu\nu} \equiv \hat{A}_{\mu\nu}$, $W^\pm_{\mu\nu} \equiv \hat{W}^\pm_{\mu\nu} \pm iW^\pm_{[\mu}(A+Z)_{\nu]}$, where $\hat{V}_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$, and θ_W is the Weinberg angle

Electroweak corrections in W^+W^-

Leading order

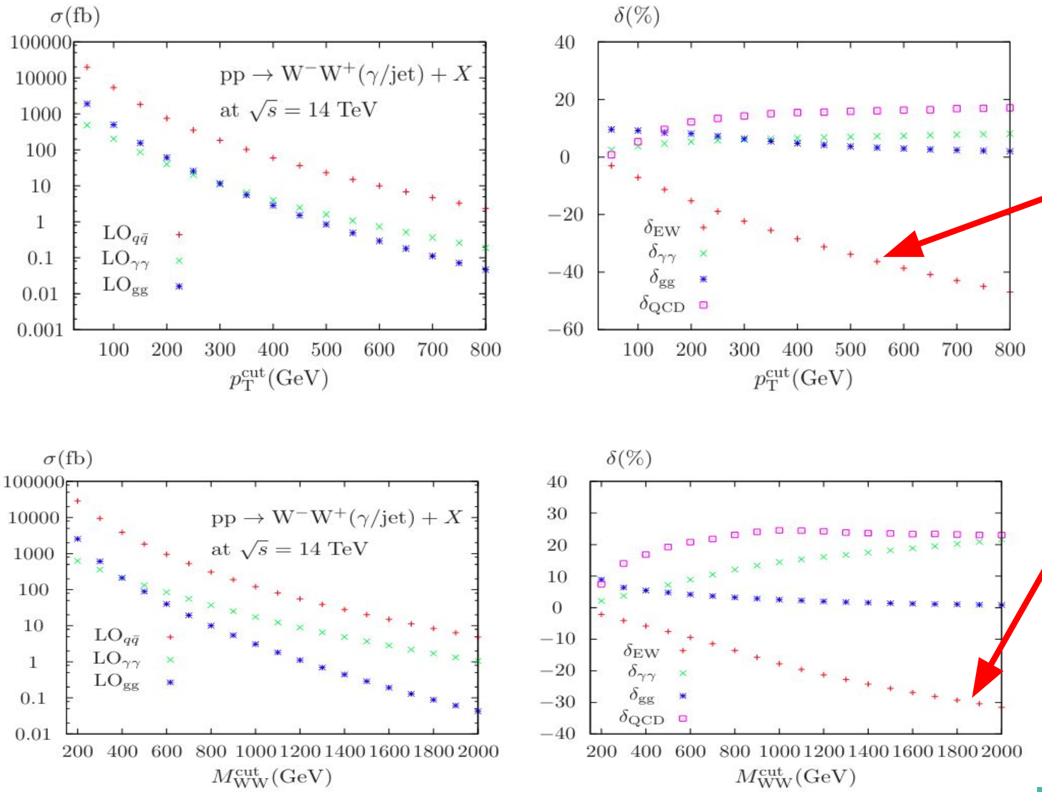


[Bierweiler et al, 2012]



Real bremsstrahlung diagrams

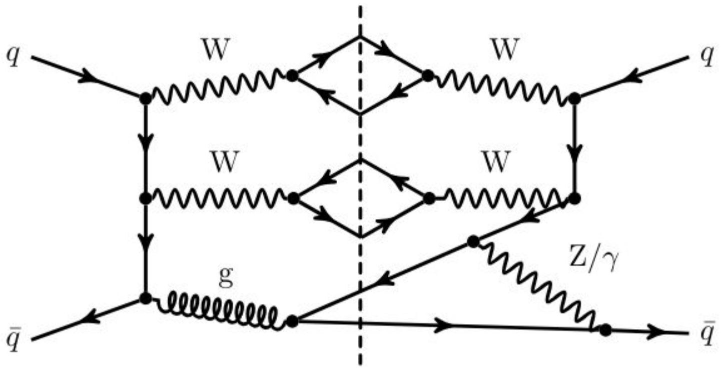
Electroweak corrections in W^+W^-



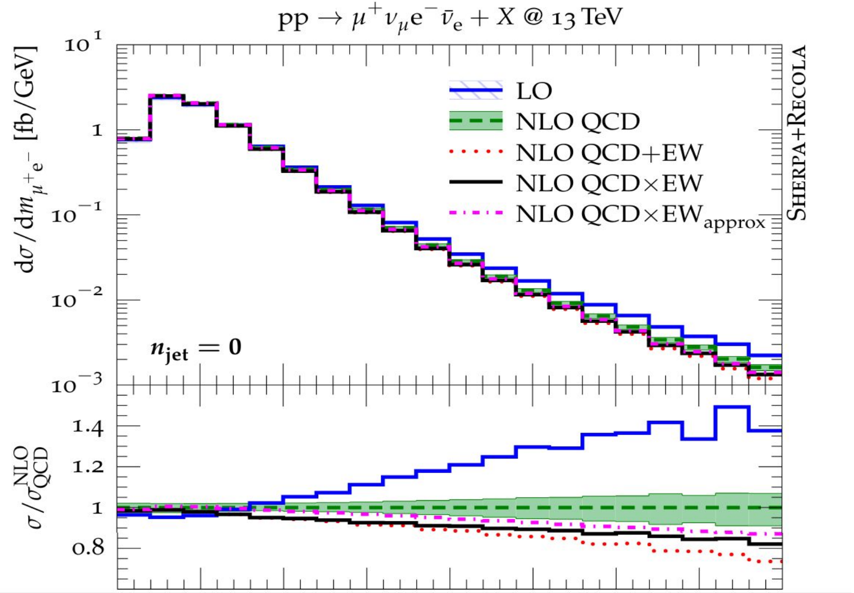
Large (negative) electroweak corrections!

[arXiv:1208.3147: Bierweiler, Kasprzik, Kühn, Uccirati]

Electroweak corrections in *W⁺W⁻(+jj)*



Squared sample diagram representing interference contributions in the real corrections at order $\mathcal{O}(\alpha_s\alpha^5)$ in the channel $pp \to \mu^+\nu_\mu e^-\bar{\nu}_e jj$.



Comparing full QCD x EW corrections with QCD x EW (approx.)

[arXiv:2005.12128: Bräuer, Denner, Pellen, Schönherr, Schumann, 2020]

Electroweak corrections

We include approximate electroweak (EW) corrections in Sherpa which includes infrared subtracted EW 1-loop corrections as additional weights to the respective Born cross sections. In those the event weight is calculated based on the expression

$$\mathrm{d}\sigma_{\mathrm{NLO,EW}_{\mathrm{approx}}} = \left[B(\Phi) + V_{\mathrm{EW}}(\Phi) + I_{\mathrm{EW}}(\Phi)\right]\mathrm{d}\Phi$$

B = Born contribution also entering the uncorrected QCD cross Section

V_{EW} = electroweak virtual corrections at 1-loop accuracy

I_{EW} = generalised Catani-Seymour insertion operator for EW NLO calculations.

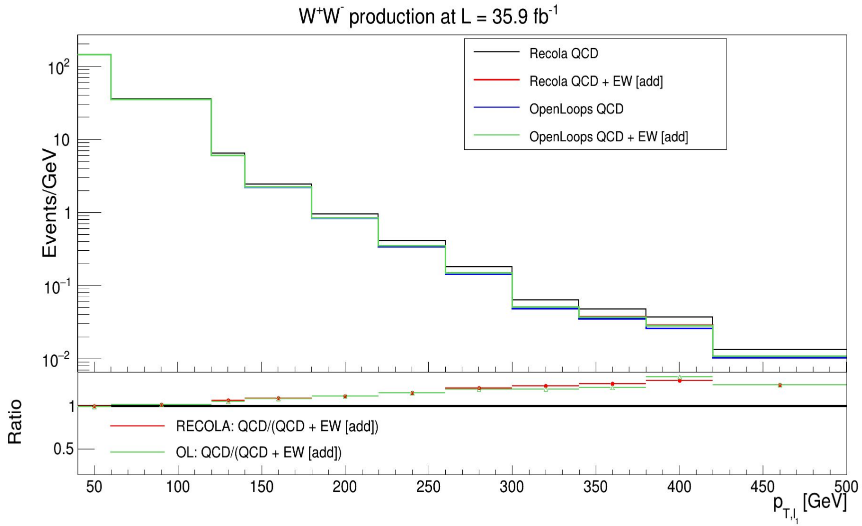
Latter subtracts all infrared singularities of the virtual corrections. This fundamentally arbitrary procedure should provide a good approximation if electroweak Sudakov logarithms are dominant.

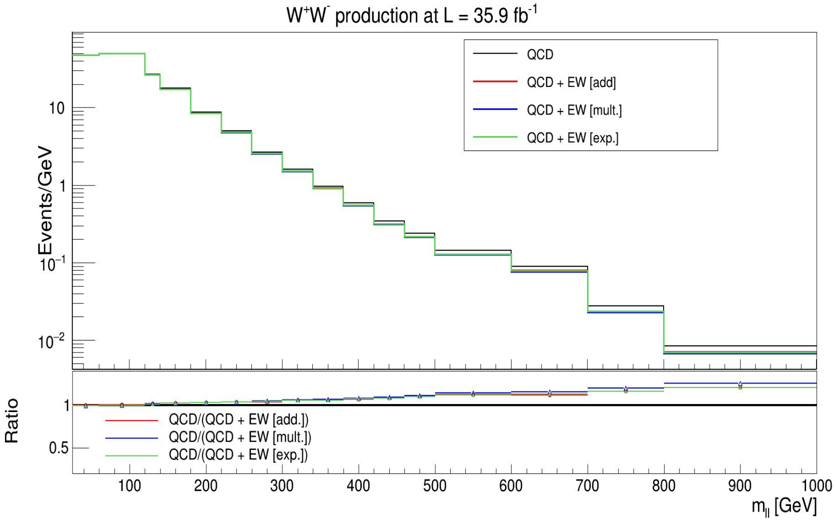
Event generation

$$pp \to W^+(l^+\nu)W^-(l^-\nu)$$

$$\mu_R^2 = \mu_F^2 = M_{\perp,W^+}^2 + M_{\perp,W^-}^2$$

[SB, Reichelt, Spannowsky, arXiv: 2406.15640]



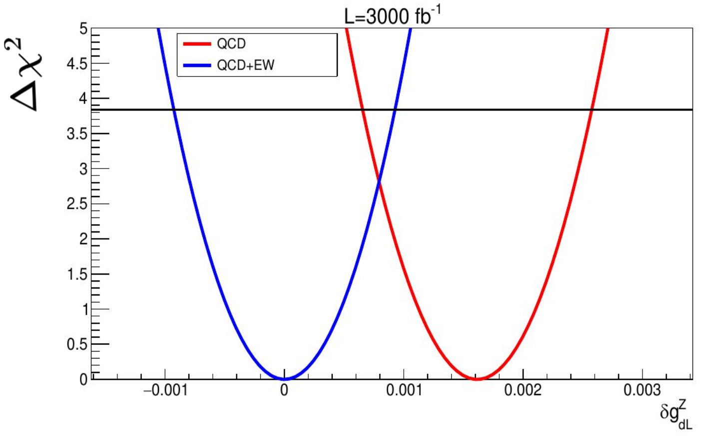


Signal: SMEFT+SM interference; Backgrounds: Drell-Yan $(pp \to \ell^+\ell^-), VZ, t\bar{t} + tW, W\ell\ell$

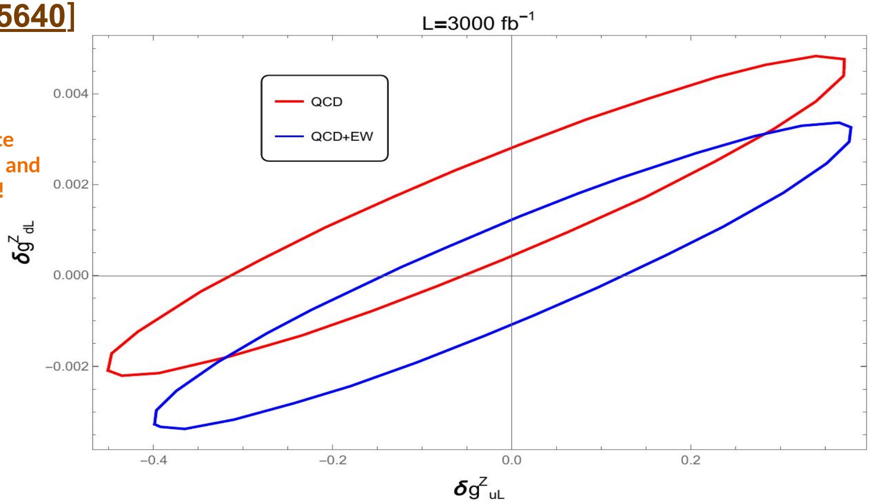
The ME W^+W^-Z is significantly suppressed because of phase-space. Moreover, the CMS analysis that is used here reduces this background even further. There are 12 VVV events when compared to ~6500 qq → W^+W^- events at 36 fb^{-1}.

Results (95% C.L. bounds) - 1 and 2 parameter fits

[SB, Reichelt, Spannowsky, arXiv:2406.15640]



Big difference between red and blue regions!

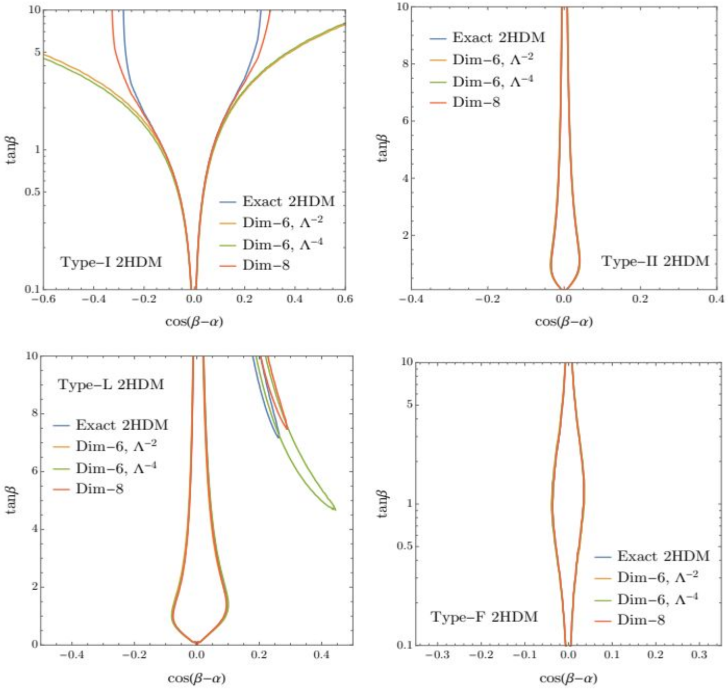


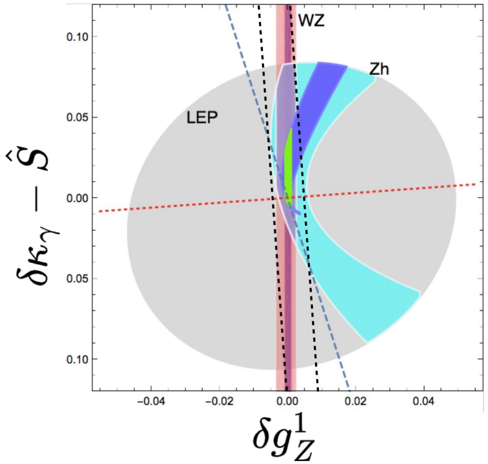
Coupling	QCD: $\mathcal{L} = 300$ fb^{-1}	QCD+EW: $\mathcal{L} = 300$ fb^{-1}	QCD: $\mathcal{L} = 3$ ab^{-1}	QCD+EW: $\mathcal{L} = 3$ ab^{-1}
$\delta g^Z_{d_R}$	[-0.2744 0.0531]	[-0.1569, 0.1569]	[-0.1611, -0.0421]	[-0.0567, 0.0567]
$\delta g^Z_{u_R}$	[-0.0180, 0.0818]	[-0.0474, 0.0474]	[0.0111, 0.0463]	[-0.0167, 0.0167]
$\delta g^Z_{d_L}$	[-0.0008, 0.0039]	[-0.0023, 0.0023]	[0.0006, 0.0026]	[-0.0010, 0.0010]
$\delta g^Z_{u_L}$	[-0.3910, 0.0927]	[-0.2383, 0.2383]	[-0.2969, -0.0702]	[-0.1104, 0.1104]

Theory uncertainties in EFT analyses: operator truncation

Example showing the importance of truncation of operators to match specific models for a top-down approach!

Dawson et al., 2022





In parameter space of interest linear term dominates the squared term!

SB, Englert, Gupta, Spannowsky, 2018

Theory uncertainties in EFT analyses: TGCs

1. EFT operators contributing to anomalous charged triple gauge couplings (cTGCs) and anomalous neutral triple gauge couplings (nTGCs) ⟶ **treated separately!**

2. For cTGCs, D8 operators are usually not considered.

3. For nTGCs, D8 operators are usually the first ones to show effects. Some such operators also contribute to cTGCs.

4. Necessary to consider TGCs through a holistic approach!

Theory uncertainties in EFT analyses: TGCs

1. **Relevant operators for TGCs at dimension-6 (D6)** $X^3 (X = W, B \text{ field strength tensor})$

2. **Relevant operators for TGCs at dimension-8 (D8)**
   $X^2\phi^2 D^2, X^2\psi^2 D \ (\phi = \text{Higgs field}, \psi = \text{fermion fields}, D = \text{covariant derivative})$

3. **These classes of operators contribute to TGCs and it is crucial to consider them in conjunction**

$$\dot{C}_W = (12c_{A,2} - 3b_{0,2})\, g_2^2 C_W \qquad \dot{C}_{\widetilde{W}} = (12c_{A,2} - 3b_{0,2})\, g_2^2 C_{\widetilde{W}}$$

	$\phi^4 D^4$
$B^2\phi^2 D^2$	g_1^2
$W^2\phi^2 D^2$	g_2^2
$WB\phi^2 D^2$	$g_1 g_2$

	$\psi^2 B\phi^3$	$\psi^2 W\phi^3$	$\psi^2 G\phi^3$	$\psi^2\phi^2 D^3$
$B^2\phi^2 D^2$	0	0	0	g_1^2
$W^2\phi^2 D^2$	0	0	0	g_2^2
$WB\phi^2 D^2$	0	0	0	$g_1 g_2$
$G^2\phi^2 D^2$	0	0	0	g_3^2

Phenomenological study! SB, Subba (in preparation)

Alonso et al., 2013

Bakshi et al., 2022