Jet Physics and Machine Learning: Lecture 2 :
Jet reconstruction & measurements

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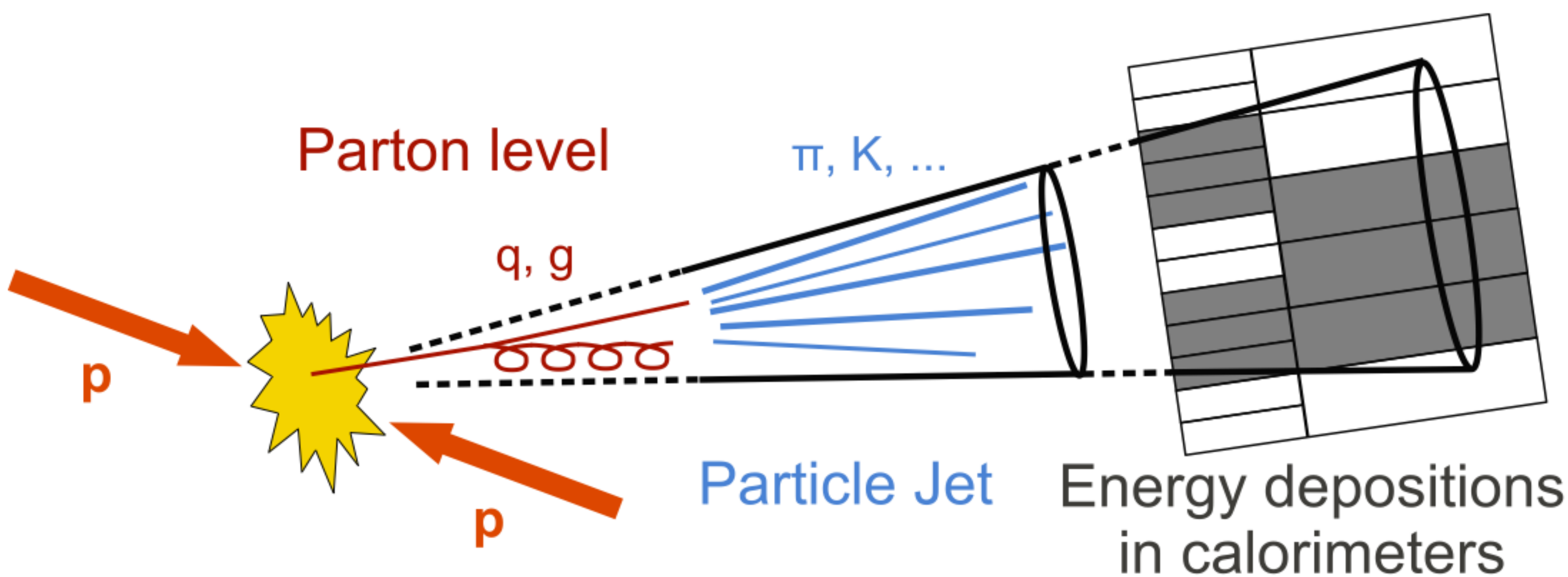
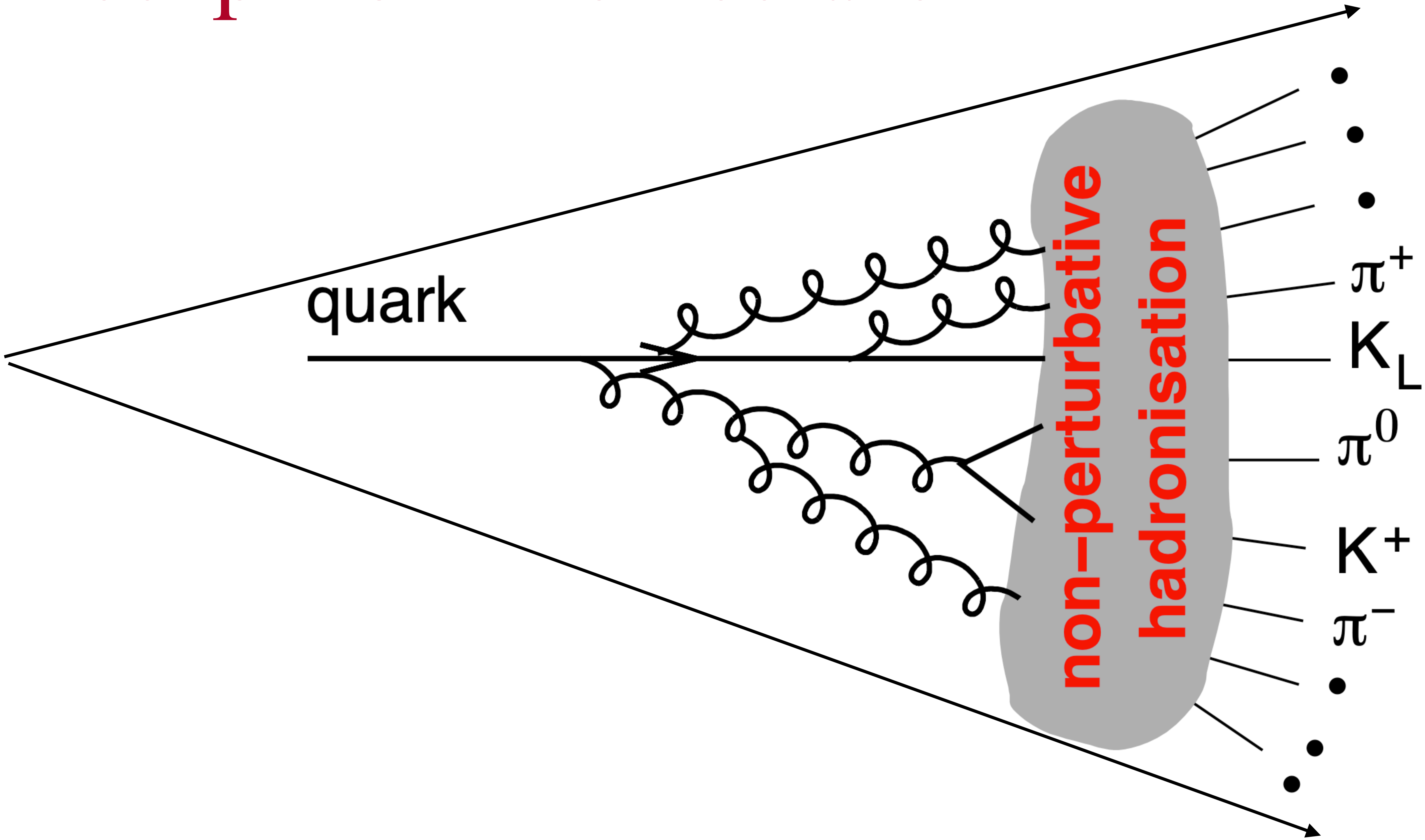
IMSc Spring School on High Energy Physics - 2025



Recap from last lecture



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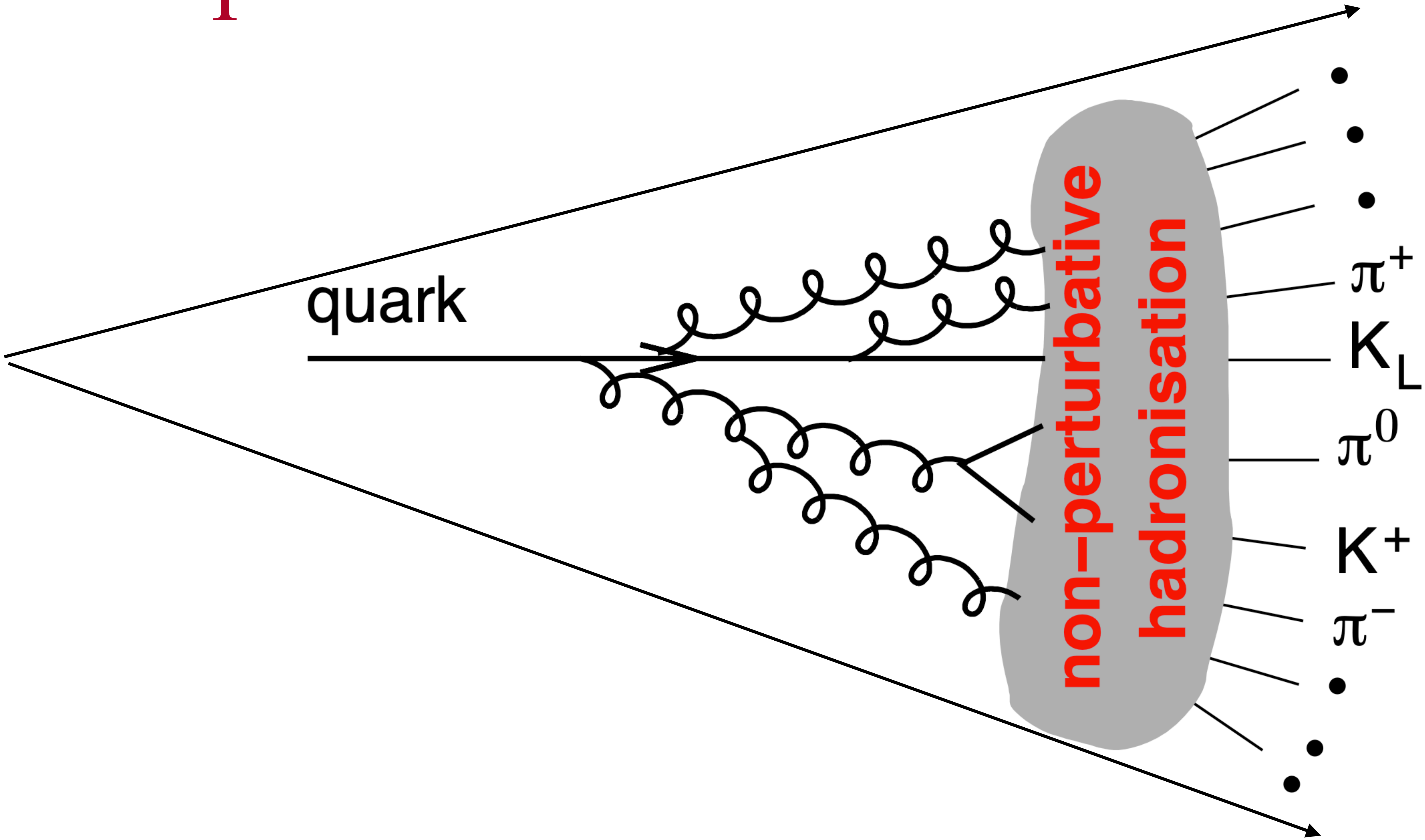
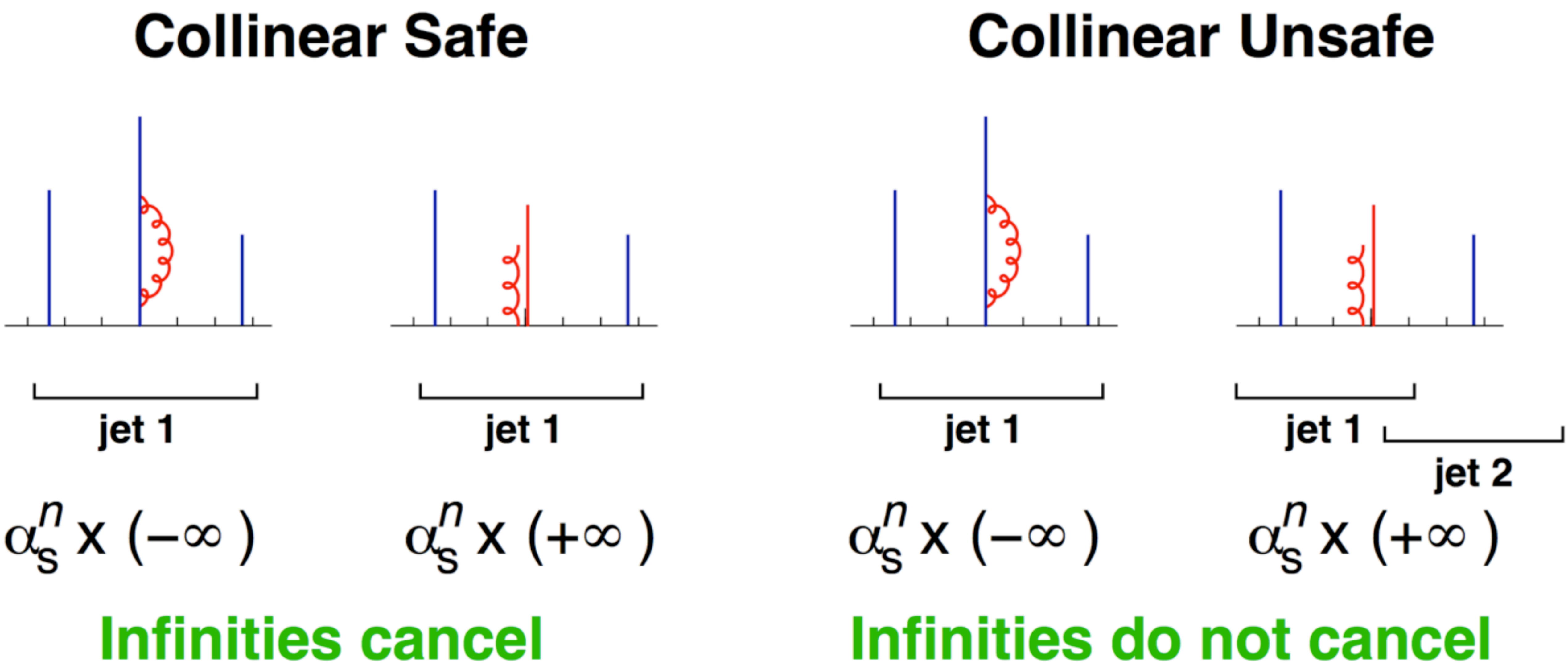


A spray of particles gets produced along the line of flight of the original parton, governed by the factor : $dS = \approx \frac{2\alpha_S\, C_{F/A}}{\pi} \frac{dE}{E} \frac{d\theta}{\theta}$.

Jet defs : which one is legal?



Perturbative calculations of jet observable will only be possible with collinear (and infrared) safe jet definitions

IRC safety

An observable is **infrared and collinear safe** if, in the limit of a **collinear splitting**, or the **emission of an infinitely soft** particle, the observable remains **unchanged:**

$$O(X; p_1, \ldots, p_n, p_{n+1} \to 0) \to O(X; p_1, \ldots, p_n)$$

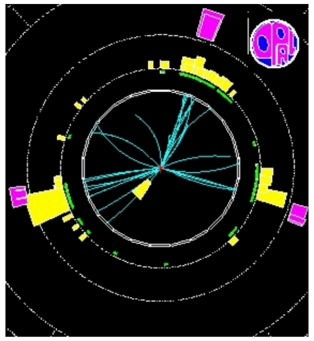
$$O(X; p_1, \ldots, p_n \parallel p_{n+1}) \to O(X; p_1, \ldots, p_n + p_{n+1})$$

This property ensures cancellation of **real** and **virtual** divergences in higher order calculations

If we wish to be able to calculate a jet rate in perturbative QCD the jet algorithm that we use must be IRC safe:
soft emissions and collinear splittings must not change the hard jets

Main approaches of jet clustering

1. Find regions where a lot of energy flows



or

2. Decide which particles are "close", aggregate them

In HEP these are usually called **cone** and **sequential recombination** algorithms respectively

(in other fields they are often called partitional-type clustering and agglomerative hierarchical clustering)

Several important properties that should be met by a jet definition are:

1. Simple to implement in an experimental analysis;
2. Simple to implement in the theoretical calculation;
3. Defined at any order of perturbation theory;
4. Yields finite cross sections at any order of perturbation theory;
5. Yields a cross section that is relatively insensitive to hadronisation.

Two main classes of algorithms

- ## Sequential recombination algorithms

Bottom-up approach: combine particles starting from **closest ones**

How? Choose a **distance measure**, iterate recombination until few objects left, call them jets

Works because of mapping closeness ⇔ QCD divergence

Examples: Jade, k_t, Cambridge/Aachen, anti-k_t, …..

Usually trivially made IRC safe, but their algorithmic complexity scales like N^3

- ## Cone algorithms

Top-down approach: find coarse regions of energy flow.

How? Find **stable cones** (i.e. their axis coincides with sum of momenta of particles in it)

Works because QCD only modifies energy flow on small scales

Examples: JetClu, MidPoint, ATLAS cone, CMS cone, SISCone…...

Can be programmed to be fairly fast, at the price of being complex and IRC unsafe

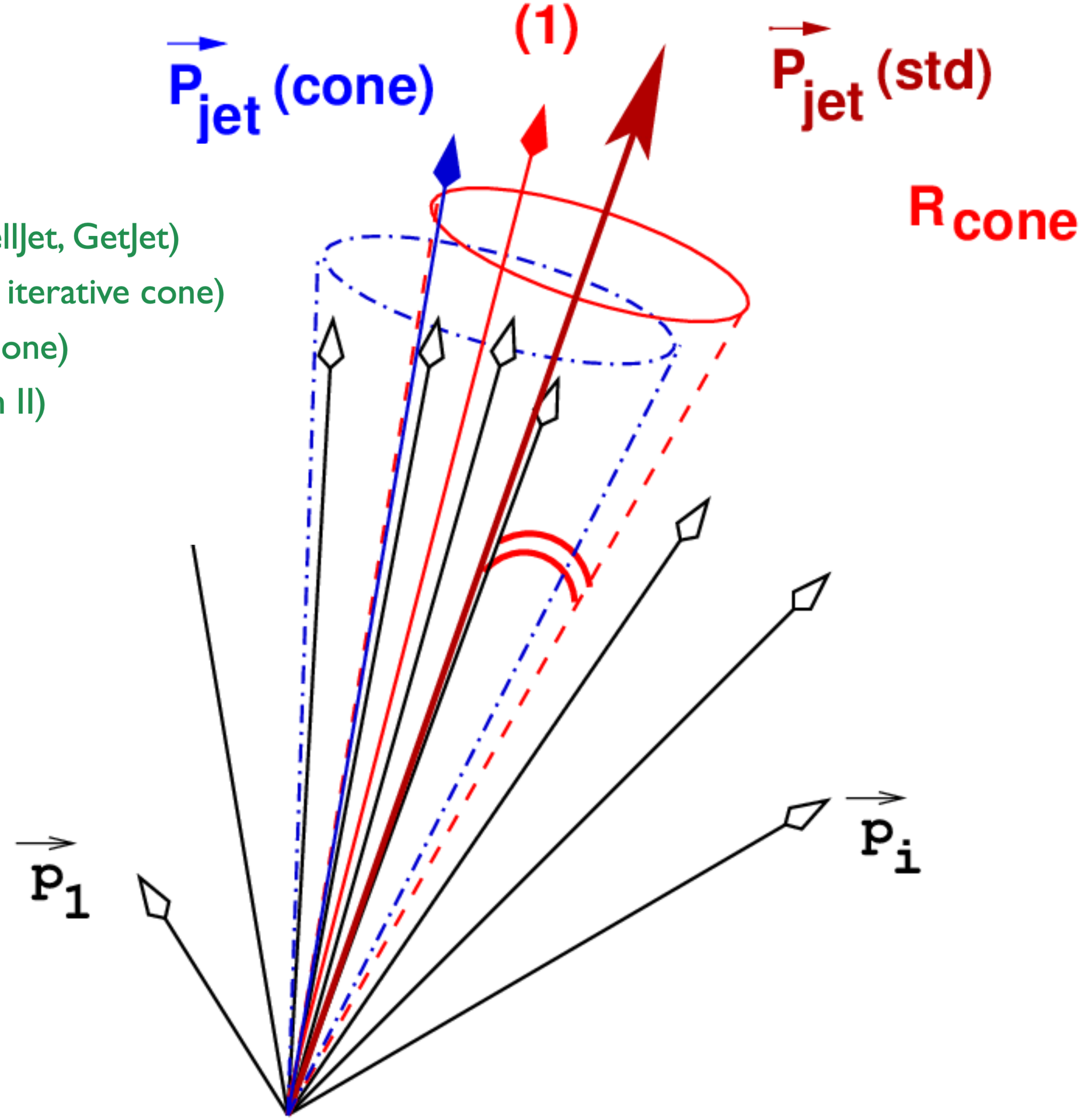
A word or two about cone algorithms

In partitional-type algorithms (i.e. cones), one wishes to find the **stable configurations**:
axis of cones coincides with sum of 4-momenta of the particles it contains

The 'safe' way of doing so is to test **all possible combinations** of N objects

The main sub-categories of cone algorithms are:

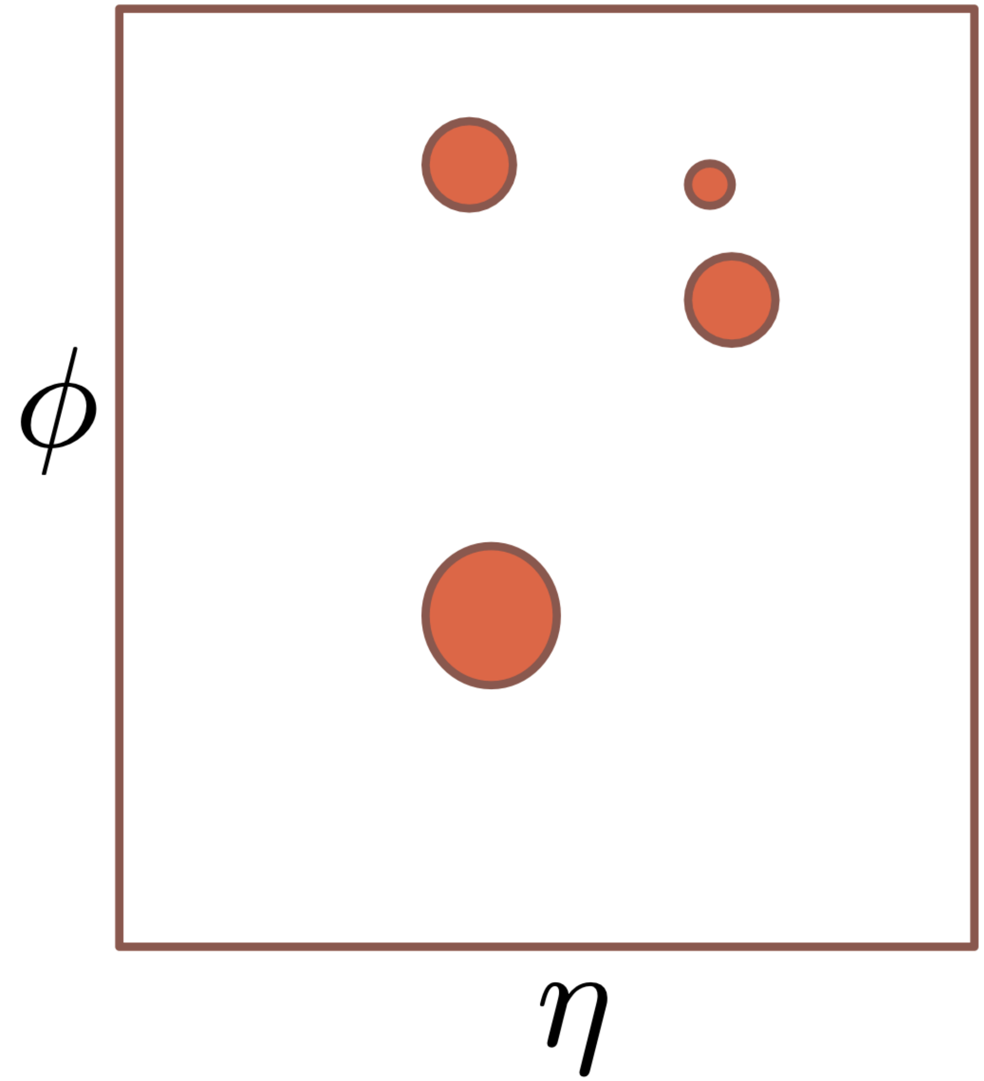
- **Fixed** cone with **progressive removal** (FC-PR) (PyJet, CellJet, GetJet)
- **Iterative** cone with **progressive removal** (IC-PR) (CMS iterative cone)
- **Iterative** cone with **split-merge** (IC-SM) (JetClu, ATLAS cone)
- **IC-SM** with **mid-points** (IC_{mp}-SM) (CDF MidPoint, D0 Run II)
- **IC_{mp}** with **split-drop** (IC_{mp}-SD) (PxCone)
- **Seedless** cone with **split-merge** (SC-SM) (SISCone)



Recombination algorithm : basic intuitions

Start with a set of 4-vectors p^i

Represent the particles in η , ϕ plane



$$\eta = -\frac{1}{2} ln\left(tan \ \frac{\theta}{2}\right)$$

$$\phi = arctan \ \left(\frac{p_y}{p_x}\right)$$

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Calculate the pairwise distance :

$$d_{ij} = \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2}$$

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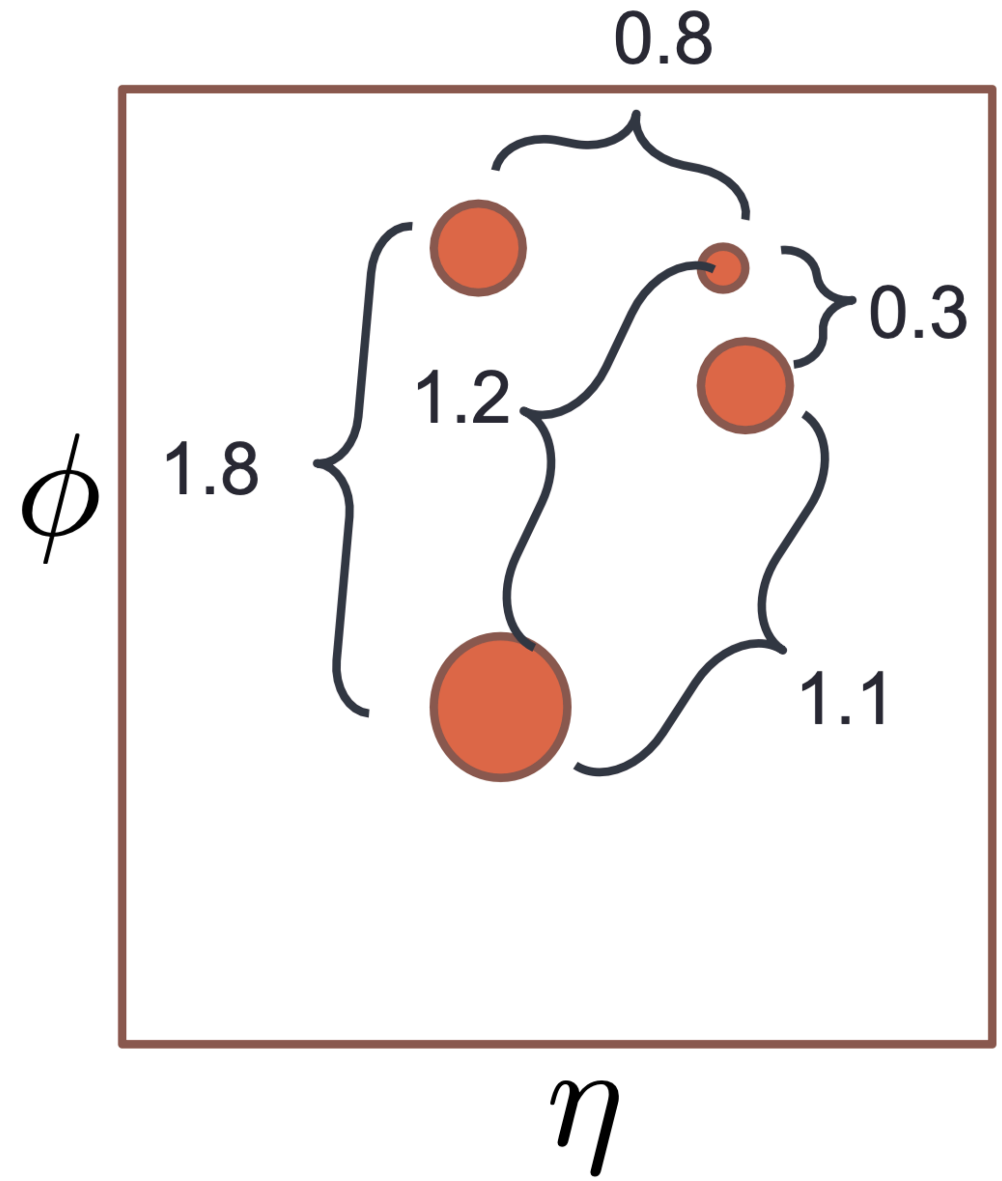

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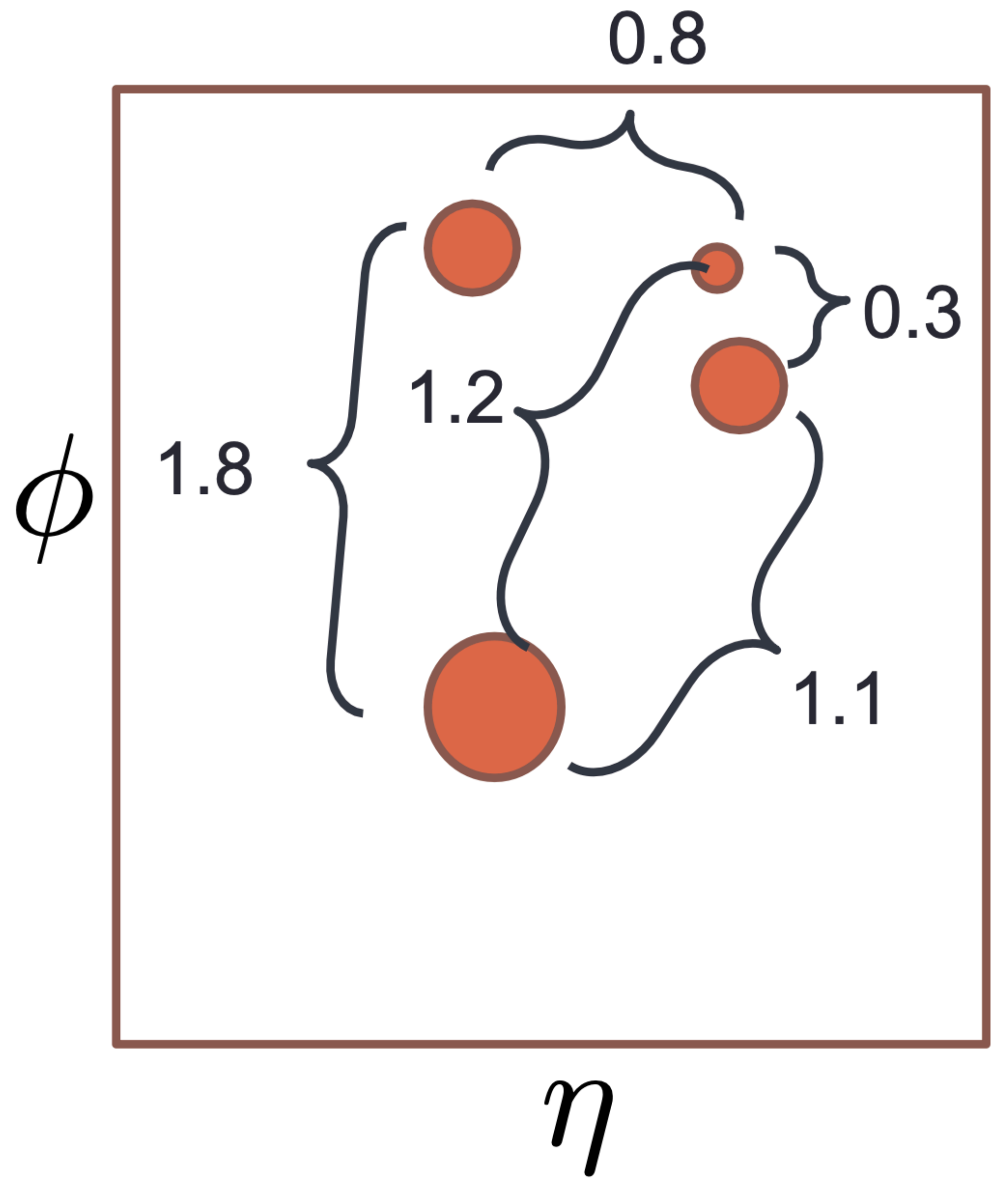
Merge the two closest particle.
i.e. replace the pair by a combined representation.



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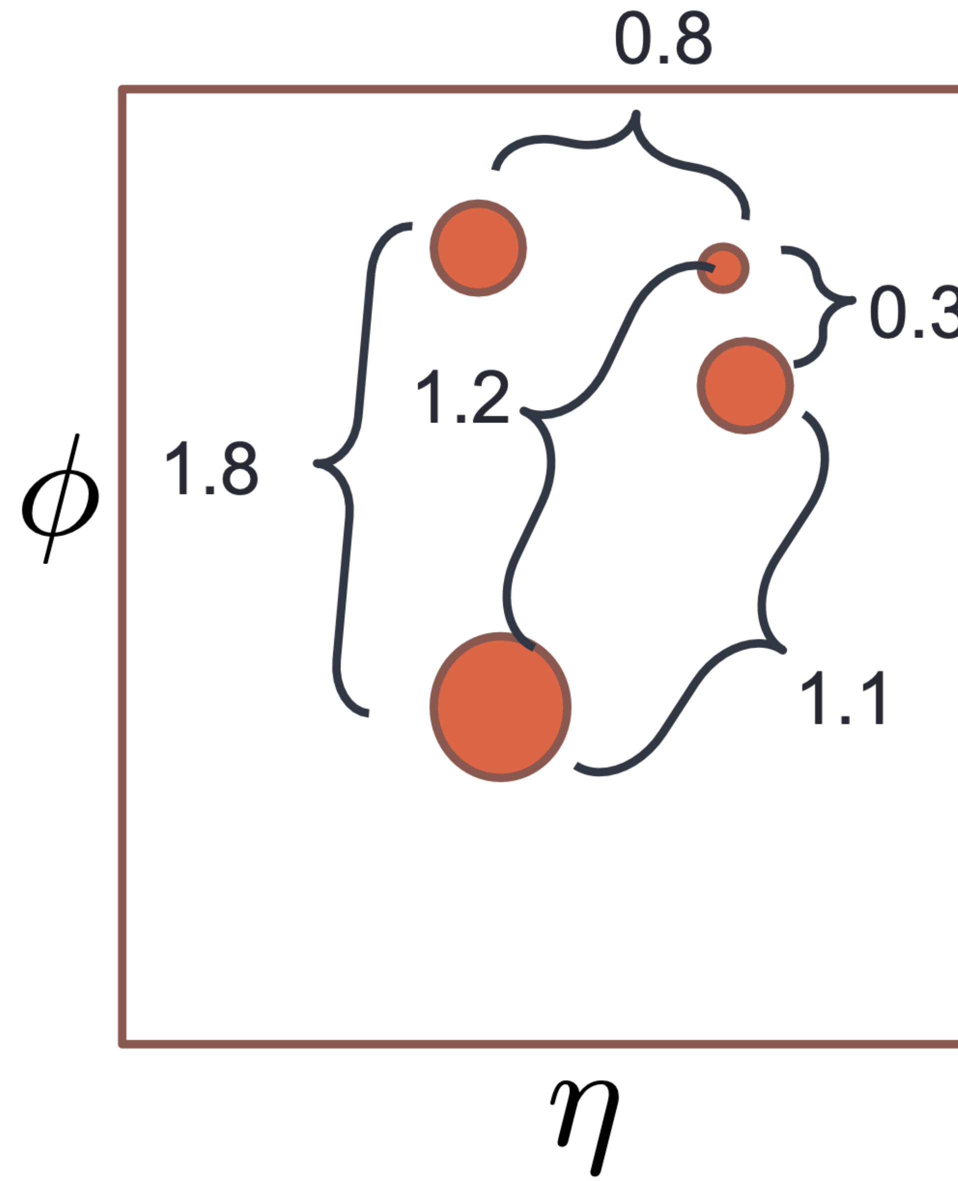
Continue until no two particles are closer than R



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e^+e^- k_T algorithm

Durham: $$y_{ij} = \frac{2\min(E_i^2, E_j^2)\left(1-\cos\theta_{ij}\right)}{Q^2},$$

Geneva: $$y_{ij} = \frac{8}{9} \cdot \frac{2E_iE_j\left(1-\cos\theta_{ij}\right)}{\left(E_i+E_j\right)^2},$$

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https://arxiv.org/pdf/1011.6247

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1. Define a resolution parameter y_{cut}
2. For every pair (p_k, p_l) of final-state particles compute the corresponding resolution variable y_{kl}.
3. If y_{ij} is the smallest value of y_{kl} computed above and $y_{ij} < y_{cut}$ then combine (p_i, p_j) into a single jet ('pseudo-particle') with momentum p_{ij} according to a recombination prescription.
4. Repeat until all pairs of objects (particles and/or pseudo-particles) have $y_{kl} > y_{cut}$.
5. Objects with $E \geq E_{min}$ are called jets.

Generalized kT (recombination) algorithm

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1. Take the particles in the event as our initial list of objects.
2. From the list of objects, build two sets of distances: an *inter-particle distance*

$$d_{ij} = \min(p_{t,i}^{2p}, p_{t,j}^{2p})\Delta R_{ij}^2, \tag{3.1}$$

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where p is a free parameter and ΔR_{ij} is the geometric distance in the rapidity-azimuthal angle plane (Eq. (2.34)), and a *beam distance*

$$d_{iB} = p_{t,i}^{2p} R^2, \tag{3.2}$$

with R a free parameter usually called the *jet radius*.

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3. Iteratively find the smallest distance among all the d_{ij} and d_{iB}
   - If the smallest distance is a d_{ij} then objects i and j are removed from the list and recombined into a new object k (using the recombination scheme) which is itself added to the list.
   - If the smallest is a d_{iB}, object i is called a *jet* and removed from the list.

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Go back to step-2

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p = 1 k_t algorithm

S. Catani, Y. Dokshitzer, M. Seymour and B. Webber, Nucl. Phys. B406 (1993) 187
S.D. Ellis and D.E. Soper, Phys. Rev. D48 (1993) 3160

p = 0 Cambridge/Aachen algorithm

Y. Dokshitzer, G. Leder, S.Moretti and B. Webber, JHEP 08 (1997) 001
M.Wobisch and T.Wengler, hep-ph/9907280

p = -1 **anti-k_t algorithm**

MC, G. Salam and G. Soyez, arXiv:0802.1189

NB: in anti-kt pairs with a **hard** particle will cluster first: if no other hard particles are close by, the algorithm will give **perfect cones**

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p > 0

New **soft** particle ($p_t \to 0$) means that d → 0 ⇒ clustered first, no effect on jets

New **collinear** particle ($\Delta y^2 + \Delta \Phi^2 \to 0$) means that d → 0 ⇒ clustered first, no effect on jets

p = 0

New **soft** particle ($p_t \to 0$) can be new jet of zero momentum ⇒ no effect on hard jets

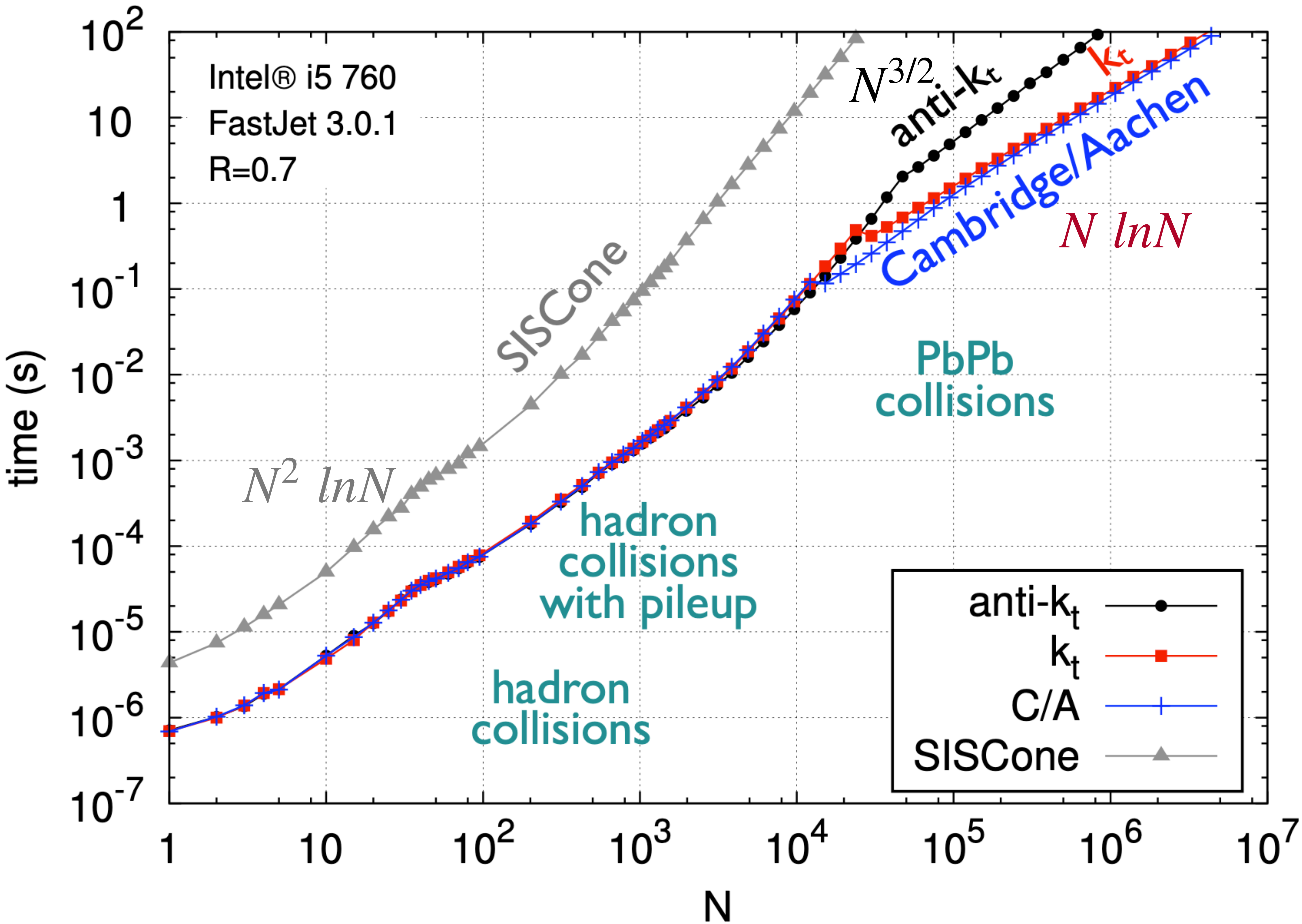
New **collinear** particle ($\Delta y^2 + \Delta \Phi^2 \to 0$) means that d → 0 ⇒ clustered first, no effect on jets

p < 0

New **soft** particle ($p_t \to 0$) means d → ∞ ⇒ clustered last or new zero-jet, no effect on hard jets

New **collinear** particle ($\Delta y^2 + \Delta \Phi^2 \to 0$) means that d → 0 ⇒ clustered first, no effect on jets

How about reconstruction time?



Let’s see how the clustering grows

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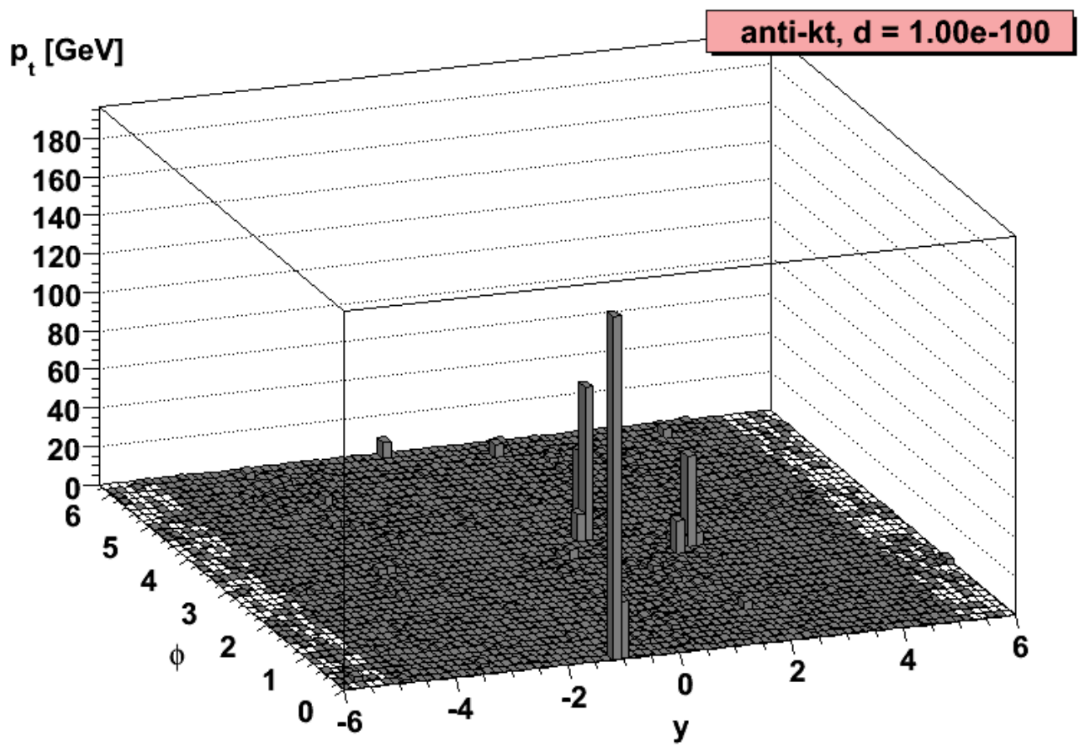

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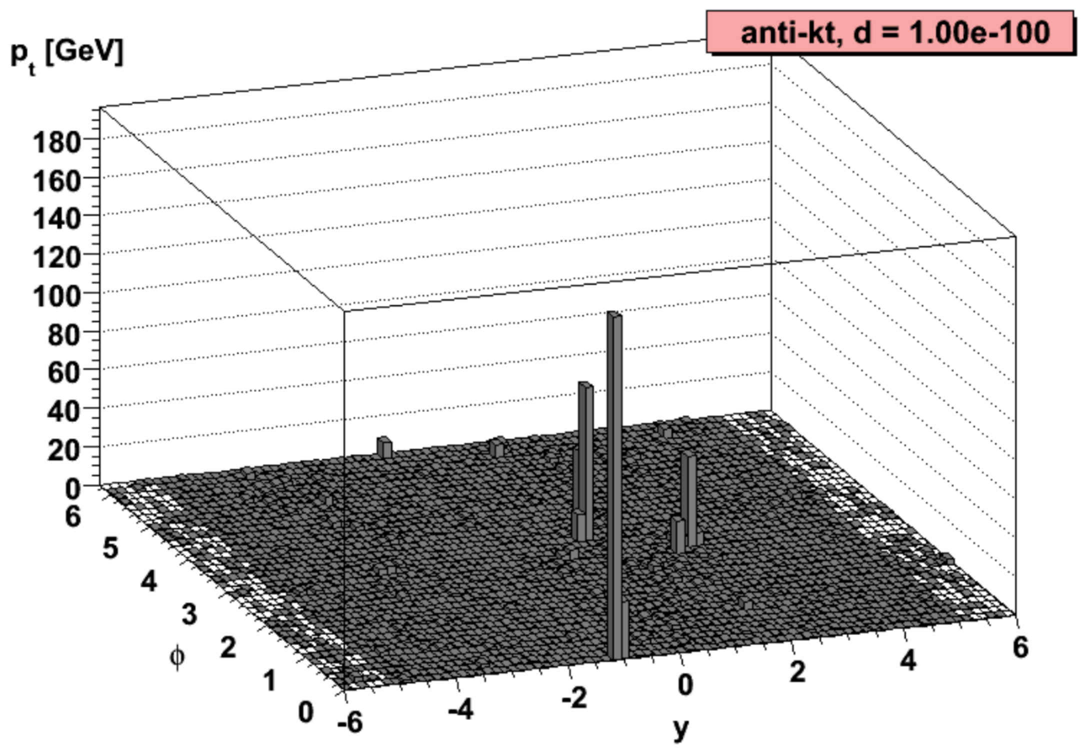

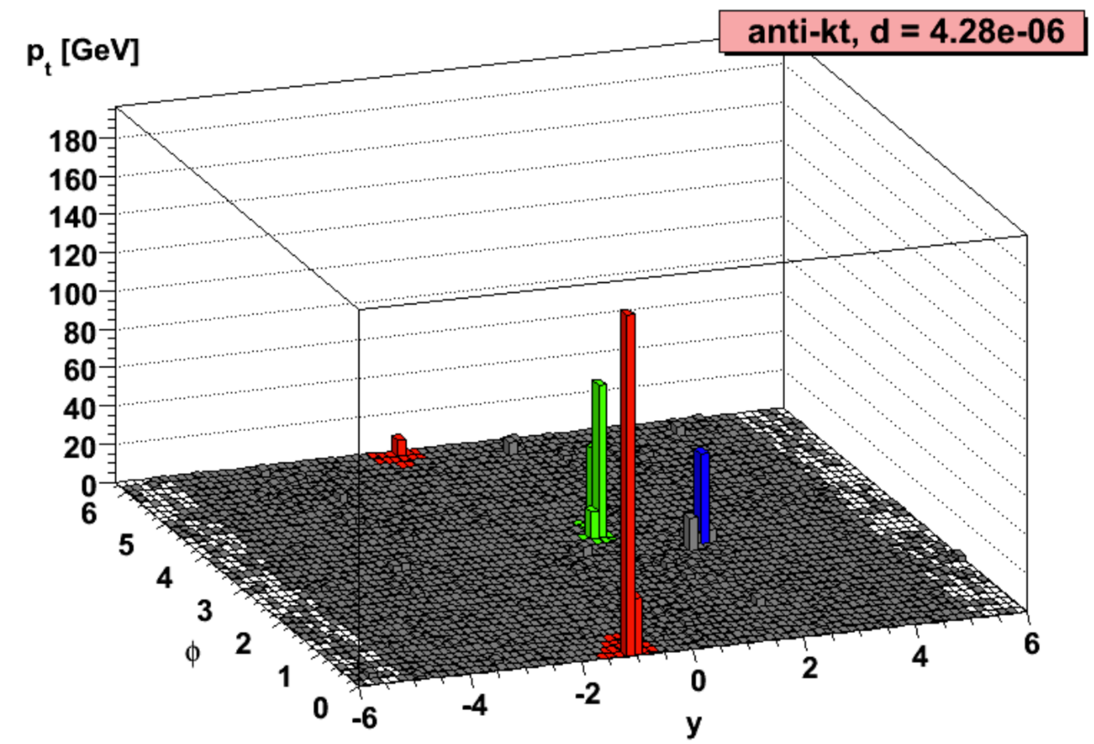

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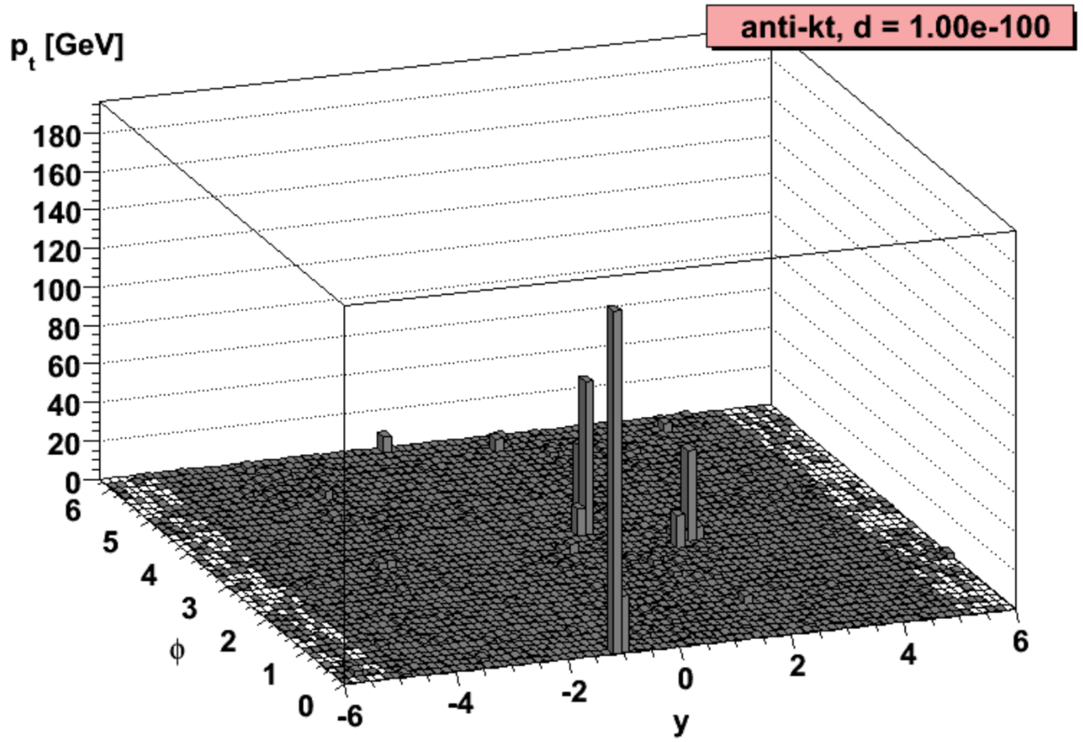

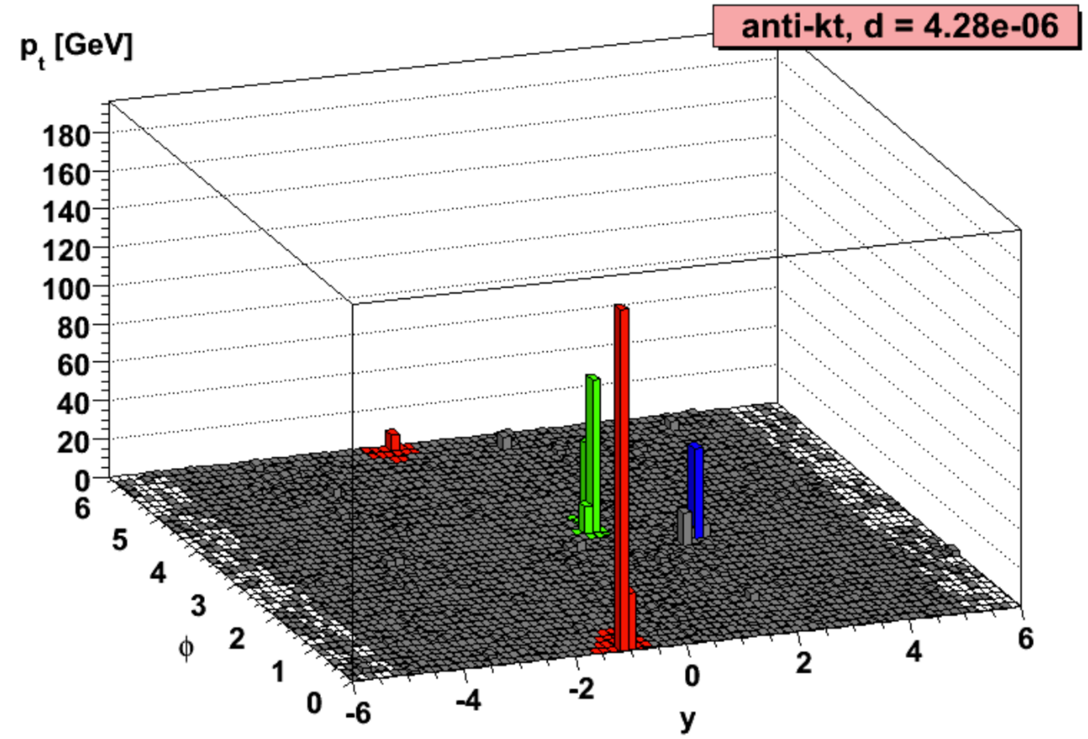

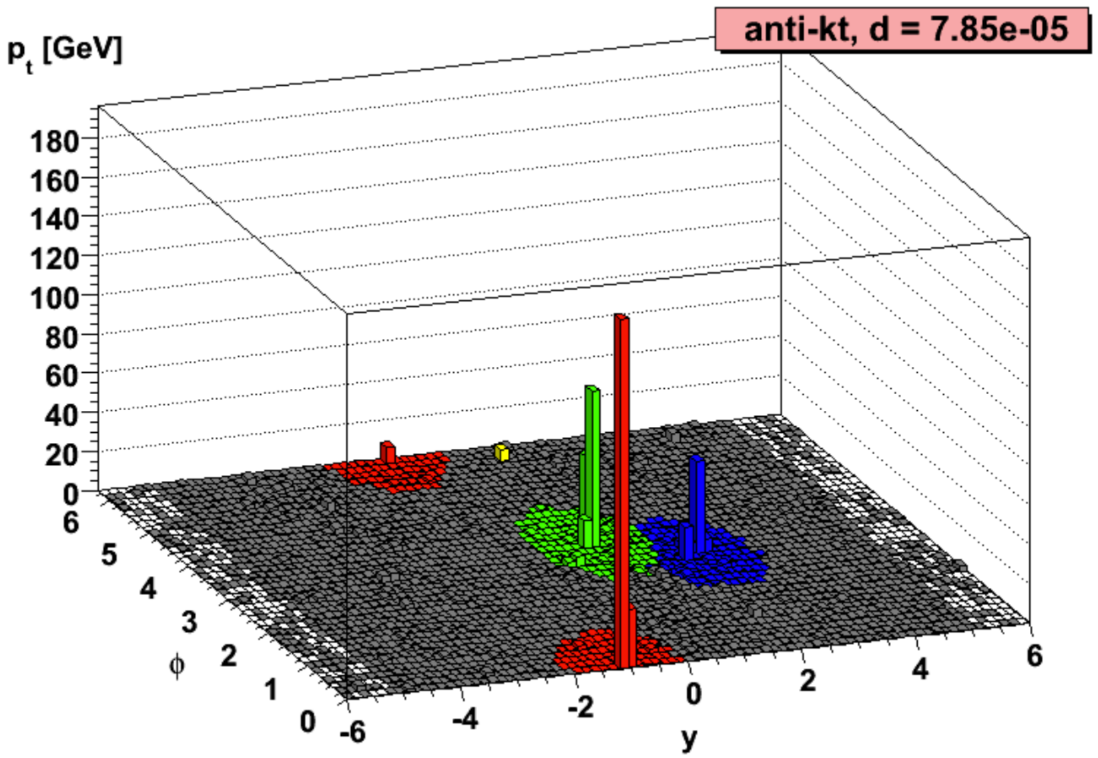

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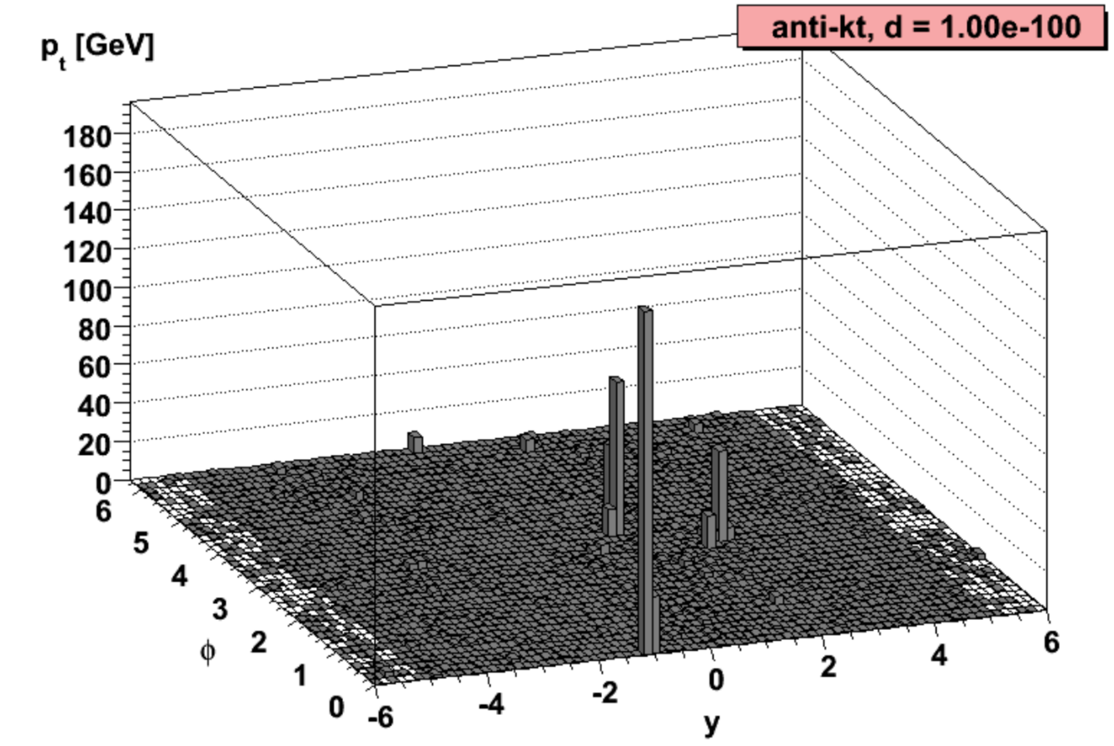
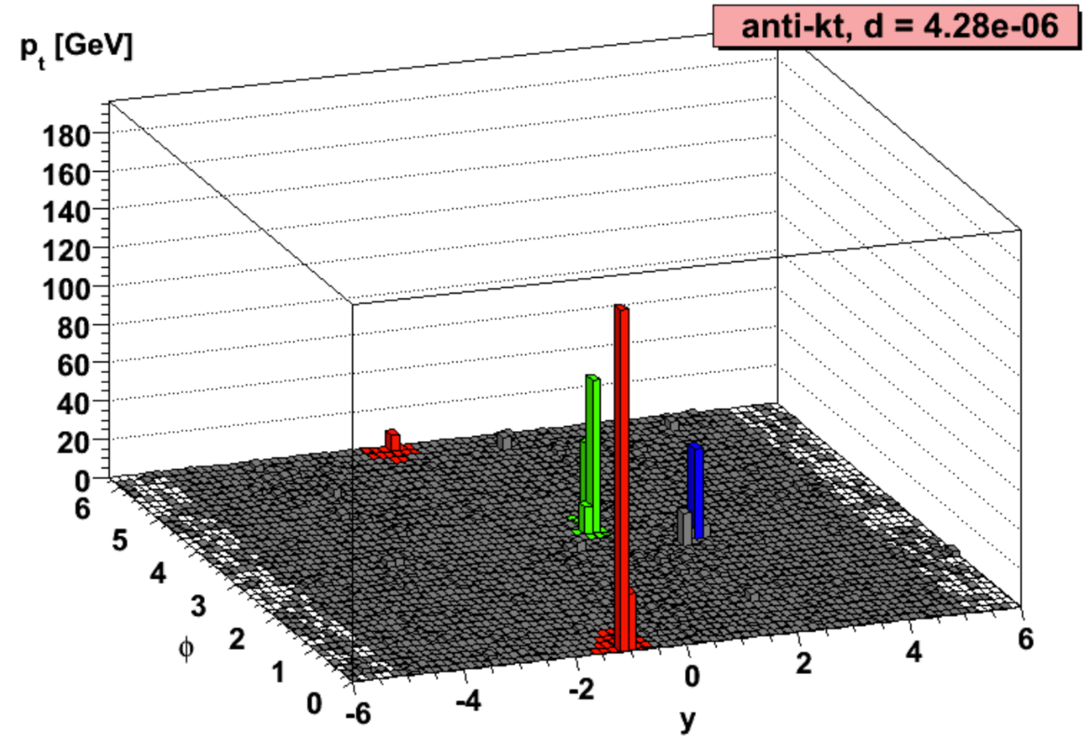
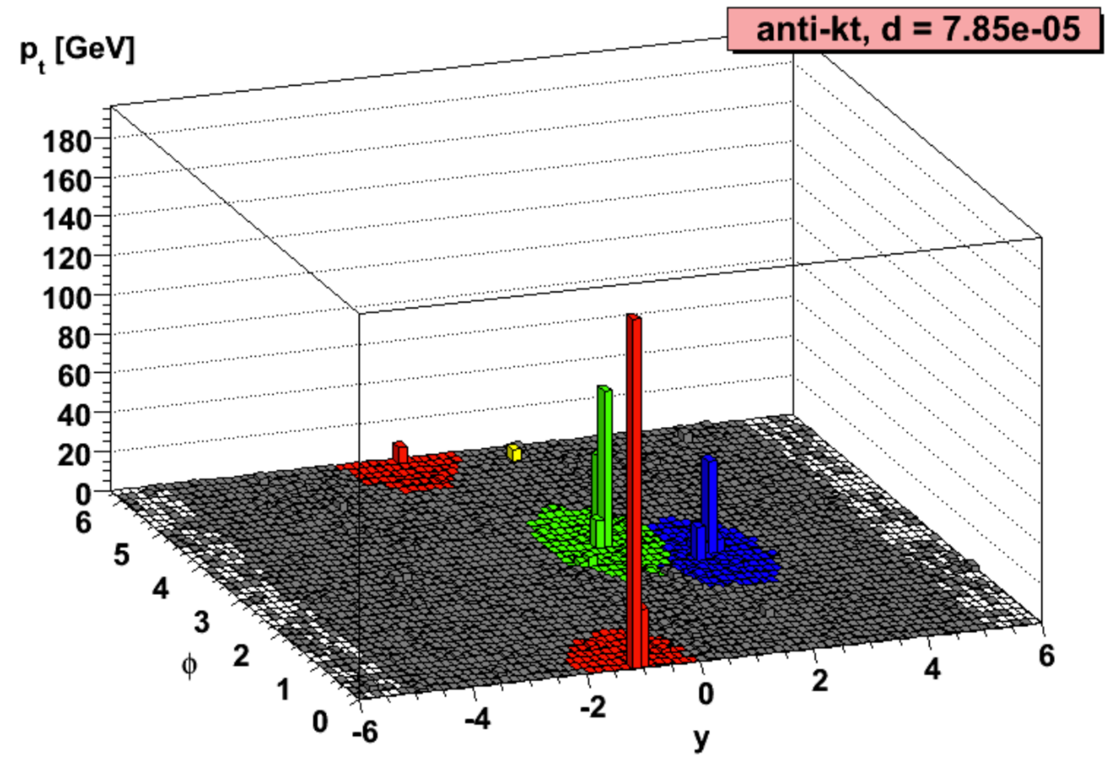
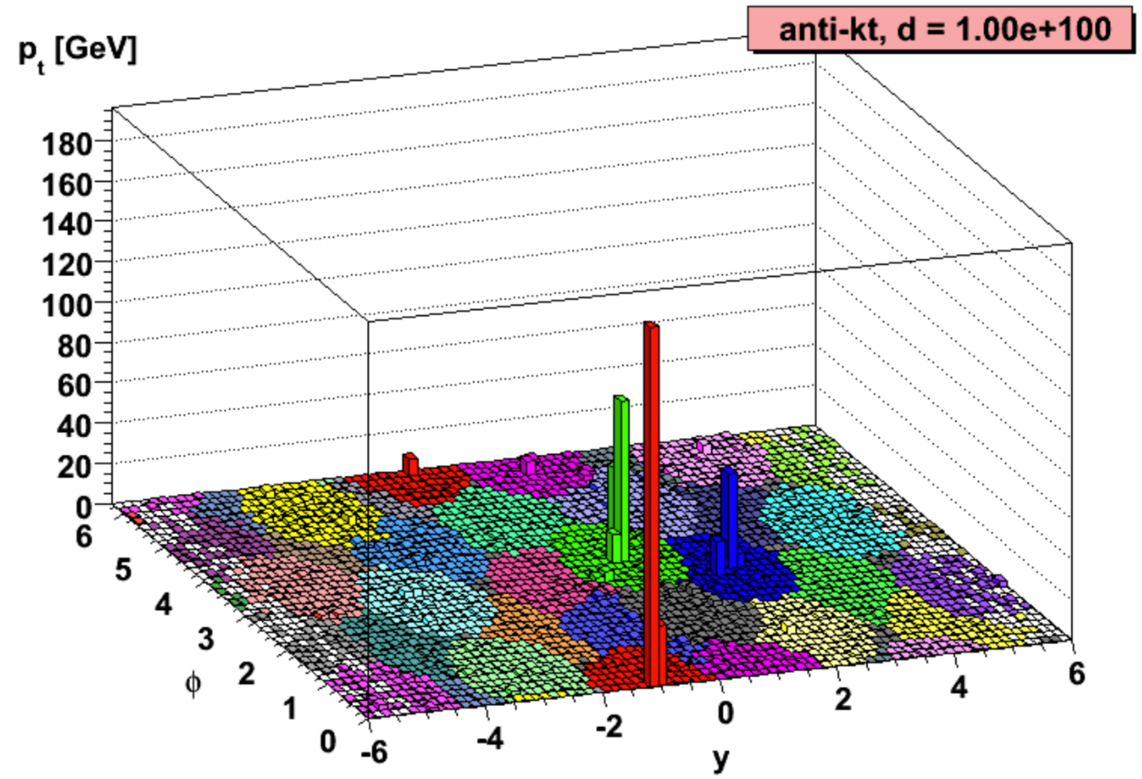
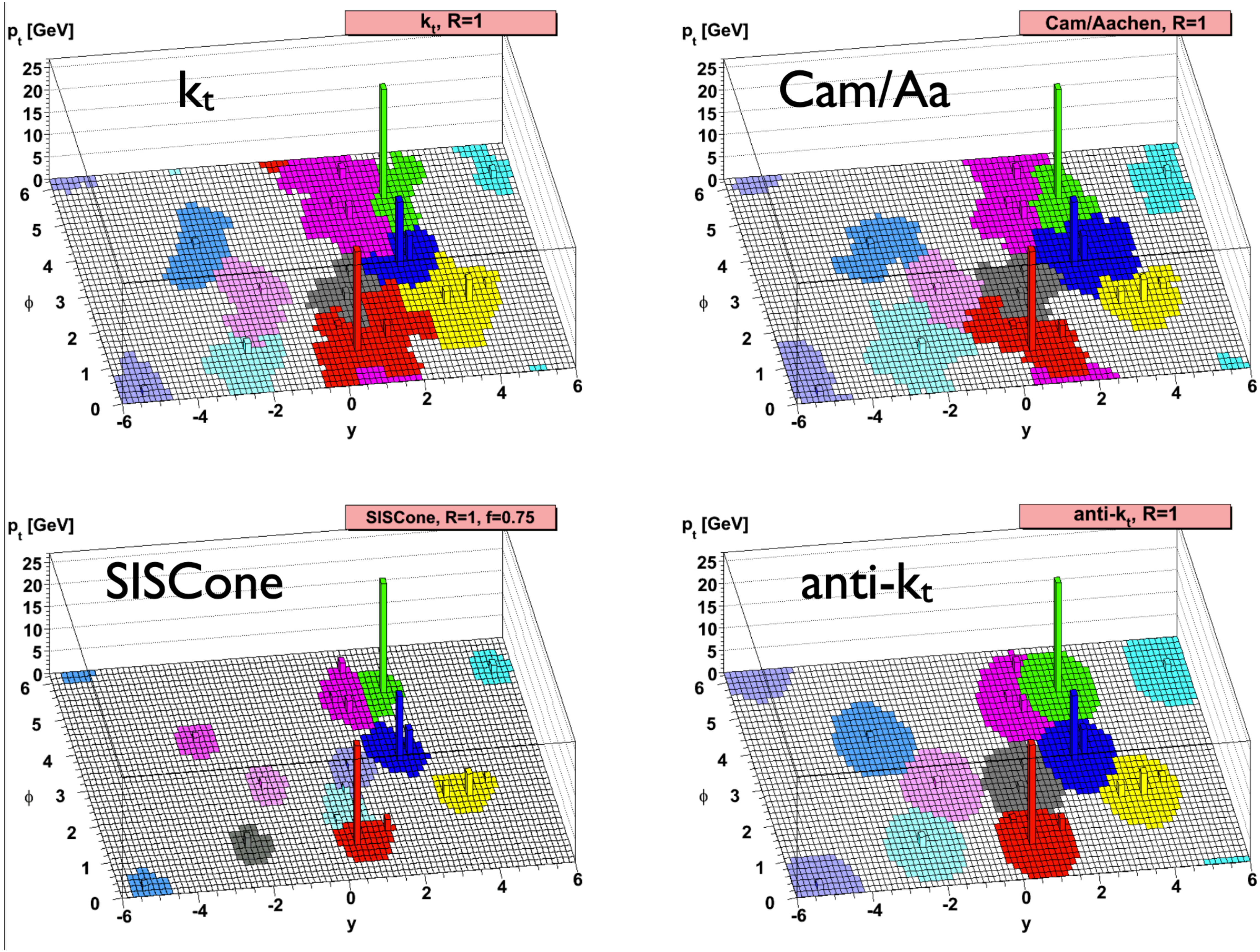








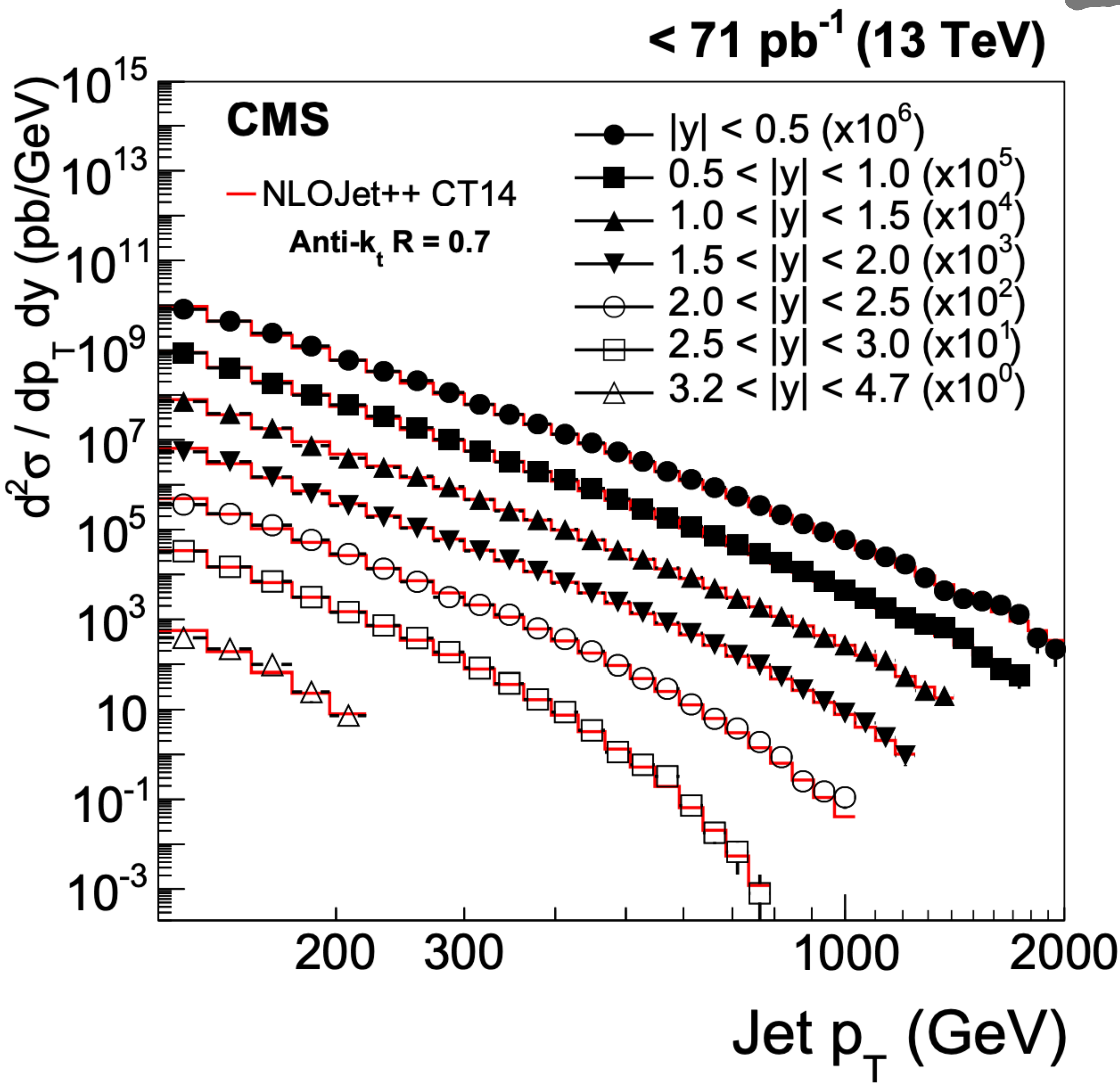
For a same set of particles, different algorithm

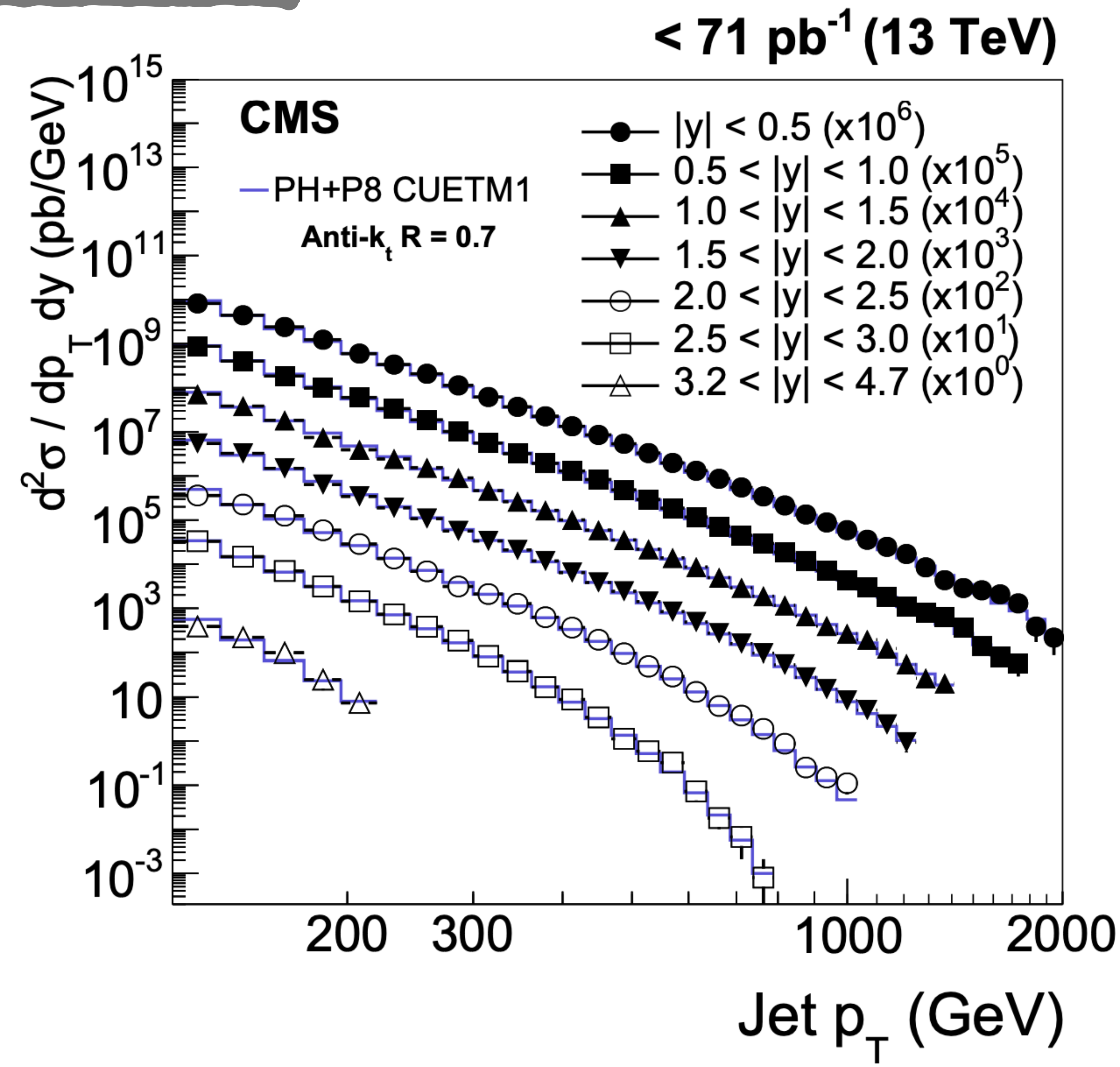


So what we can measure?

$$\boxed{\frac{d^2\sigma}{dp_T\,dy} = \frac{1}{\mathcal{L}}\frac{N_{\rm jets}^{\rm eff}}{\Delta p_T\,\Delta y}}$$

arXiv: 1605.04436





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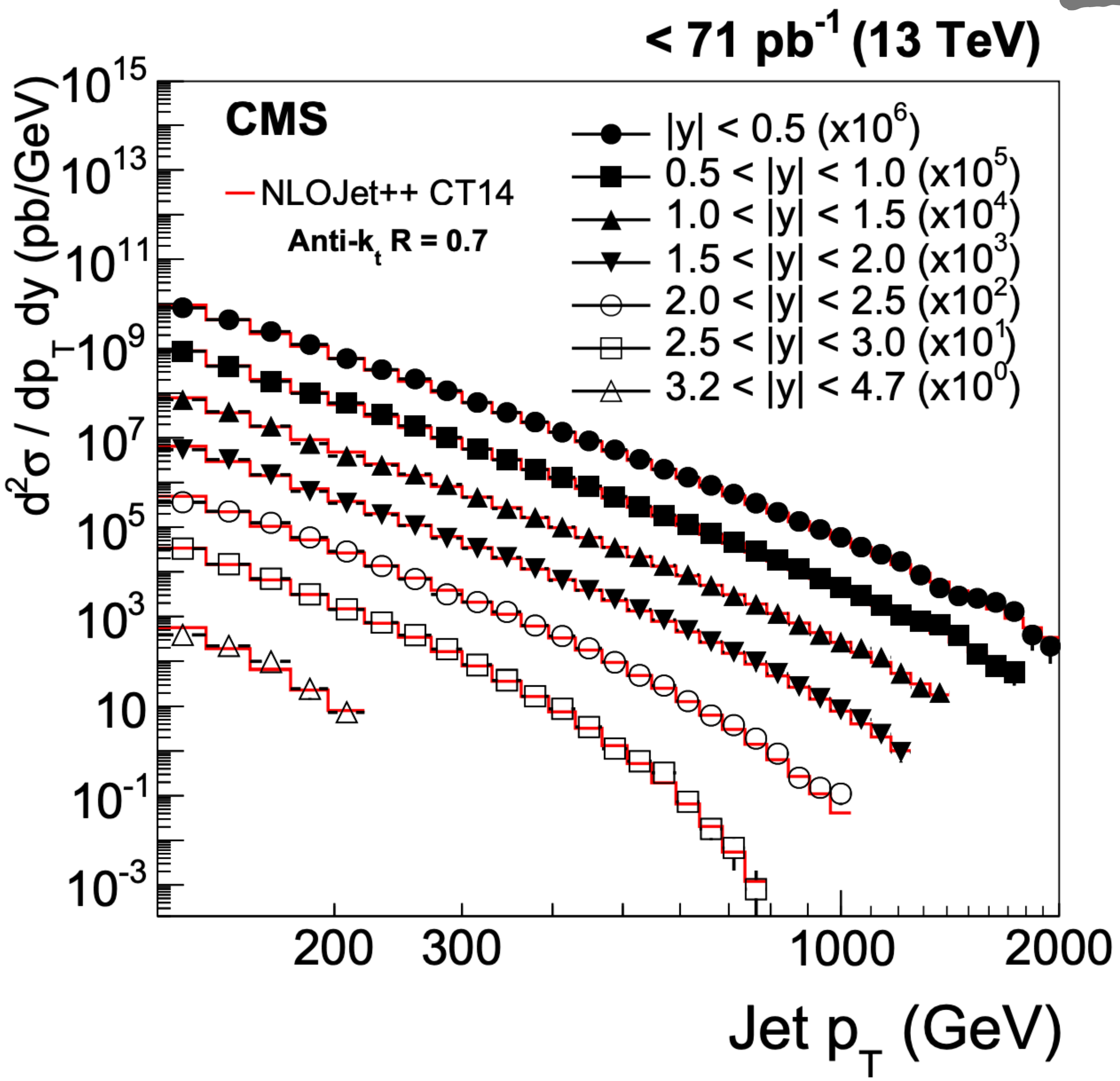
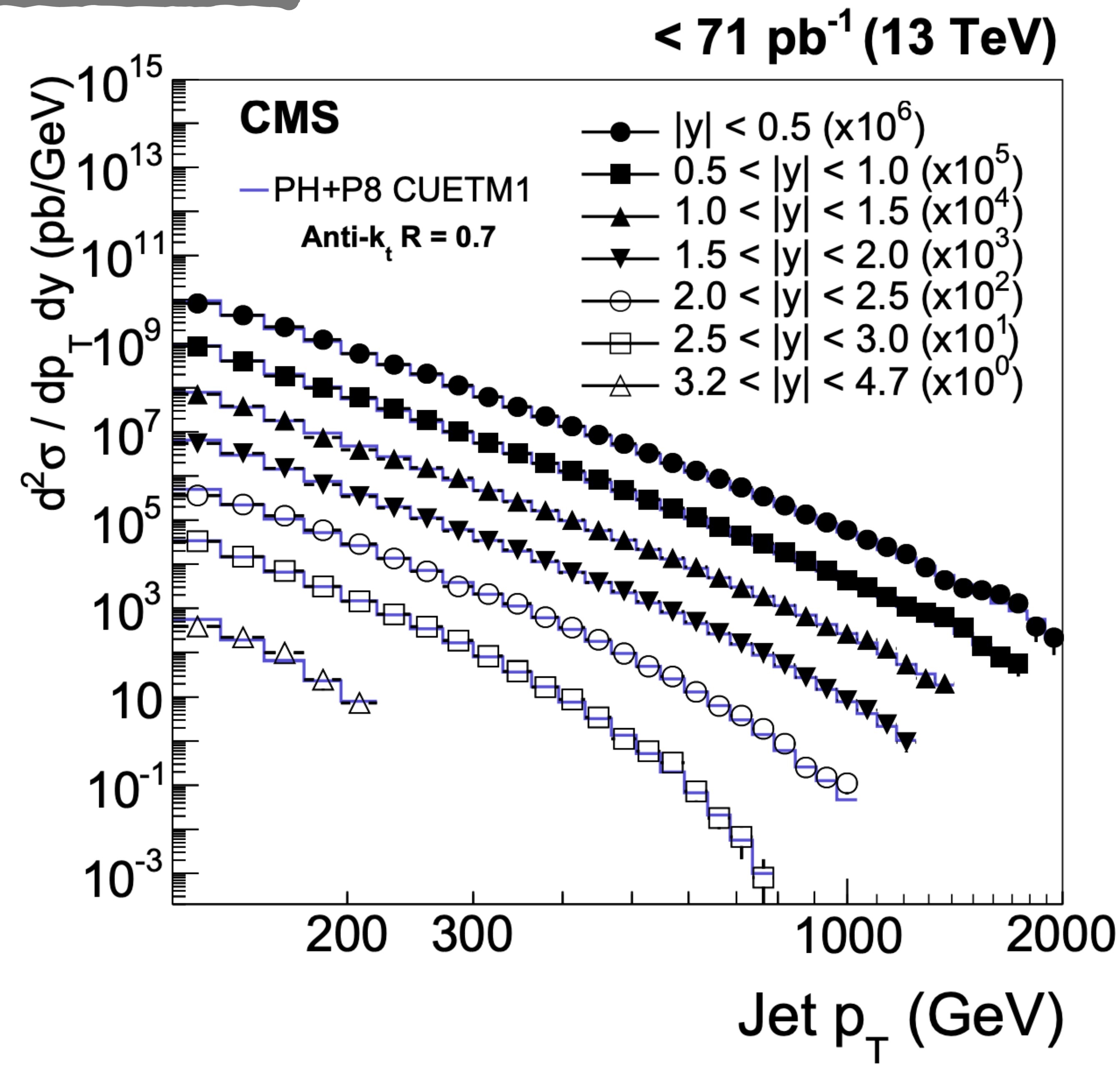
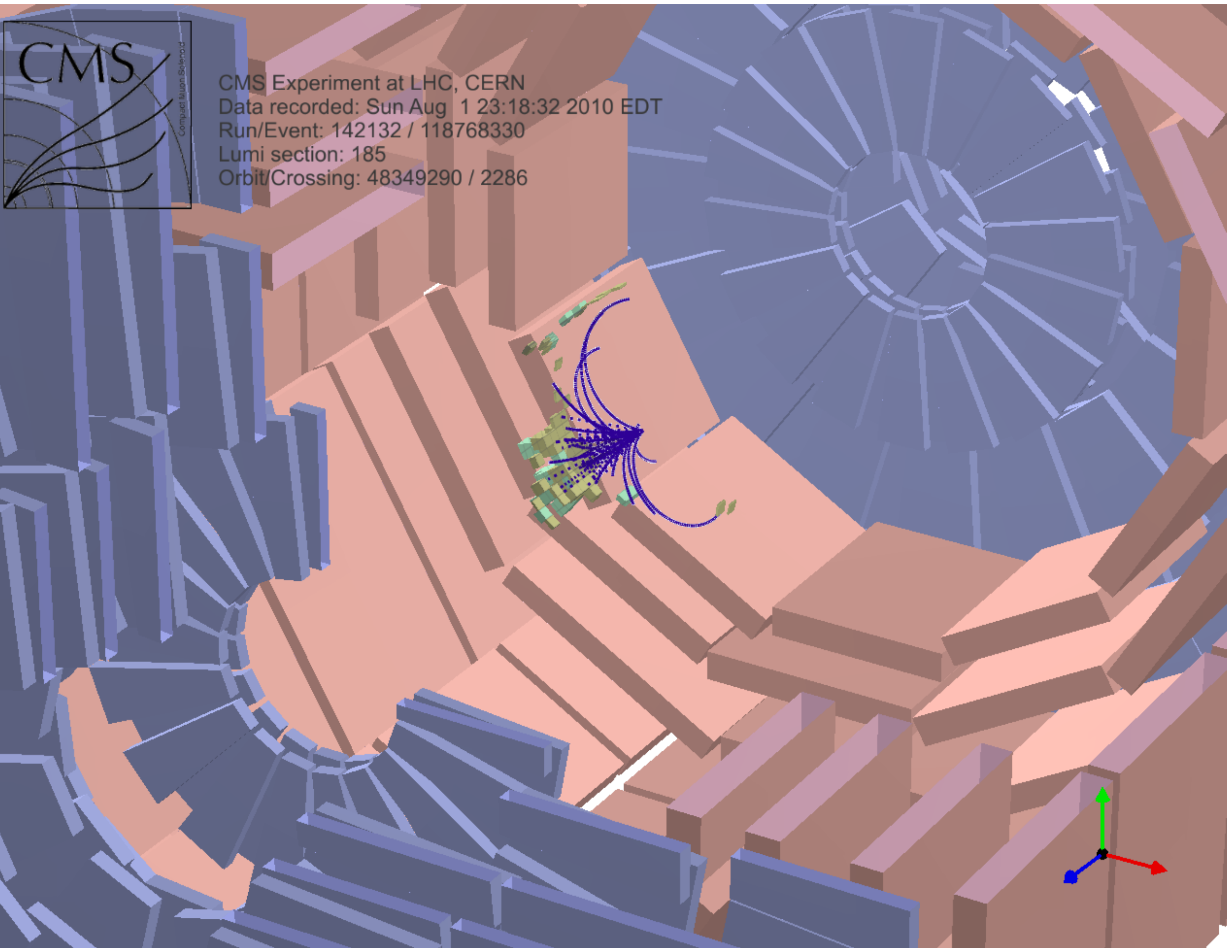
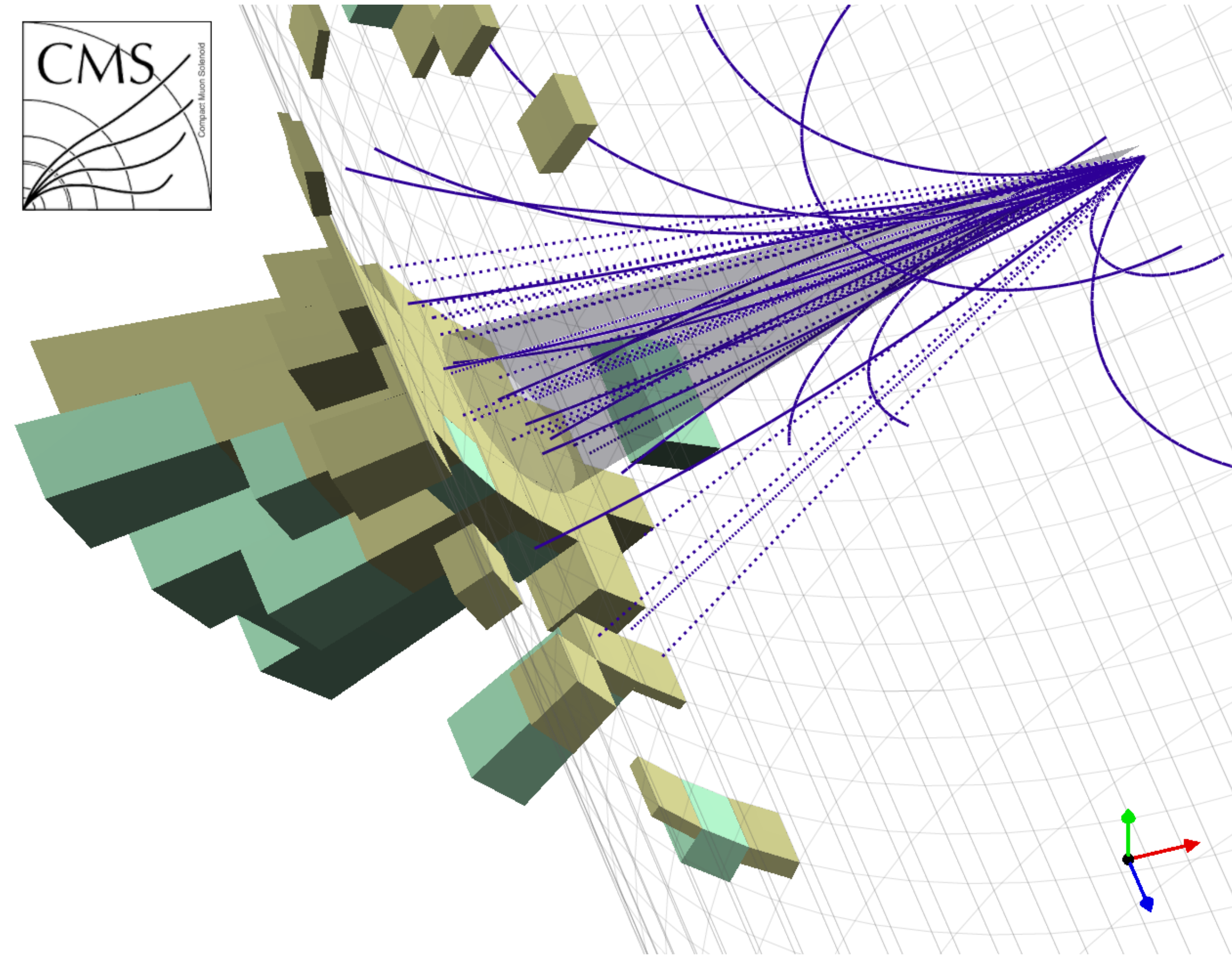


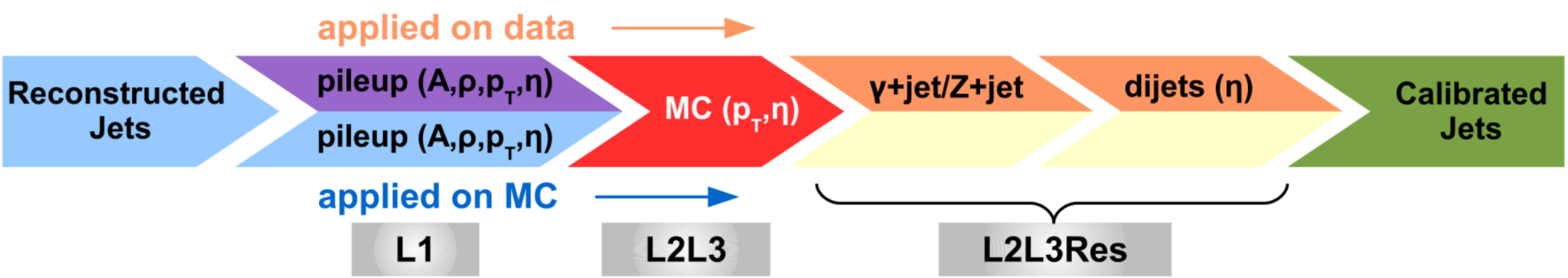


What does it to take to perform the simplest of measurements ?

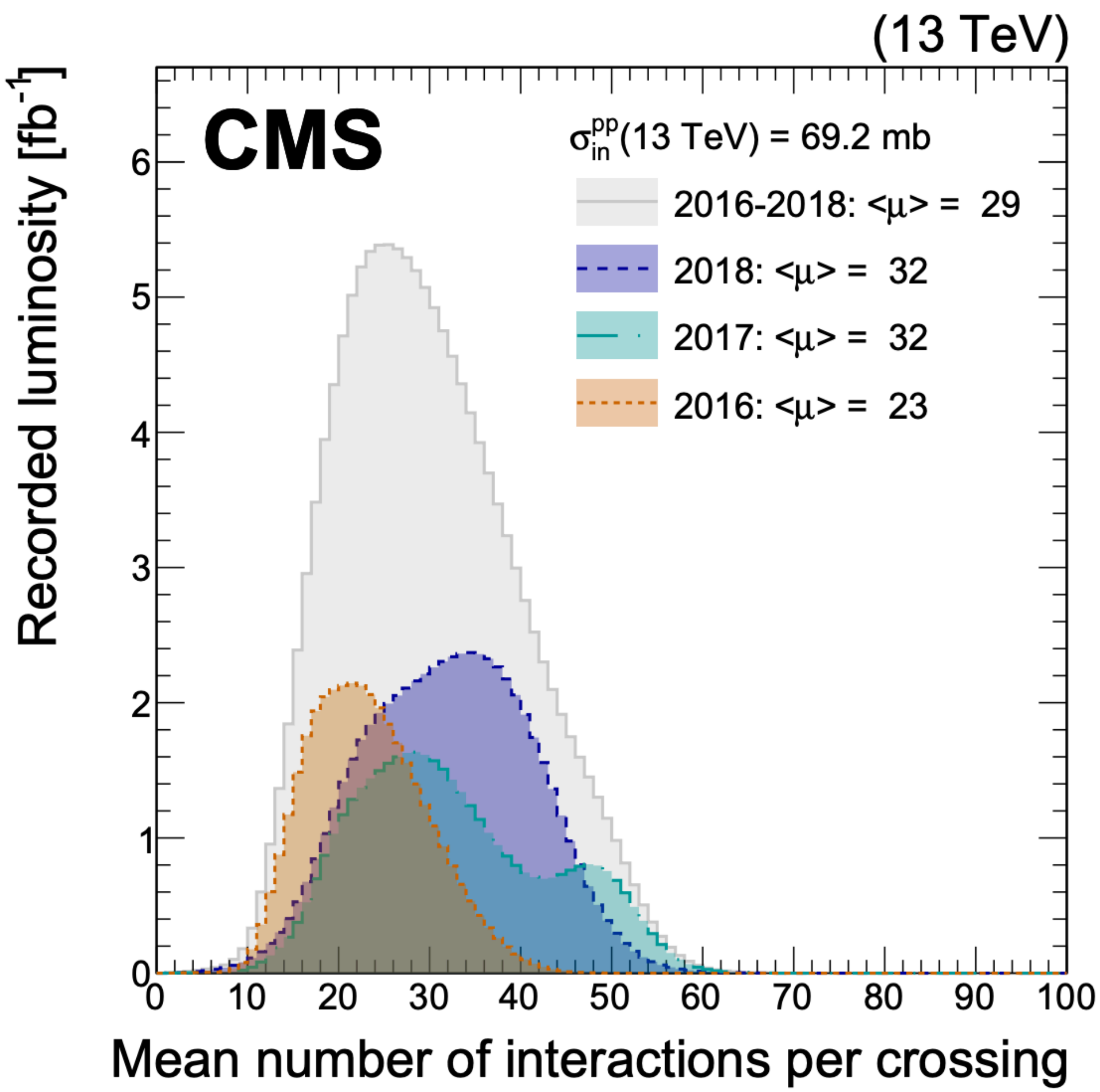
Where we start from

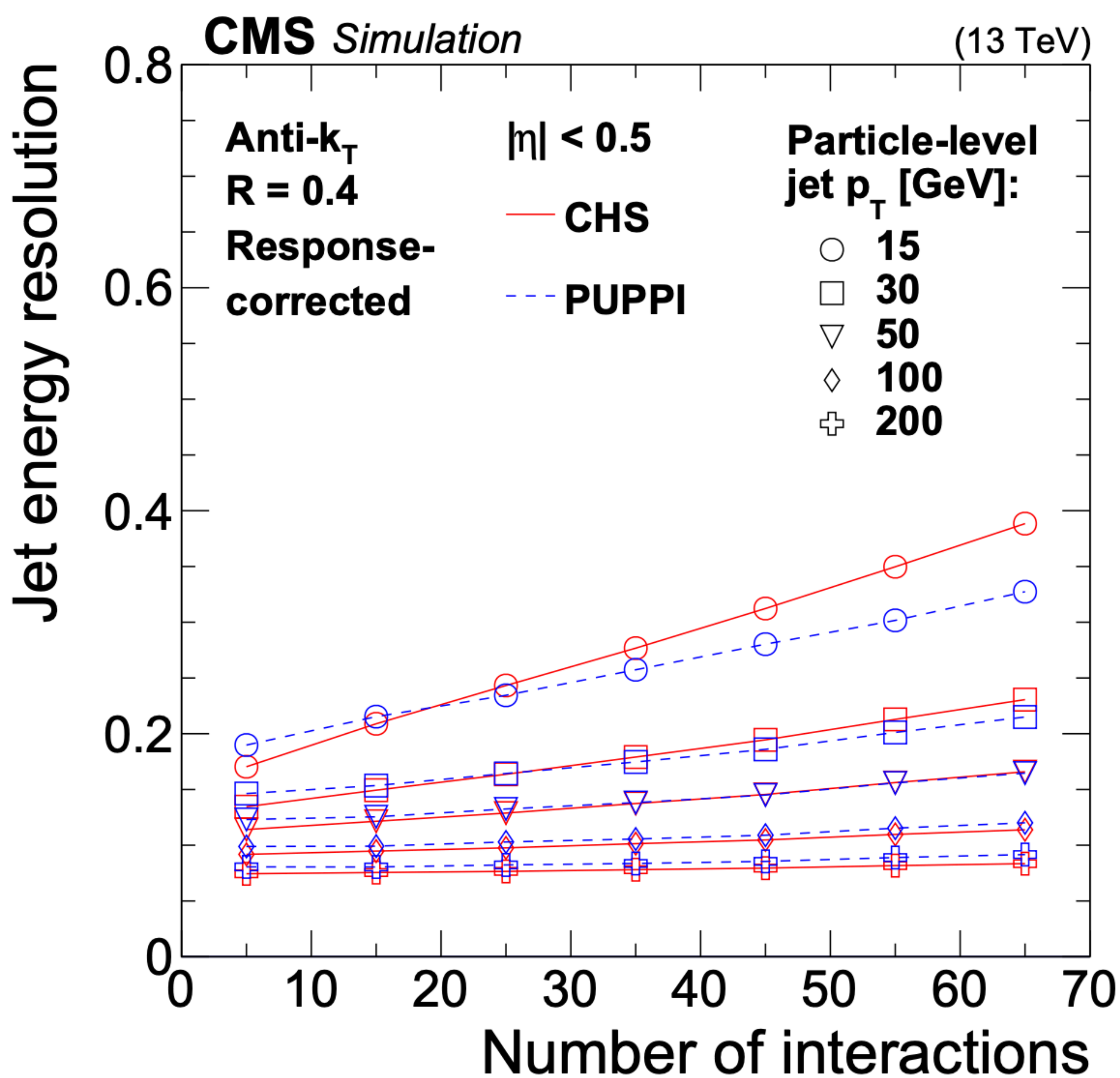






Pileup correction





https://arxiv.org/pdf/2003.00503

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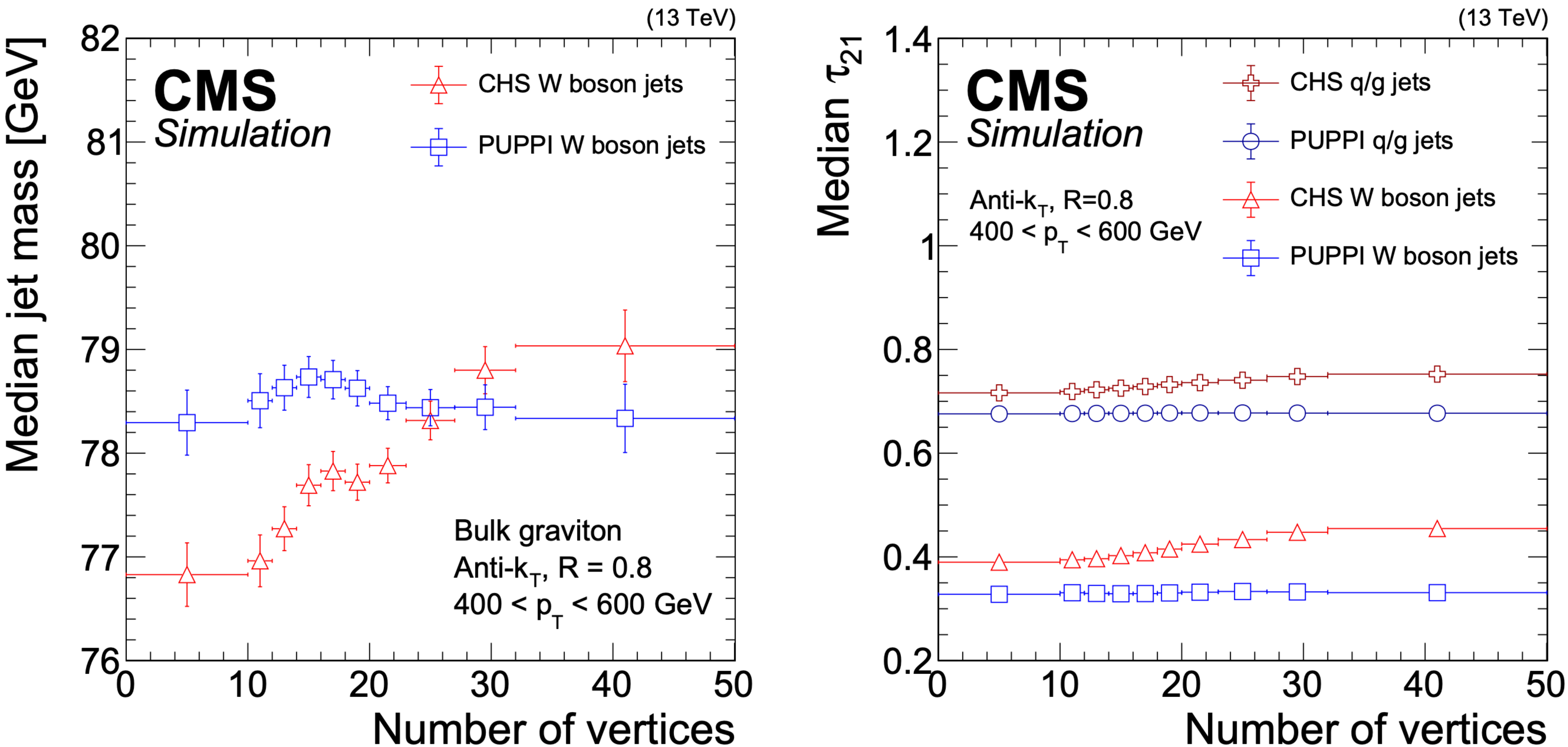

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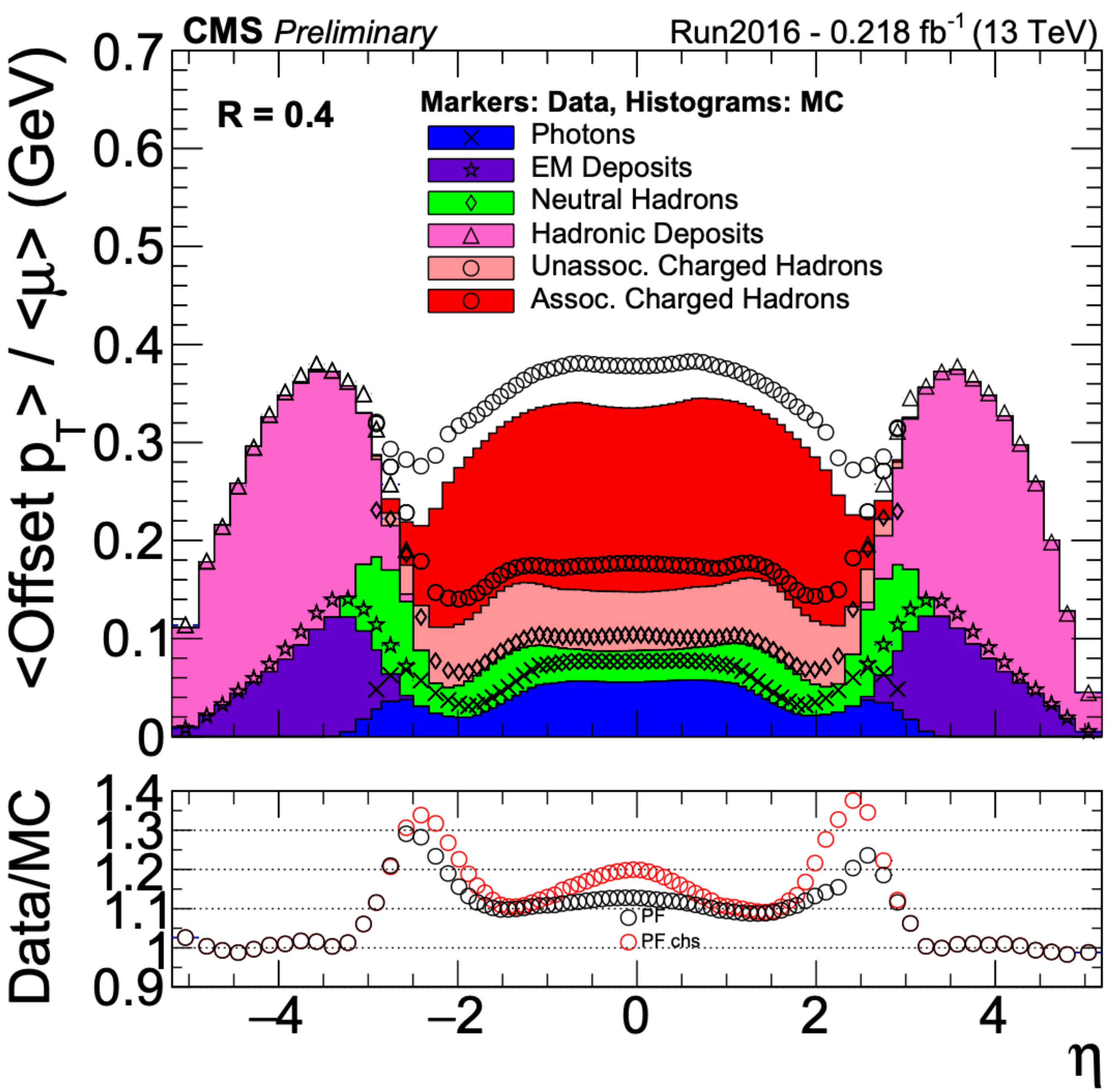
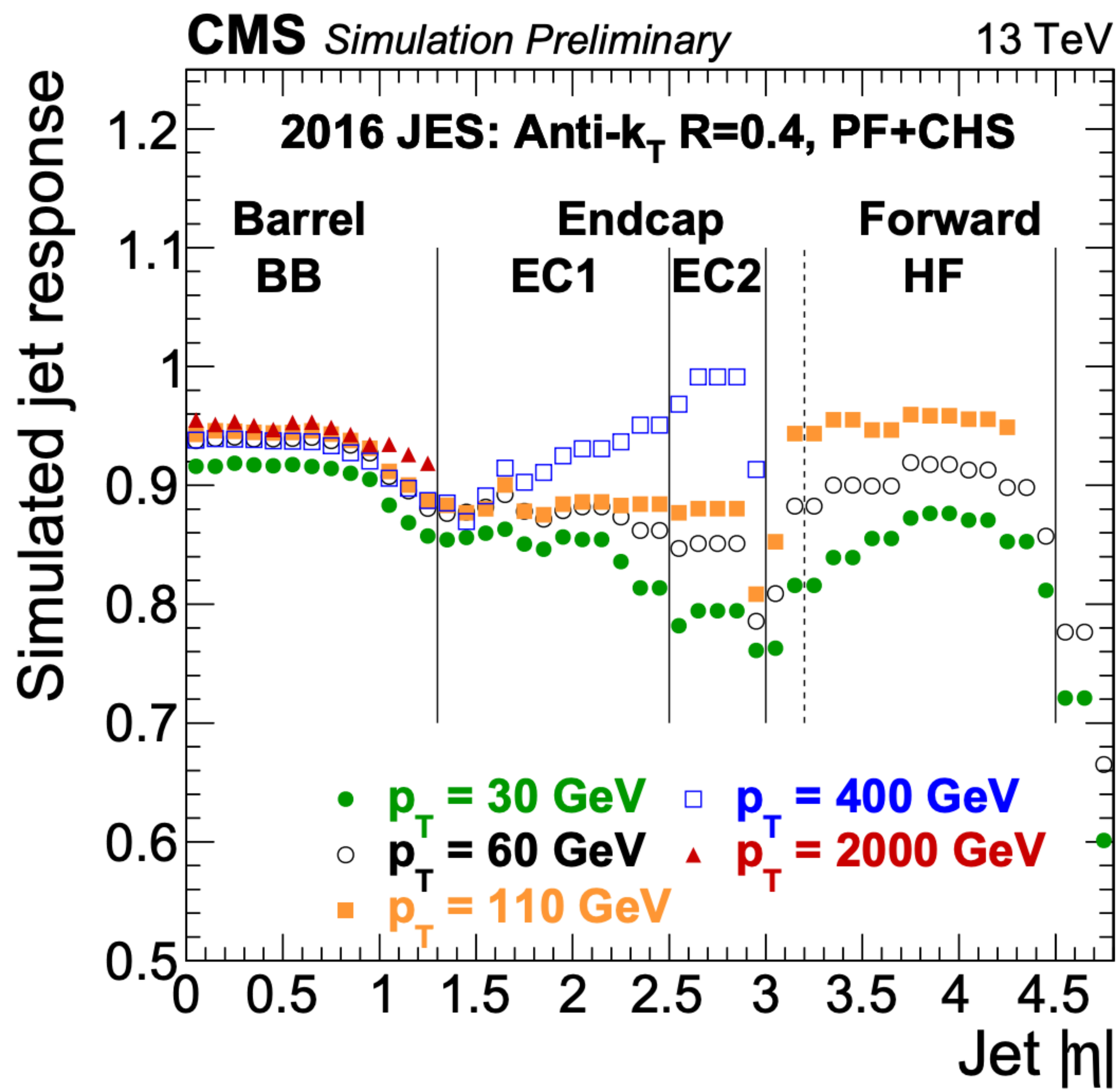
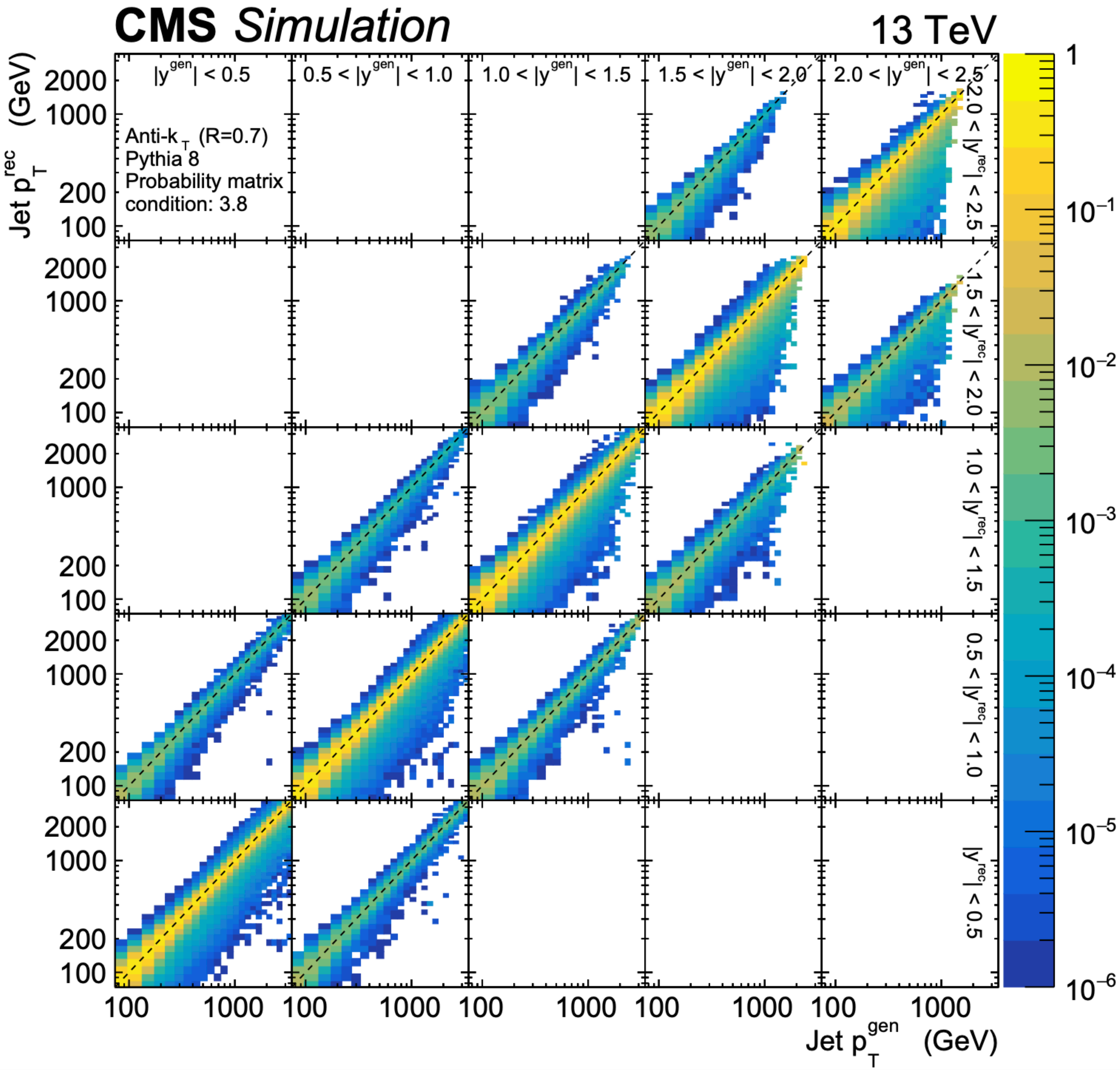
pT/eta dependent JEC



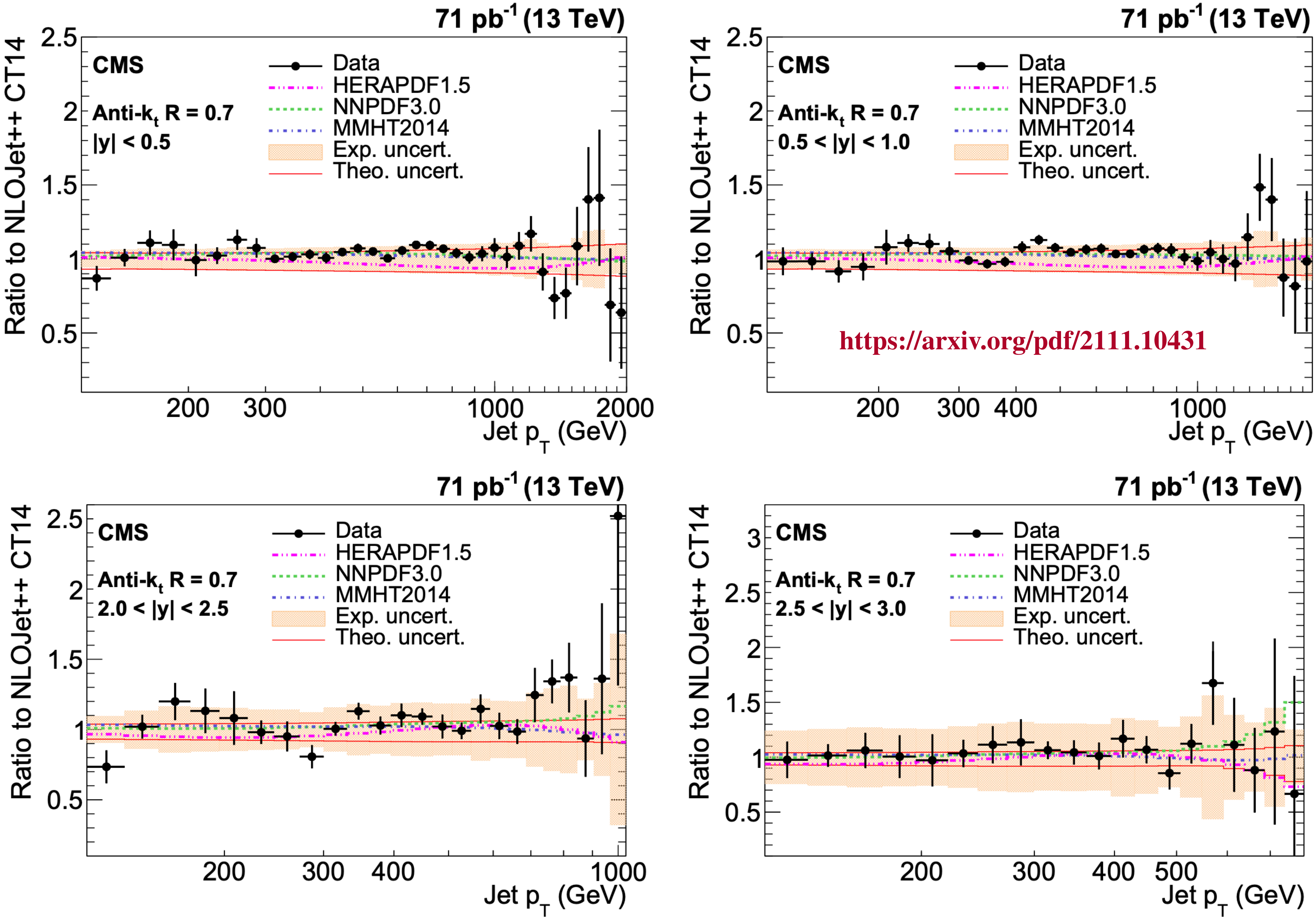


https://inspirehep.net/files/0ac1e2e157965e3491f289c6b86f18ce

Unfolding a pT spectra

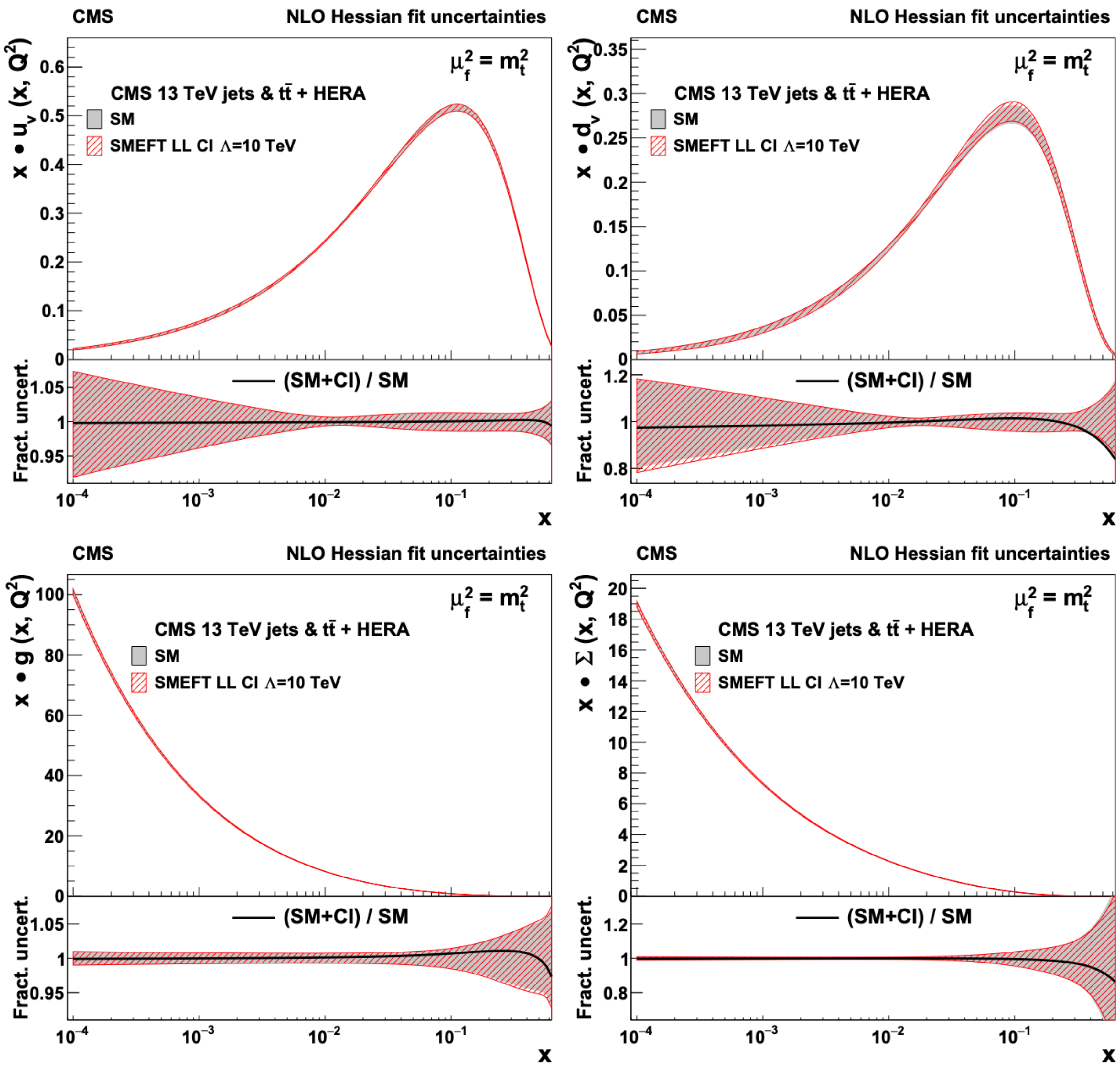


How reliable are these measurements?



PDF extraction from the measurement

https://arxiv.org/pdf/2111.10431



α_S extraction from the measurement

https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsCombined https://arxiv.org/pdf/2111.10431