Jet Physics and Machine Learning: Lecture 1 :What & Why of Jets

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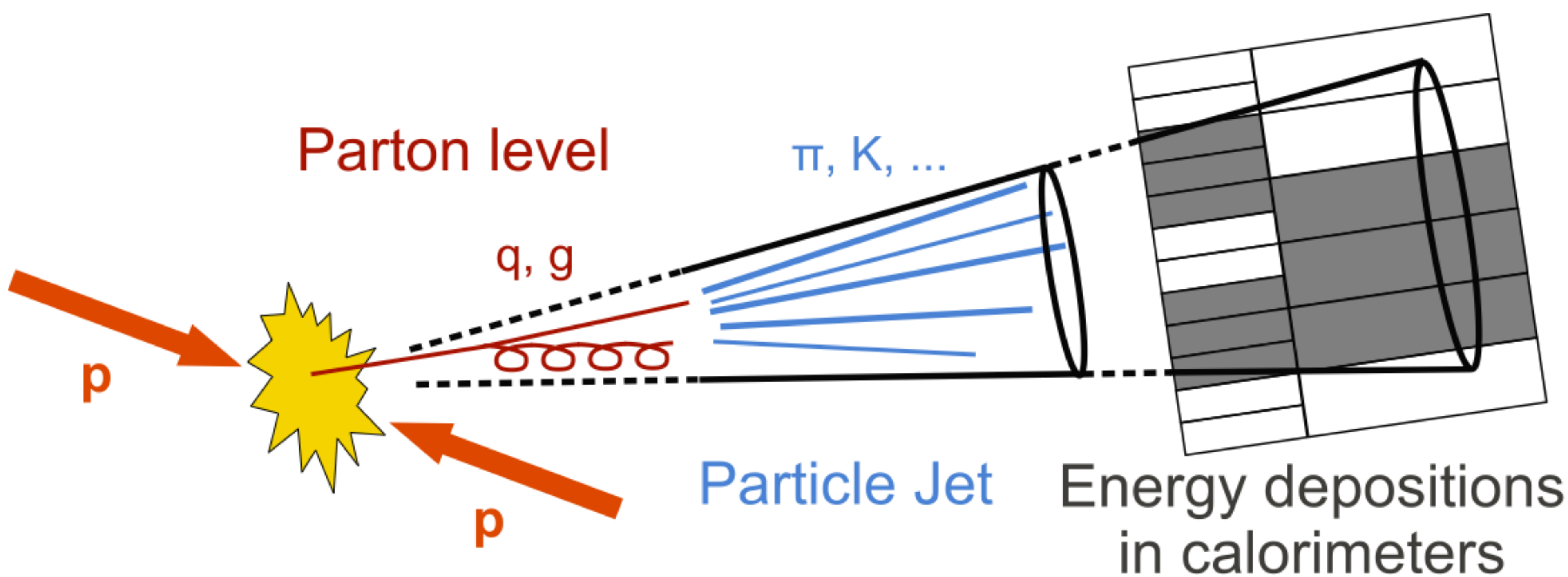
IMSc Spring School on High Energy Physics - 2025



References

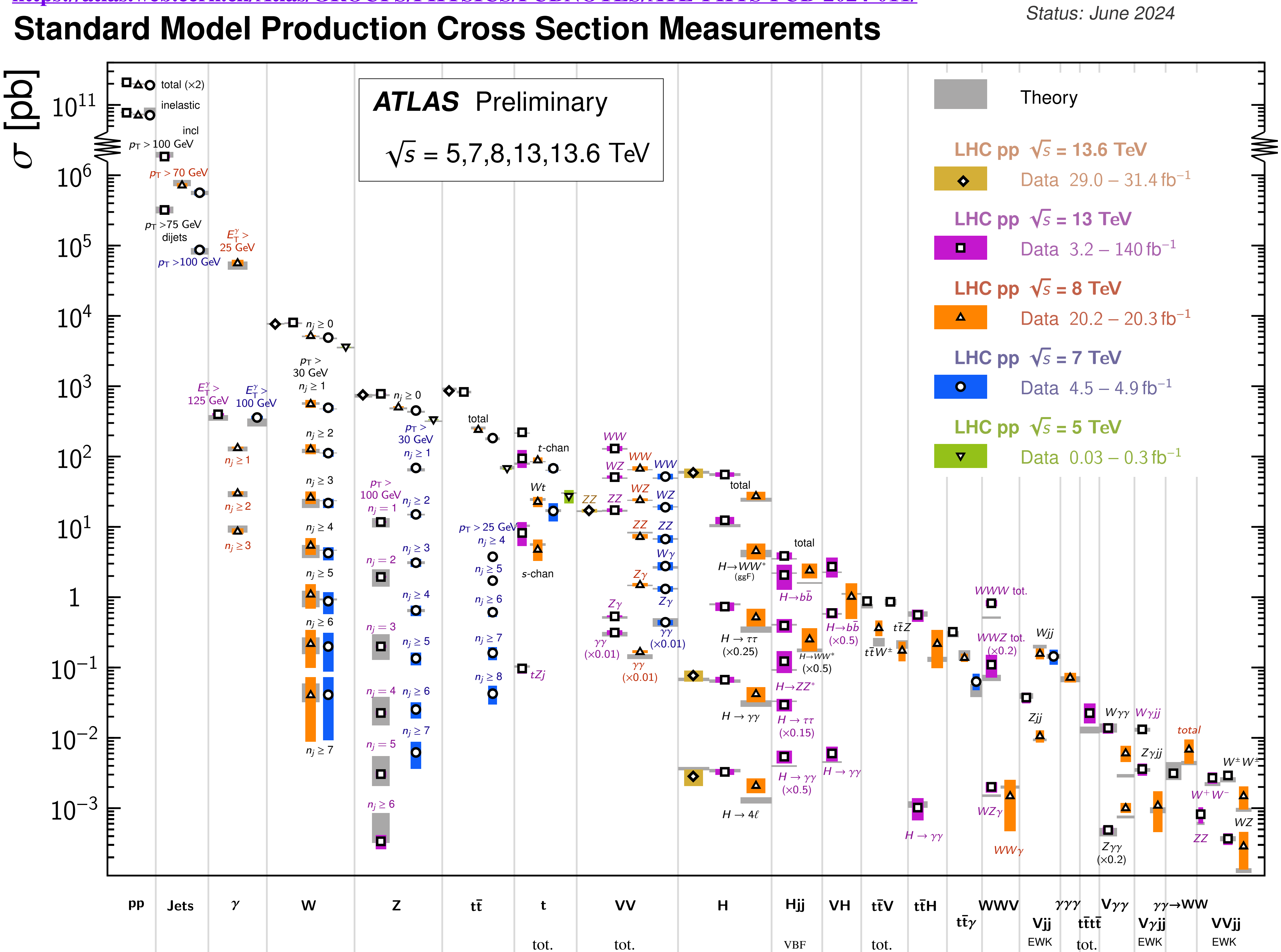
https://gsalam.web.cern.ch/teaching/PhD-courses.html

https://indico.ihep.ac.cn/event/7822/contributions/98092/attachments/52114/60078/lecture1and2.pdf (By Mateo Cacciari)

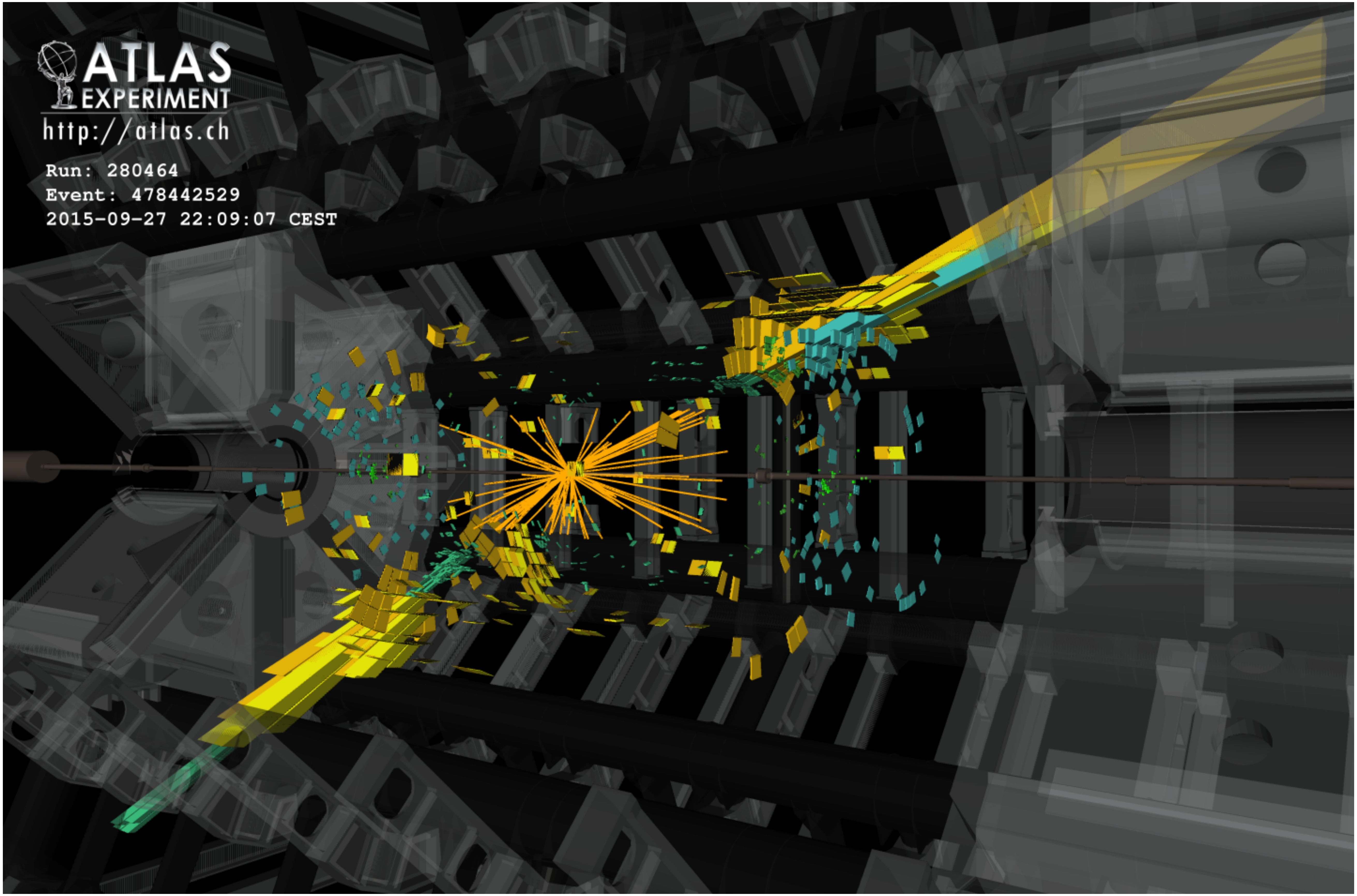
Andrew Larkoski : QCD master-class 2407.04897

What does raw measurement reveals?

https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/PUBNOTES/ATL-PHYS-PUB-2024-011/



What precisely these "JET" events are?



What is our best explanation?

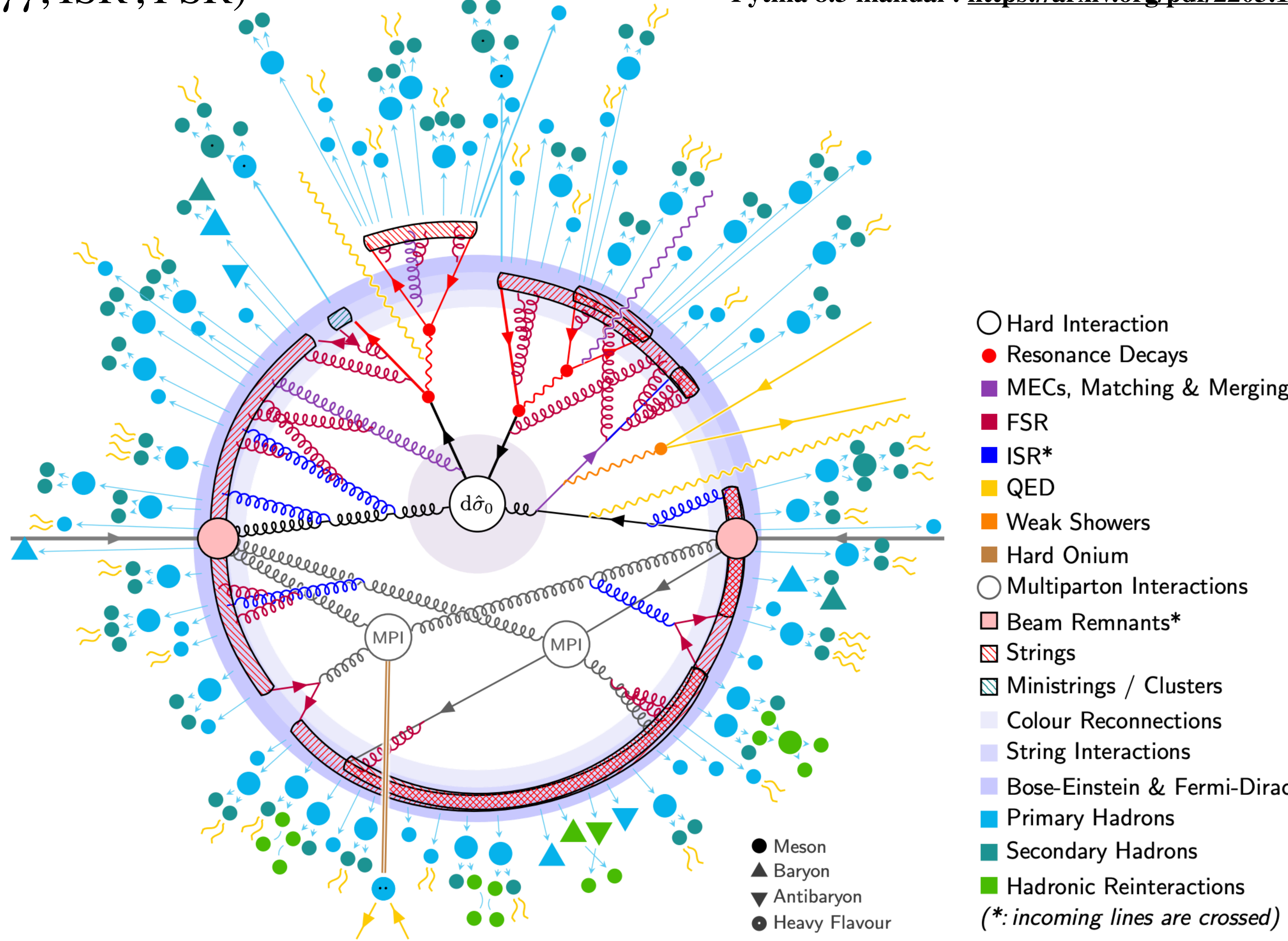
$\gamma\ (\pi^0 \rightarrow \gamma\gamma, \mathrm{ISR}, \mathrm{FSR})$

Pythia 8.3 manual : https://arxiv.org/pdf/2203.11601

K_L, K_S

$\pi^{\pm}$

$l^{\pm}$



Let's just look into a back-to-back emission

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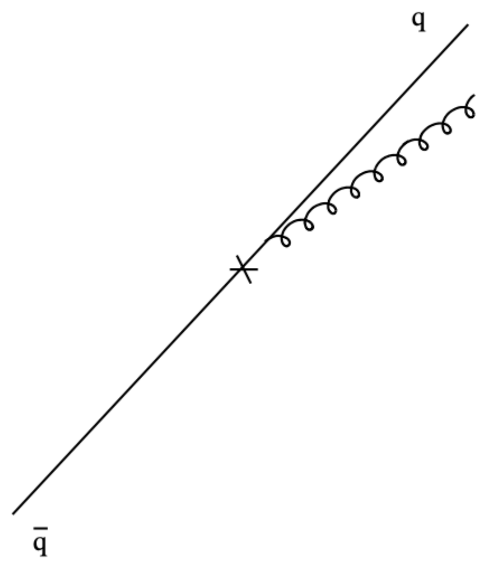

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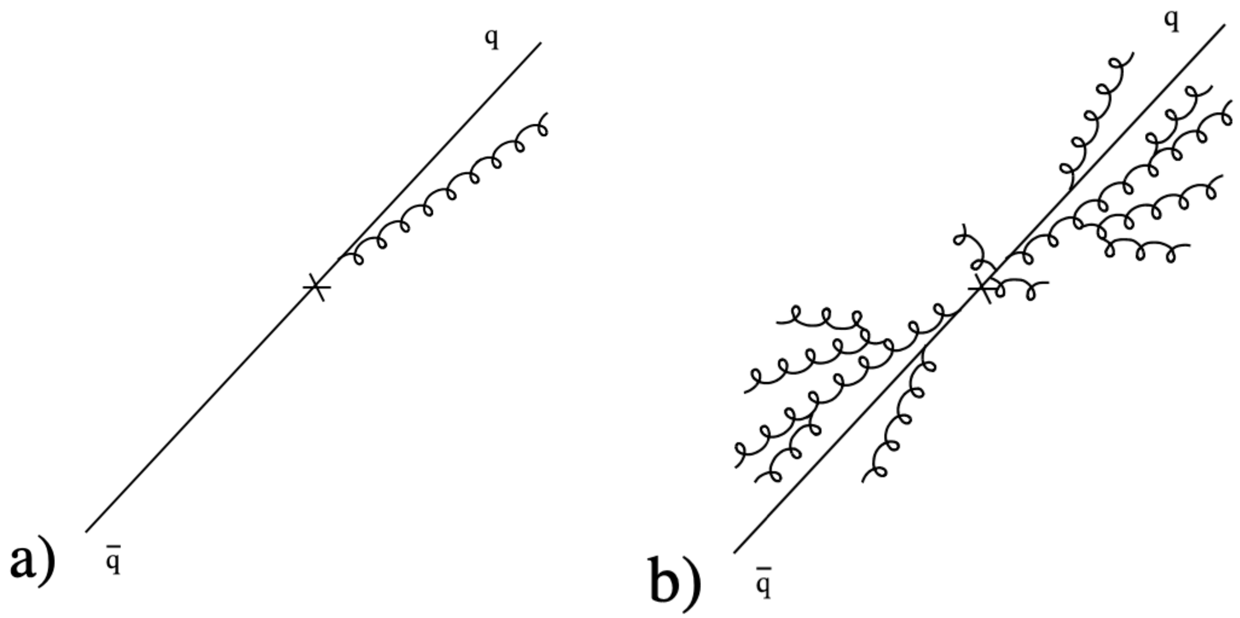

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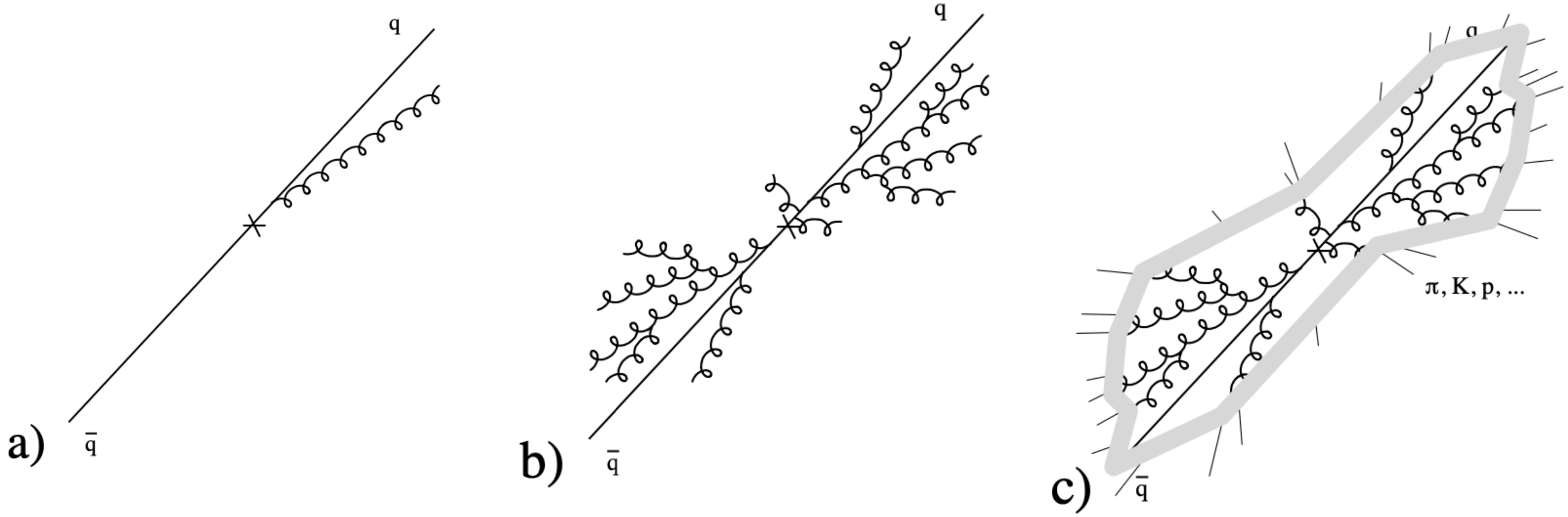

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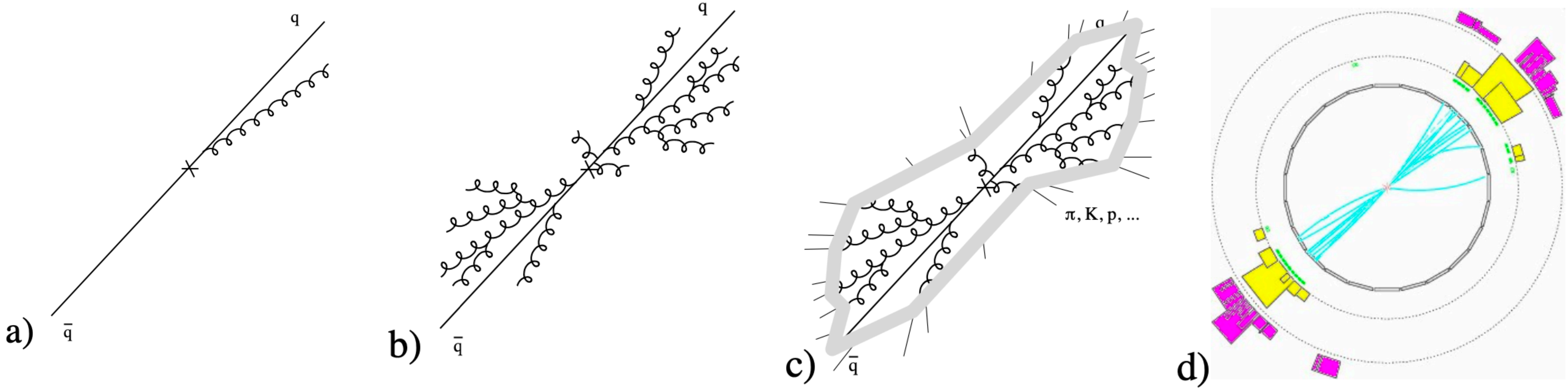
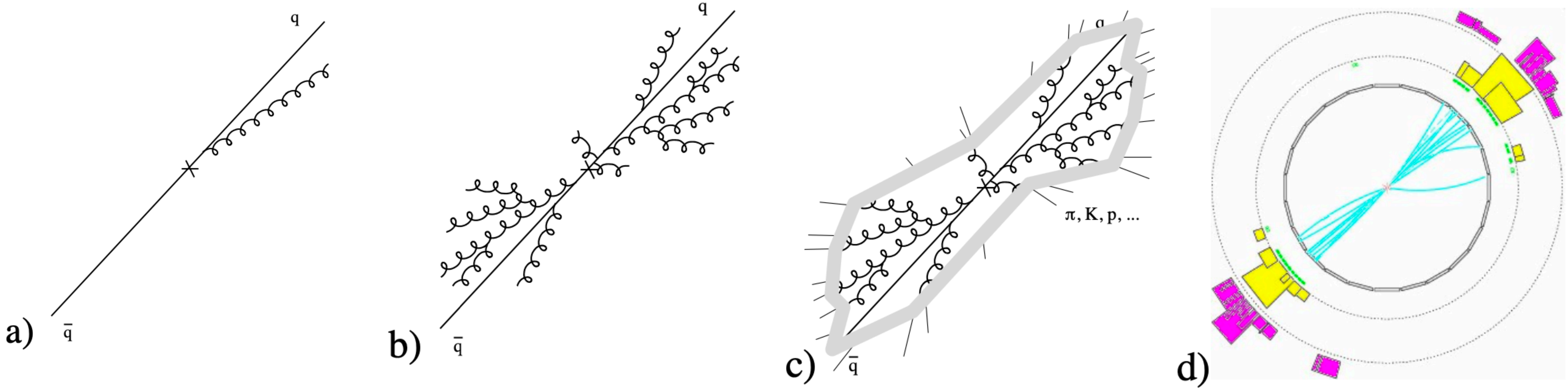


Let’s just look into a back-to-back emission

Figure source : Gavin Salam QCD lectures; 1011.5131



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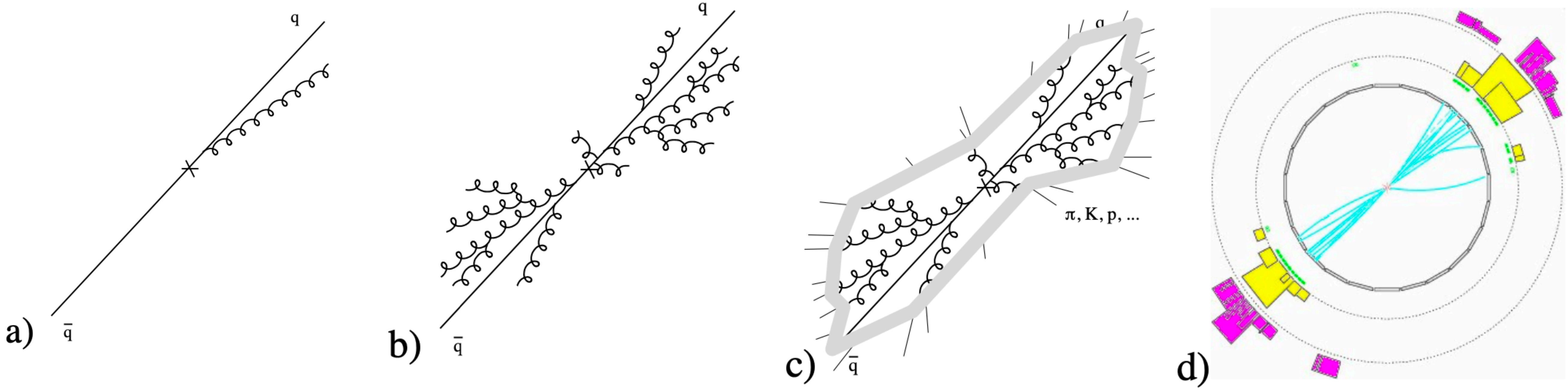

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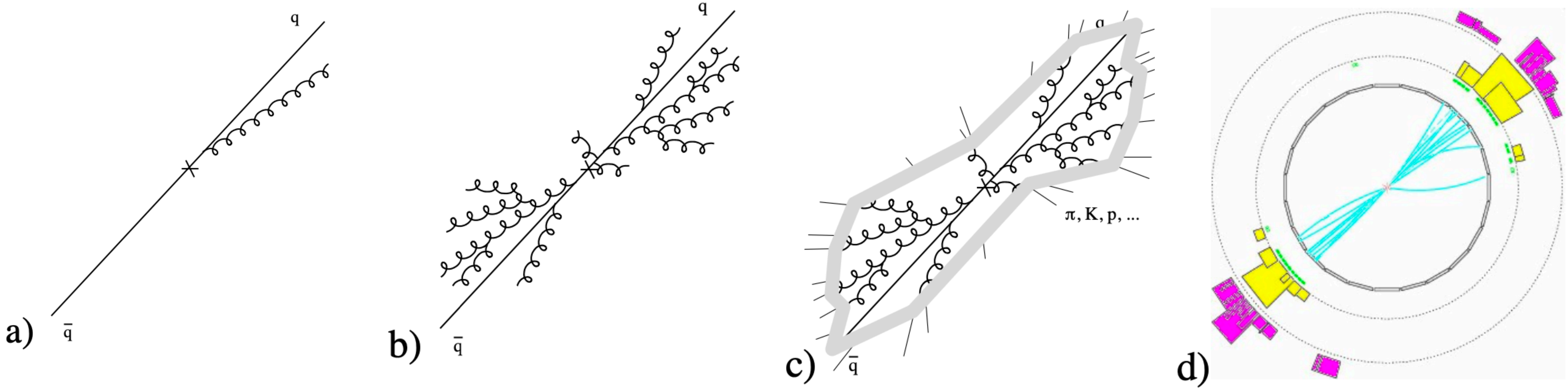

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Thus we get a stream of particles, as an end state of fragmentation and hadronization, in the direction of the initial hard parton.

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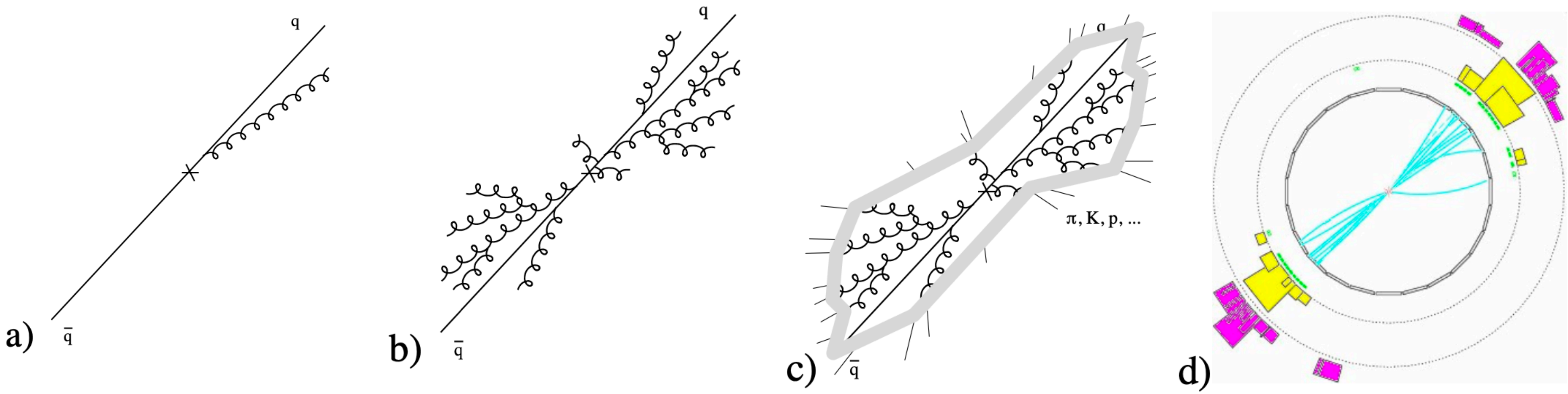

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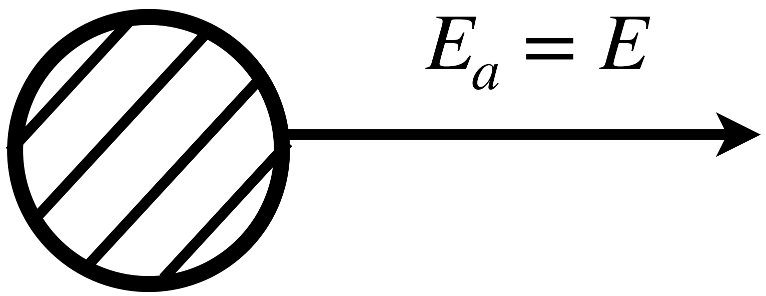
Thus we get a stream of particles, as an end state of fragmentation and hadronization, in the direction of the initial hard parton.

How do we explain this phenomena with our beloved lagrangian :

$$\mathscr{L} = -\frac{1}{4}\sum_{A=1}^{N_C^2-1} F^A_{\mu\nu}F^{A;\mu\nu} + \sum_{a,b=1}^{N_F} \bar{\psi}^a\left[i\gamma^\mu(\partial_\mu\delta_{ab} - g_s\sum_{C=1}^{N_C^2-1} T^C_{ab}A^C_\mu) - m\right]\psi^b$$

How about some general analytic structures?

<u>**Larkoski : 2407.04897**</u>



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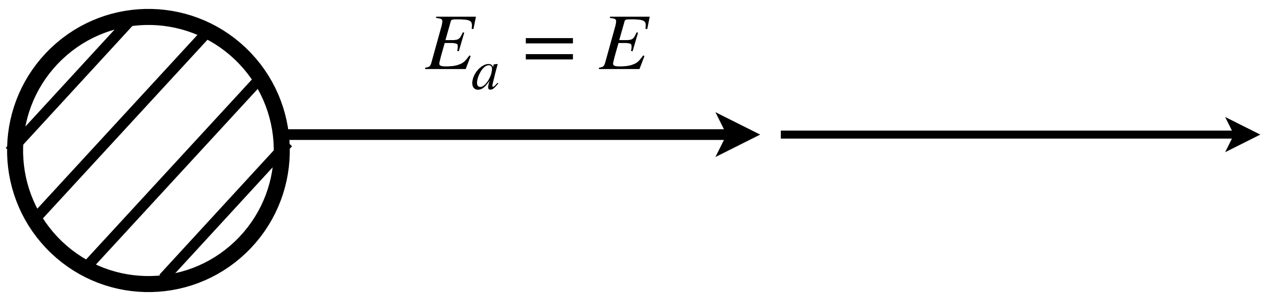

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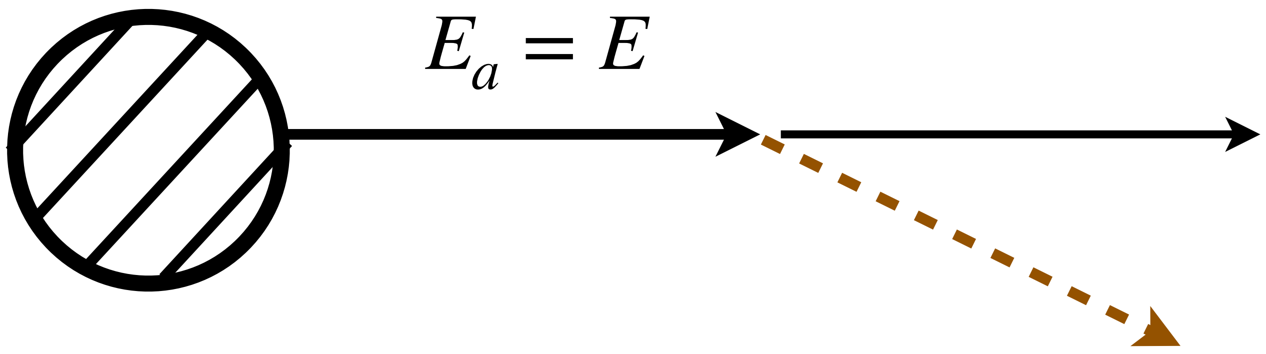

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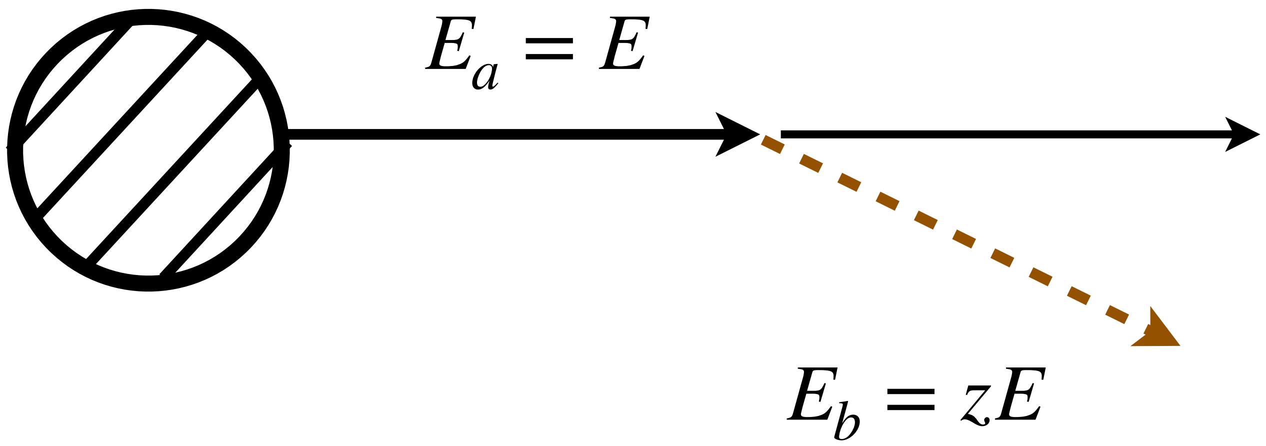

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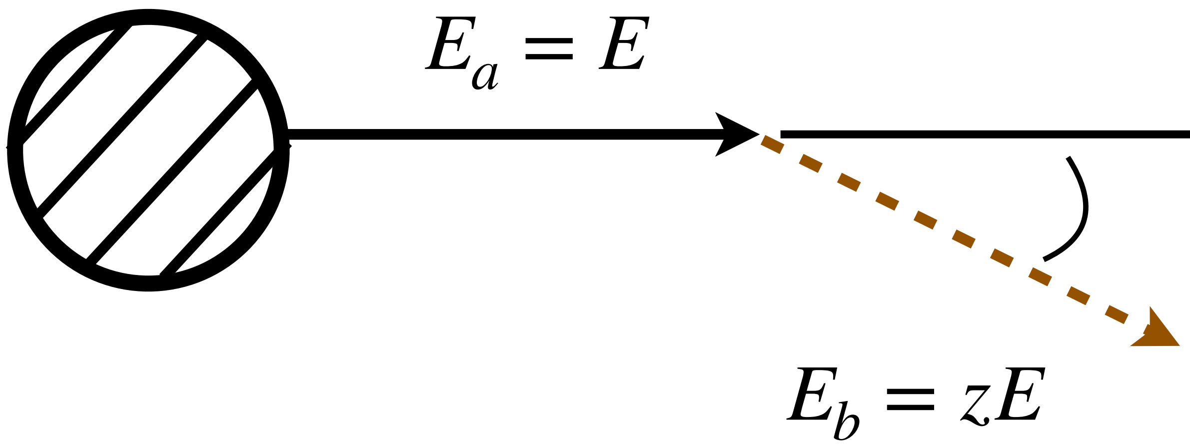

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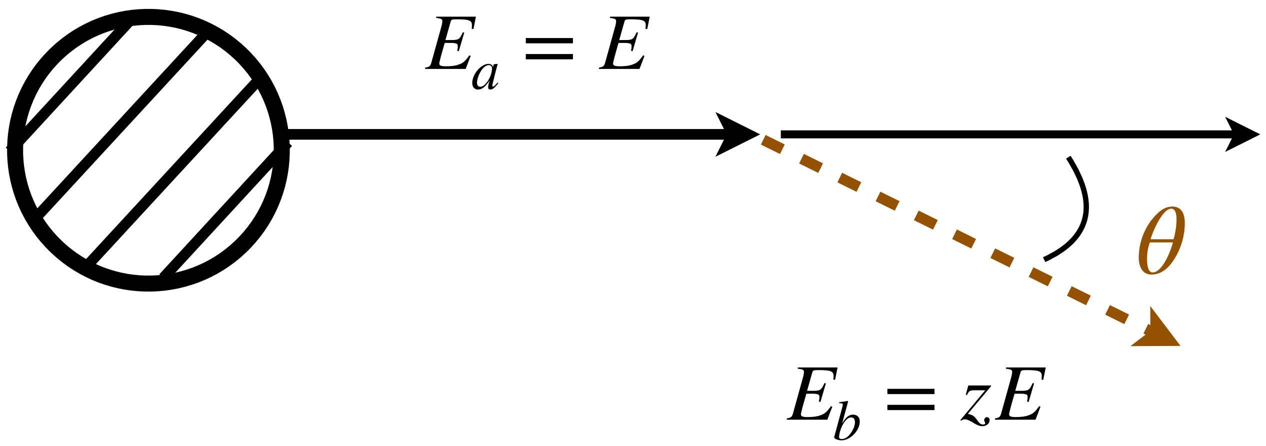

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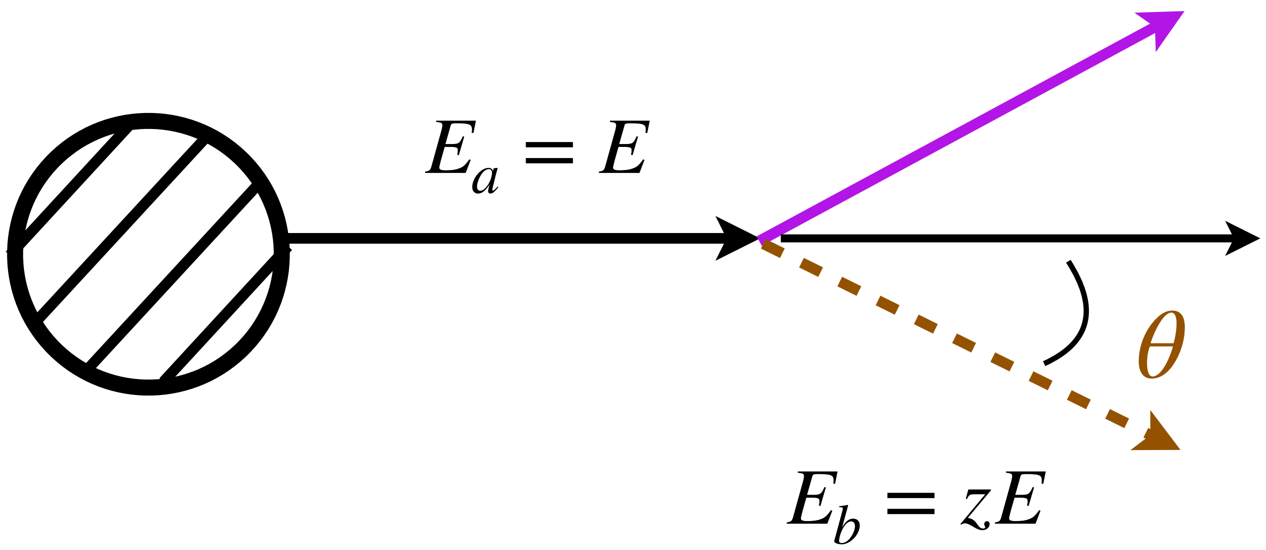

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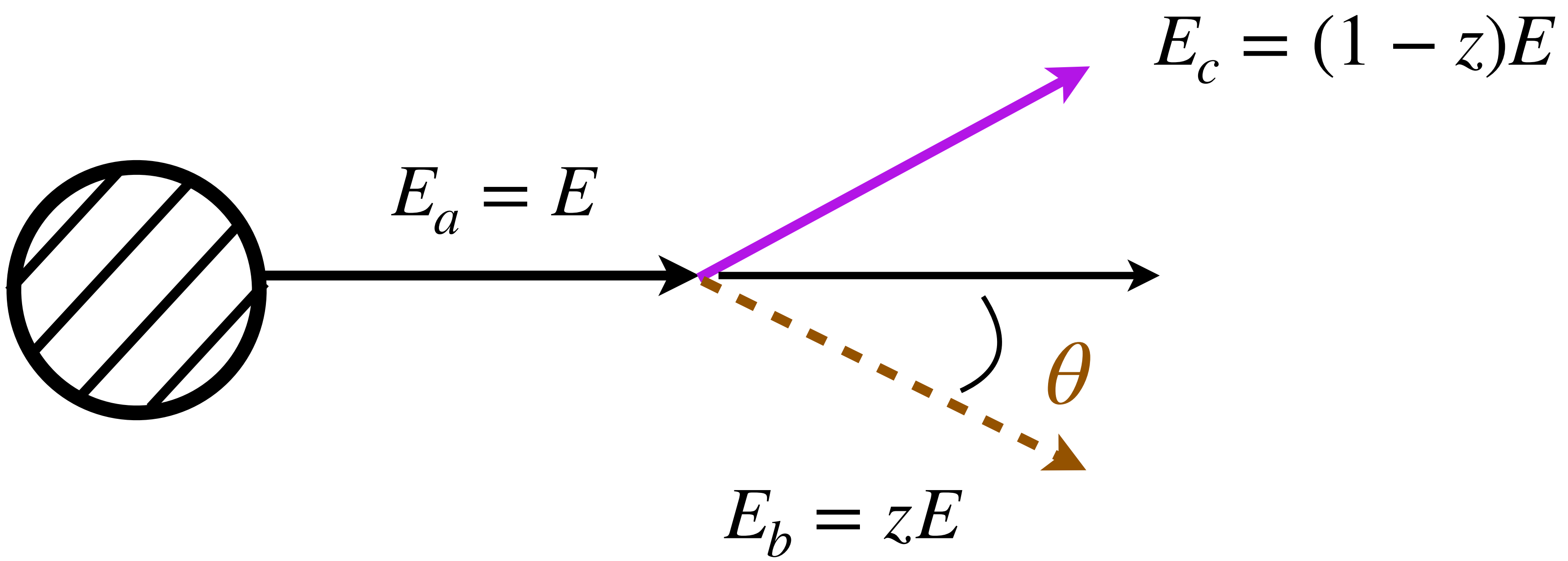

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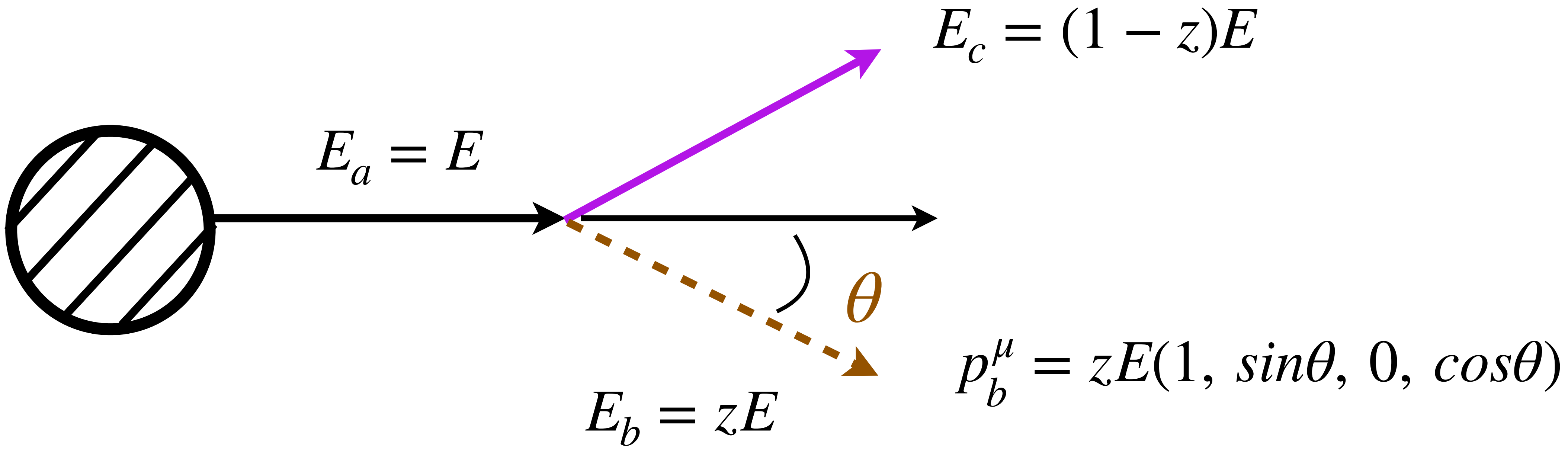

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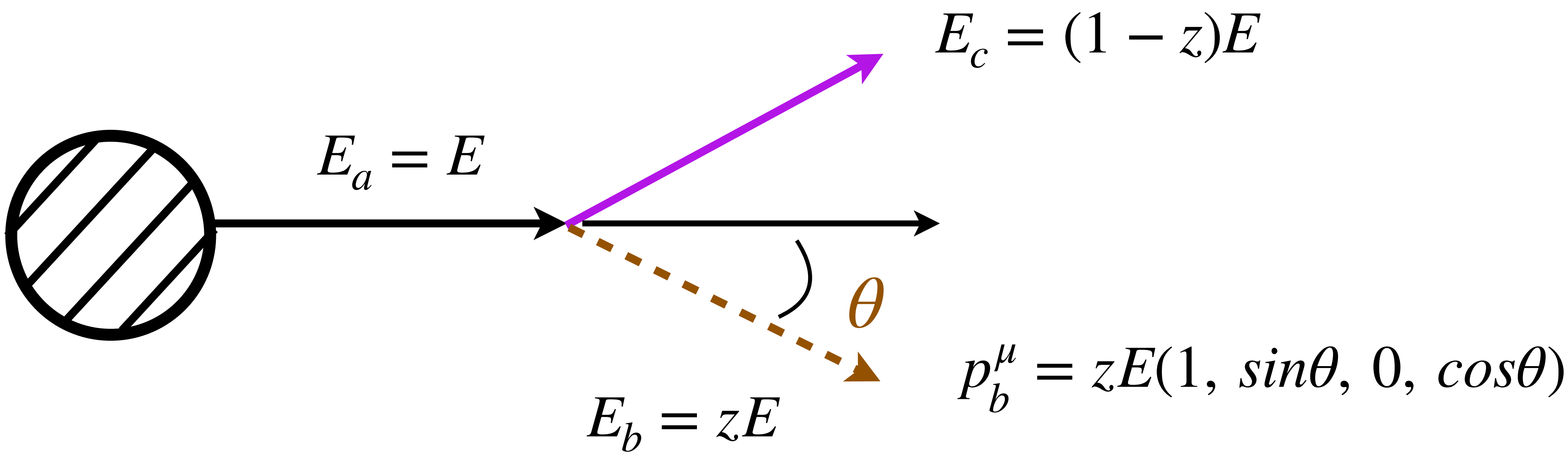

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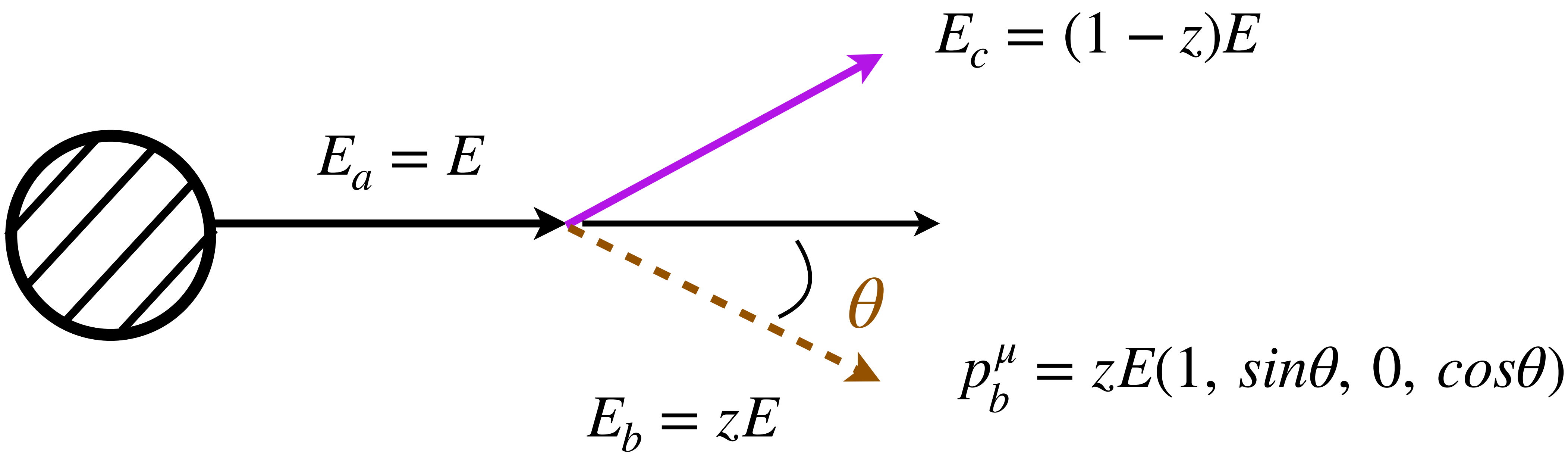

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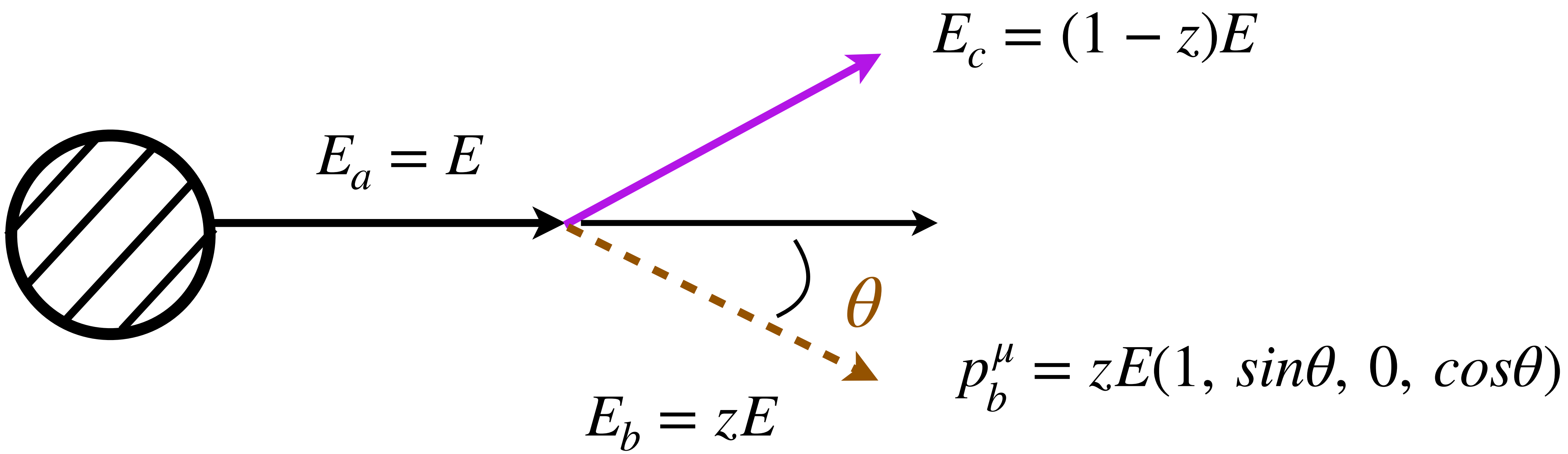

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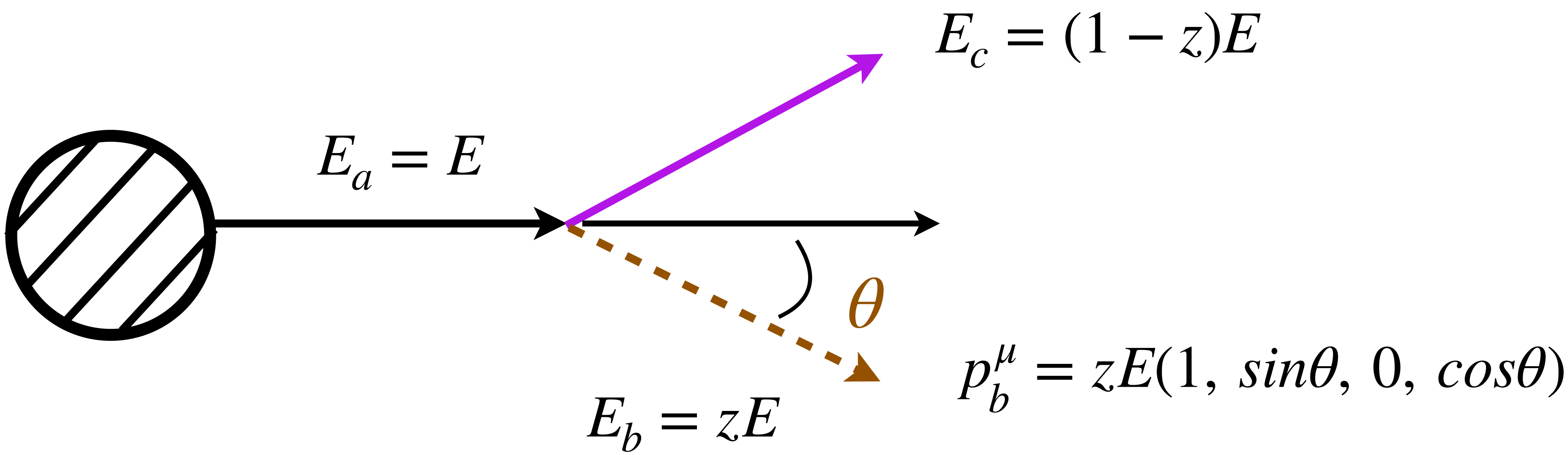

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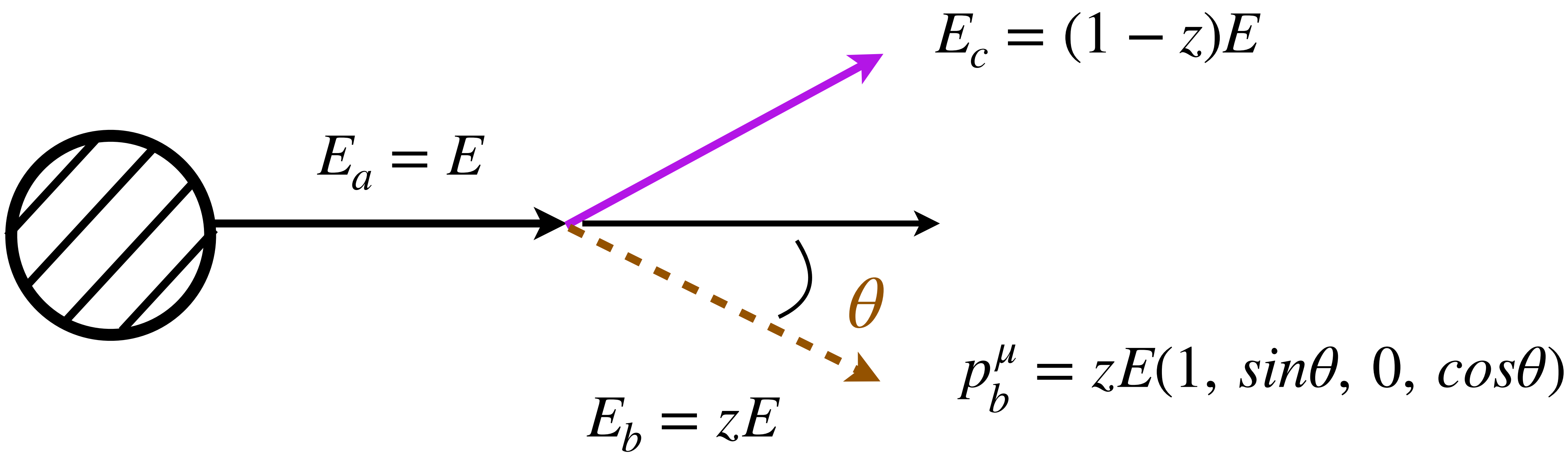

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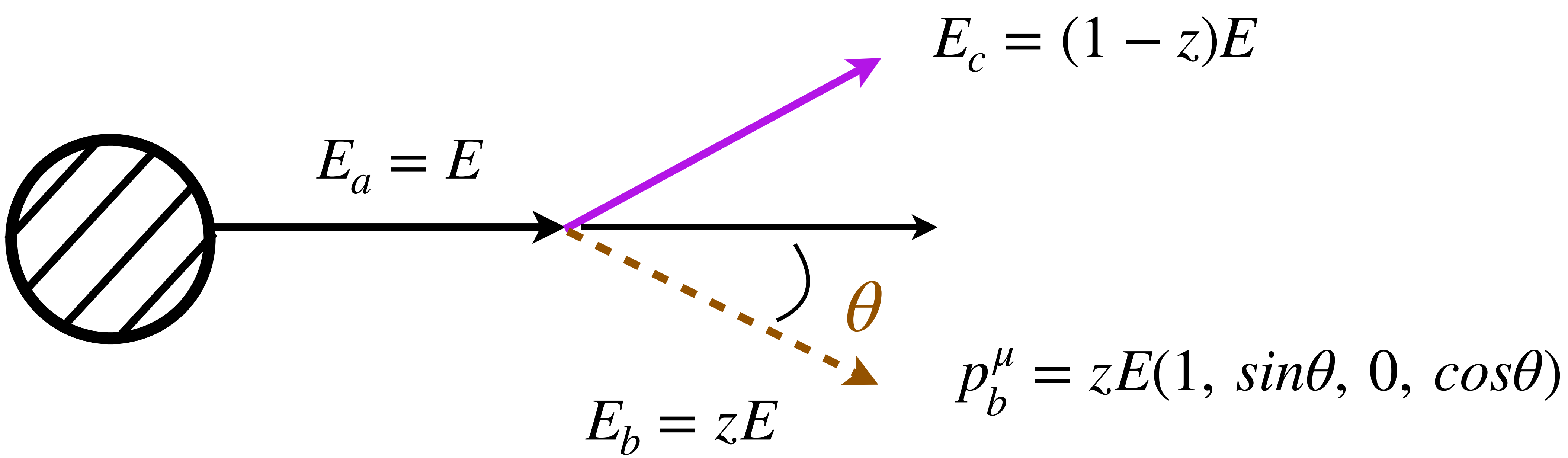

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Under this scaling $\mathscr{P}(E, z, \theta) \, dz \, d\theta \to \mathscr{P}'(E, z', \theta') \, dz' \, d\theta' = \mathscr{P}'(E, z, \theta') \, dz \, d\theta'$

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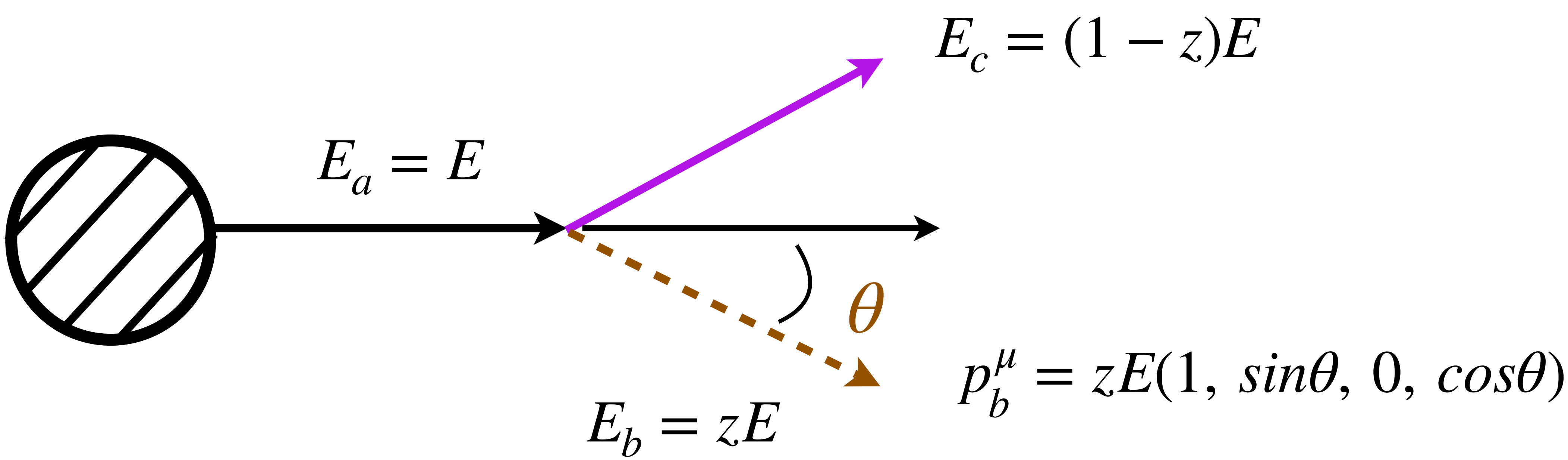

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Let's expand $\mathscr{P}'(E, z, \theta') \, dz \, d\theta' = \mathscr{P}(E, z, e^{\epsilon}\theta) \, dz \, d(e^{\epsilon}\theta)$, in the limit $\epsilon \to 0$.

How about some general analytic structures?

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Now $\mathscr{P}(E, z, e^{\epsilon}\theta) = \mathscr{P}(E, z, \theta) \; + \; \epsilon\theta \frac{\partial \mathscr{P}}{\partial \theta} \; + \; \mathcal{O}(\epsilon^2)$

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Hence

$$\mathscr{P}(E, z, e^{\epsilon}\theta)\ dz\ d(e^{\epsilon}\theta) = \left(\mathscr{P}(E, z, \theta) \ + \ \epsilon\theta\frac{\partial\mathscr{P}}{\partial\theta} \ + \ \mathcal{O}(\epsilon^2)\right)\left(dz\ d\theta \ + \ \epsilon dz\ d\theta \ + \ \mathcal{O}(\epsilon^2)\right)$$

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$$\begin{aligned}\mathscr{P}(E, z, e^{\epsilon}\theta) \; dz \; d(e^{\epsilon}\theta) &= \left(\mathscr{P}(E, z, \theta) \; + \; \epsilon\theta \frac{\partial \mathscr{P}}{\partial \theta} \; + \; \mathcal{O}(\epsilon^2)\right) \left(dz \; d\theta \; + \; \epsilon dz \; d\theta \; + \; \mathcal{O}(\epsilon^2)\right) \\ &= \mathscr{P}(E, z, \theta) \; dz \; d\theta.\end{aligned}$$

Comparing the coefficient of ϵ, we have :

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The solution to above differential equation is : $\mathscr{P}(E, z, \theta) \sim \dfrac{1}{\theta}$.

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Keeping θ fixed, if we scale z, we have : $\mathscr{P}(E, z, \theta) \sim \dfrac{1}{z}$.

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Since z and θ can be varied independently : $\mathscr{P}(E, z, \theta) \sim \dfrac{1}{z} \dfrac{1}{\theta}$; the lowest order singularity.

How about some general analytic structures?

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This translates to the emission probability : $\mathscr{P}(E, z, \theta)\ dz\ d\theta \sim \frac{dz}{z}\frac{d\theta}{\theta} \sim \frac{dE_b}{E_b}\frac{d\theta}{\theta}$

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Sudakov double logarithm

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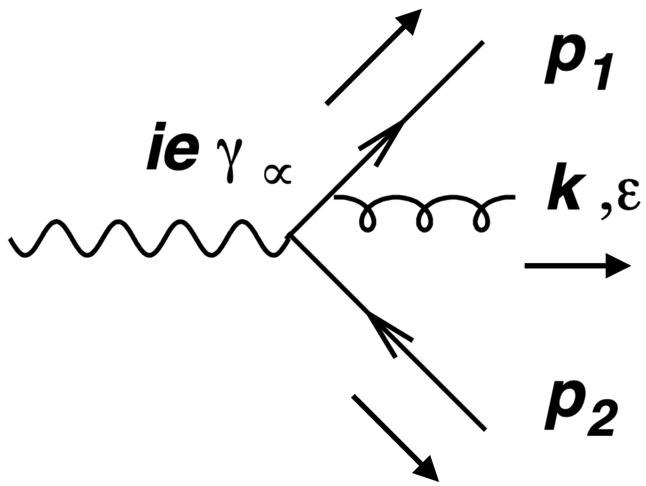
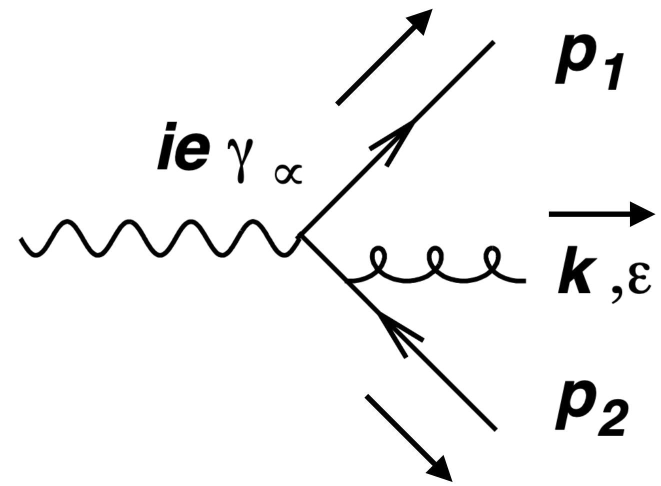
Small quiz :

1. If we match higher orders of ϵ, will we get correction to the scaling laws?
2. Angle θ is always dimensionless for any radiation. Will the logarithmic divergence appear for massive particle radiation as well?

Let's do the analysis for a gluon radiation

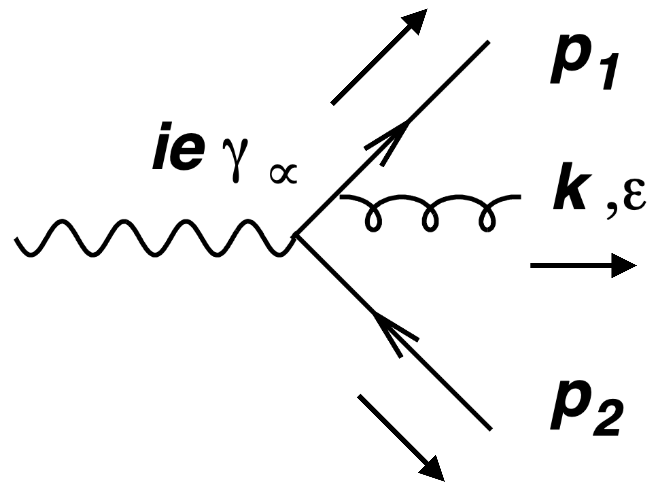
$\mathcal{M}_{q\bar{q}g} =$ 

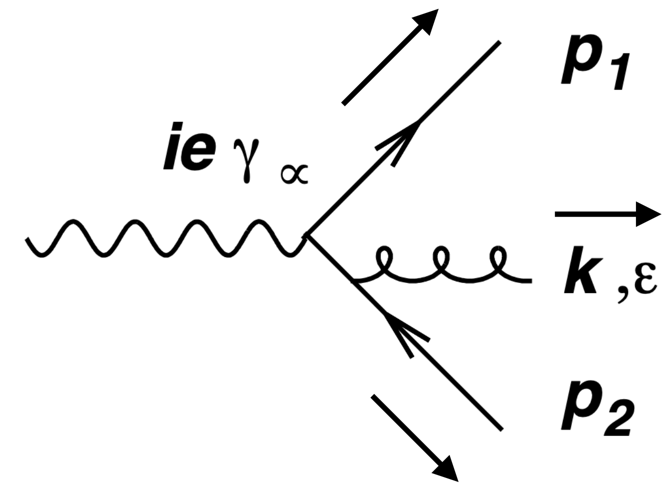
$+$ 

$$i\mathcal{M}^{1}_{q\bar{q}g} = \bar{u}(p_1)\ (ig_s T^a)\ \not{\epsilon}(k)^* \left(\frac{i(\not{p}_1 + \not{k} + m)}{(p_1+k)^2 - m^2}\right) (-ie_q \gamma^{\alpha})\ v(p_2)$$

$$i\mathcal{M}^{2}_{q\bar{q}g} = -\ \bar{u}(p_1)\ (-ie_q \gamma^{\alpha}) \left(\frac{i(\not{p}_2 + \not{k} + m)}{(p_2+k)^2 - m^2}\right) (ig_s T^a)\ \not{\epsilon}(k)^*\ v(p_2)$$

Let's do the analysis for a gluon radiation

$\mathcal{M}_{q\bar{q}g} =$ 

$+$ 

$$i\mathcal{M}^1_{q\bar{q}g} = \bar{u}(p_1)\ (ig_s T^a)\ \not{\epsilon}(k)^*\left(\frac{i(\not{p}_1 + \not{k} + m)}{(p_1+k)^2 - m^2}\right)(-ie_q\gamma^\alpha)\ v(p_2)$$

$$i\mathcal{M}^2_{q\bar{q}g} = -\ \bar{u}(p_1)\ (-ie_q\gamma^\alpha)\left(\frac{i(\not{p}_2 + \not{k} + m)}{(p_2+k)^2 - m^2}\right)(ig_s T^a)\ \not{\epsilon}(k)^*\ v(p_2)$$

Let's use the following identities to simplify the above amplitudes :

$\bar{u}(p)\ \not{\epsilon}(k)^*(\not{p} + m) = 2\ \bar{u}(p)\ \epsilon_\mu(k)^*\ p^\mu\ ;\ (\not{p} + m)\ \ \not{\epsilon}(k)^*\ v(p) = 2\ \ \epsilon_\mu(k)^*\ p^\mu\ v(p)$

$\bar{u}(p)\ \not{\epsilon}(k)^*(\not{k}) = -\,i\ \bar{u}(p)\ \epsilon_\mu(k)^*\ \sigma^{\mu\nu}\ k_\nu\ ;\ \not{k}\ \ \not{\epsilon}(k)^*\ v(p) = -\,i\ \epsilon_\mu(k)^*\ \sigma^{\mu\nu}\ k_\nu\ v(p)$

$$(p+k)^2 - m^2 = 2\ (p\,.\,k)$$

Let’s do the analysis for a gluon radiation

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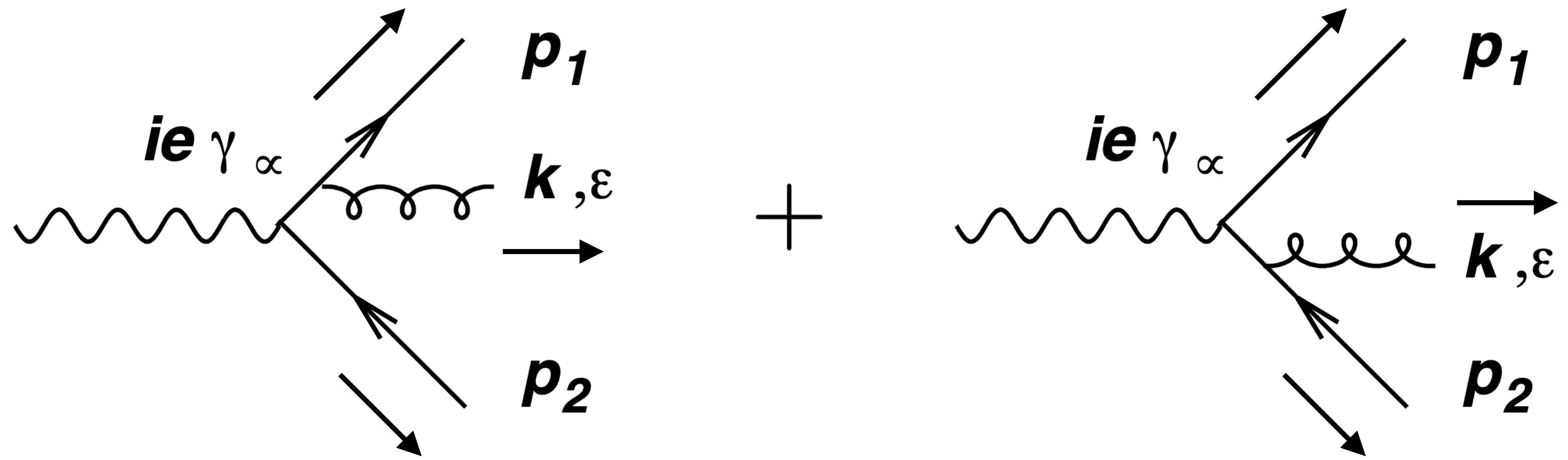

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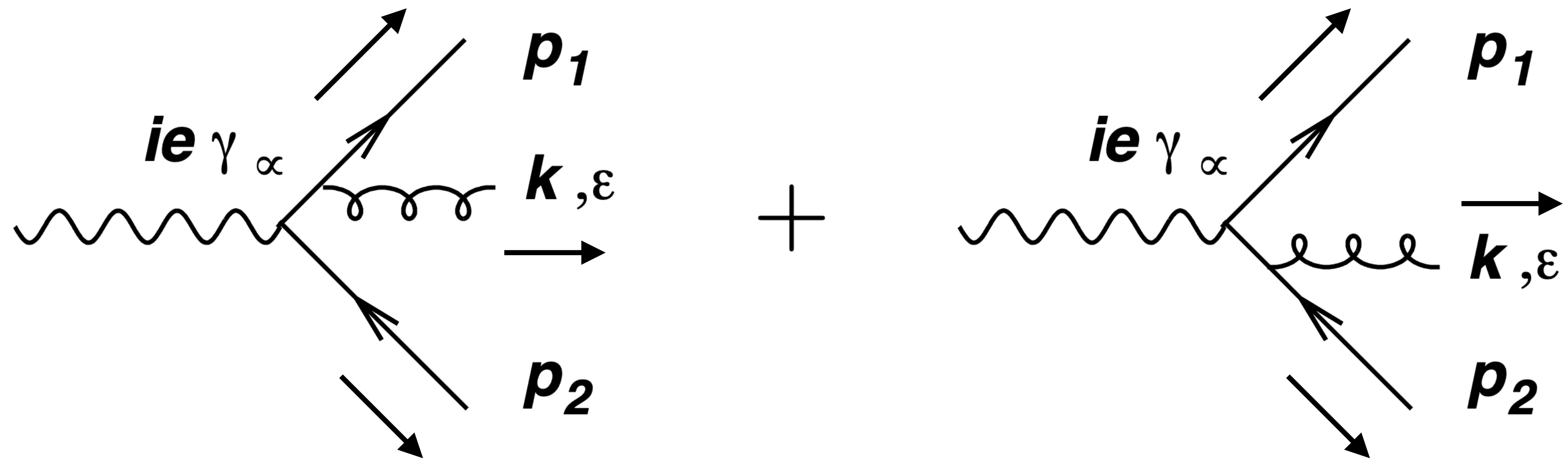

$$\mathscr{M}^{1}_{q\bar{q}g} = \bar{u}(p_1)\ (g_s T^a)\ \epsilon_\mu(k) * \left(\frac{(2\ p_1^\mu - i\sigma^{\mu\nu}\ k_\nu)}{2\ p_1 \,.\, k}\right)\ (e_q \gamma^\alpha)\ v(p_2)$$

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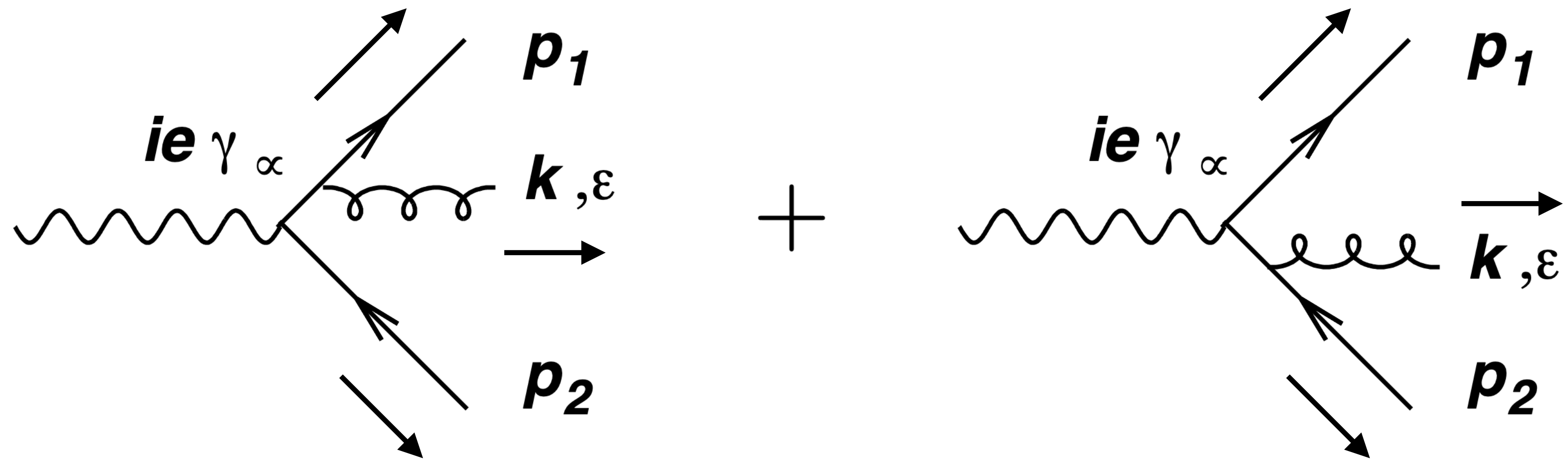

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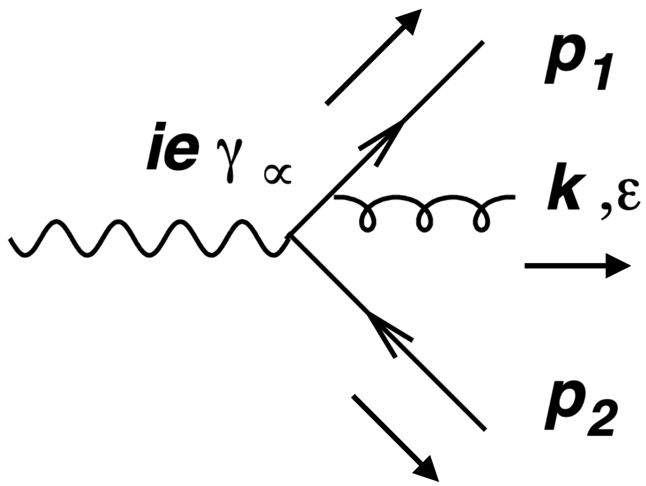
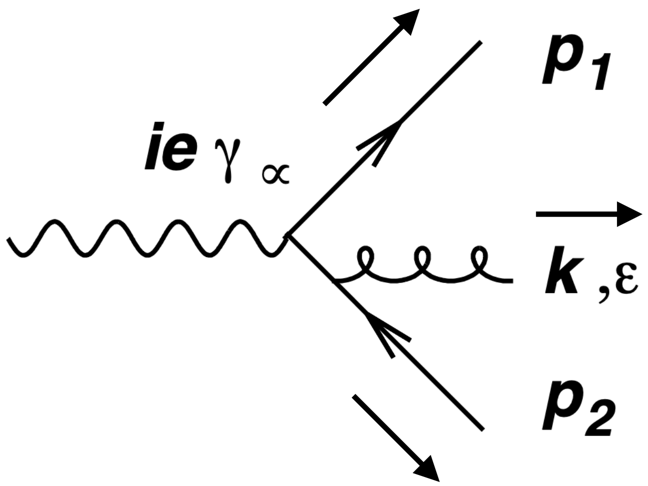
Let's do the analysis for a gluon radiation

$\mathcal{M}_{q\bar{q}g} =$  $+$ 

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In the limit $k \to 0$:

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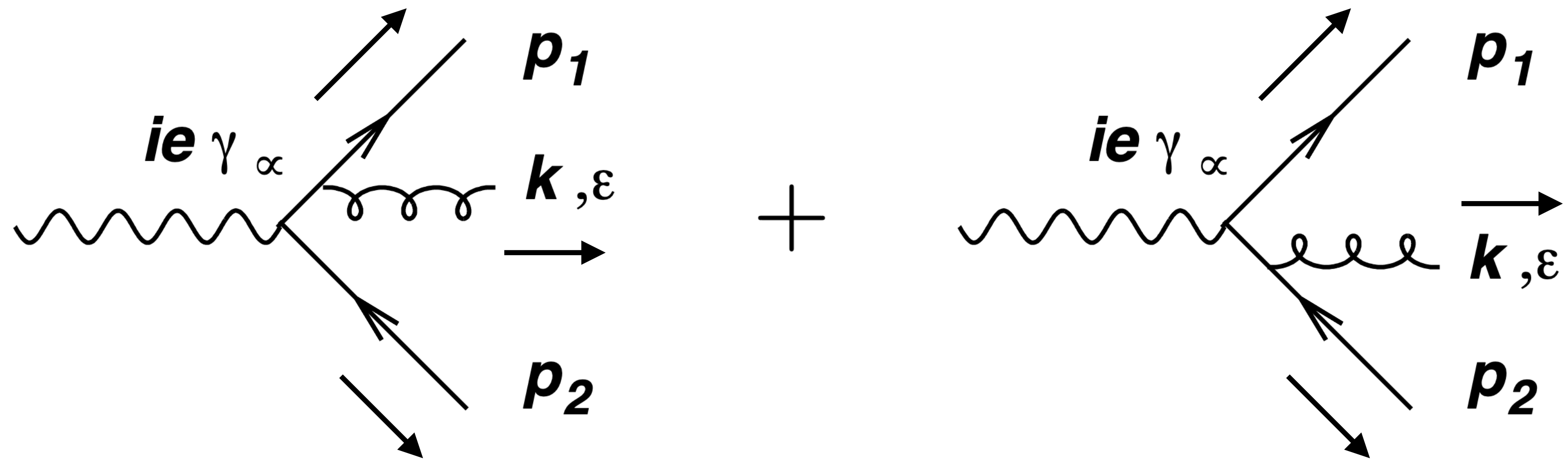

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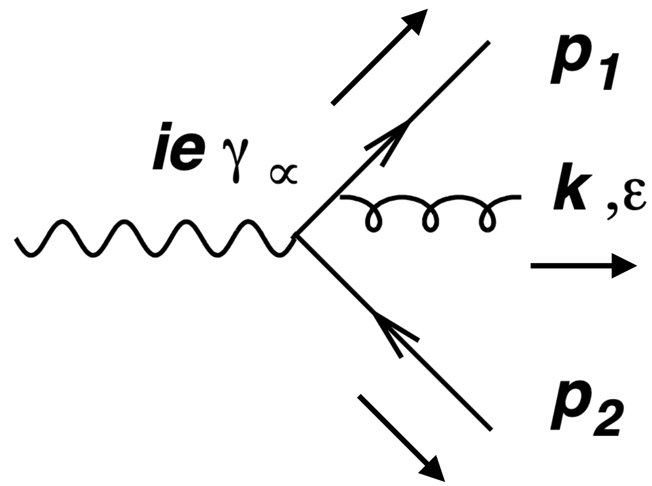
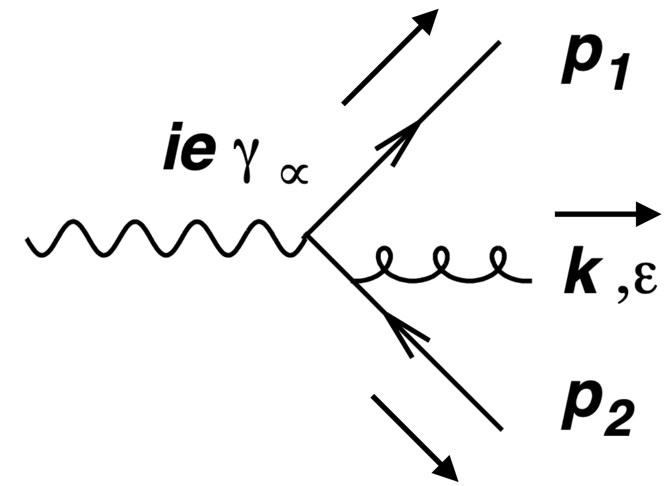
Let's do the analysis for a gluon radiation

$\mathcal{M}_{q\bar{q}g} =$ 

$+$ 

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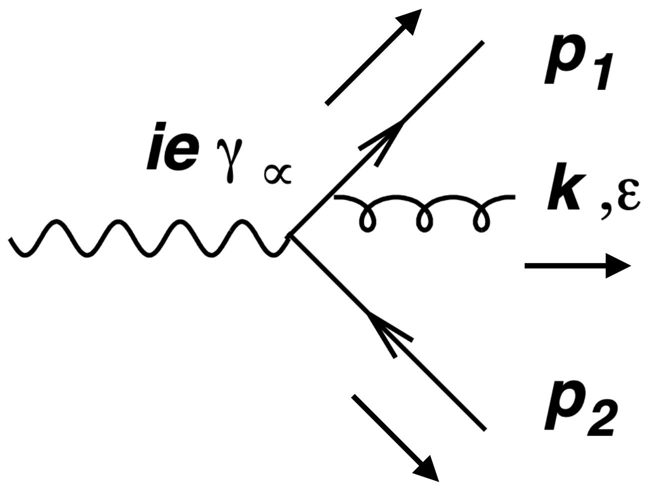
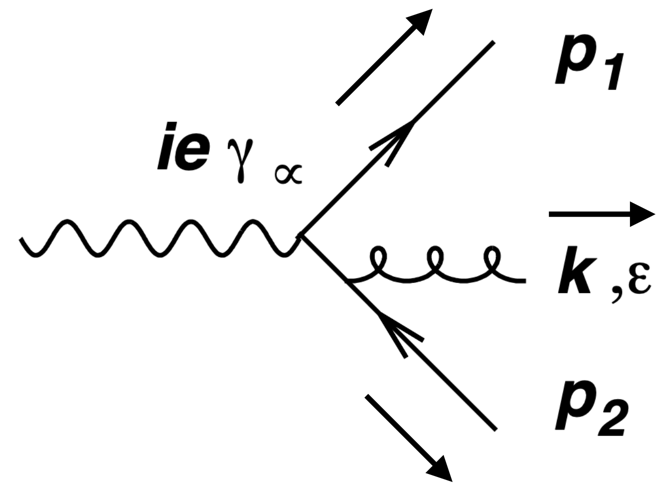


$$+$$



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$$= \ \bar{\mathcal{M}}^2{}_{q\bar{q}}\ g_s^2 C_F\left[2\ \frac{p_1 . p_2}{(p_1 . k)\ (p_2 . k)} - \frac{m^2}{(p_2 . k)^2} - \frac{m^2}{(p_1 . k)^2}\right]$$

Let's do the analysis for a gluon radiation

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For computing the x-section of the emission we need the 3-body phase space :

$$d\,\Phi_{q\bar{q}g} \approx d\,\Phi_{q\bar{q}}\,\frac{d^3\vec{k}}{(2\pi)^3}\,\frac{1}{2k^0}\,.$$

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Hence the x-section of soft gluon emission, with $k^0 = E$:

$$\bar{\mathscr{M}}^2{}_{q\bar{q}g}\ d\ \Phi_{q\bar{q}g} \approx \bar{\mathscr{M}}^2{}_{q\bar{q}}\ d\ \Phi_{q\bar{q}}\ \frac{2\alpha_S\ C_F}{\pi}\ E\ dE\ d(cos\theta)\ \frac{d\phi}{2\pi}\ \left(\frac{2\ p_1\,.\,p_2}{(p_1\,.\,k)\ (p_2\,.\,k)}\right)\ .$$

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If we try to carry out the P.S. integral in $q\bar{q}$ COM frame :

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When a gluon is emitted from a quark line, in collinear limit, the additional piece in the x-section :

$$dS = \frac{2\alpha_S\,C_F}{\pi}\,\frac{dE}{E}\,\frac{d\theta}{sin\theta}\,\frac{d\phi}{2\pi} \approx \frac{2\alpha_S\,C_F}{\pi}\,\frac{dE}{E}\,\frac{d\theta}{\theta}\,.$$

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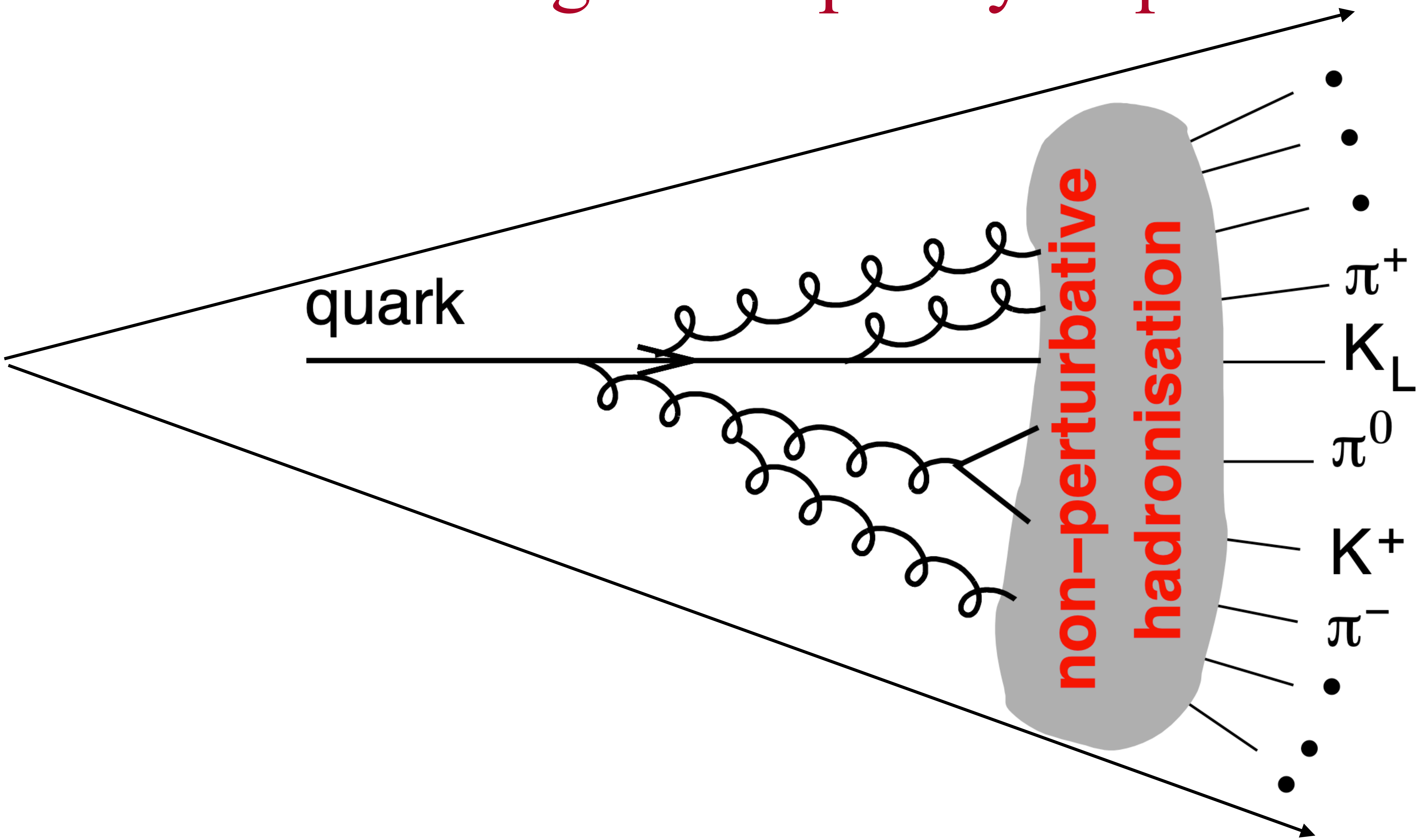
So we have a large multiplicity of particles



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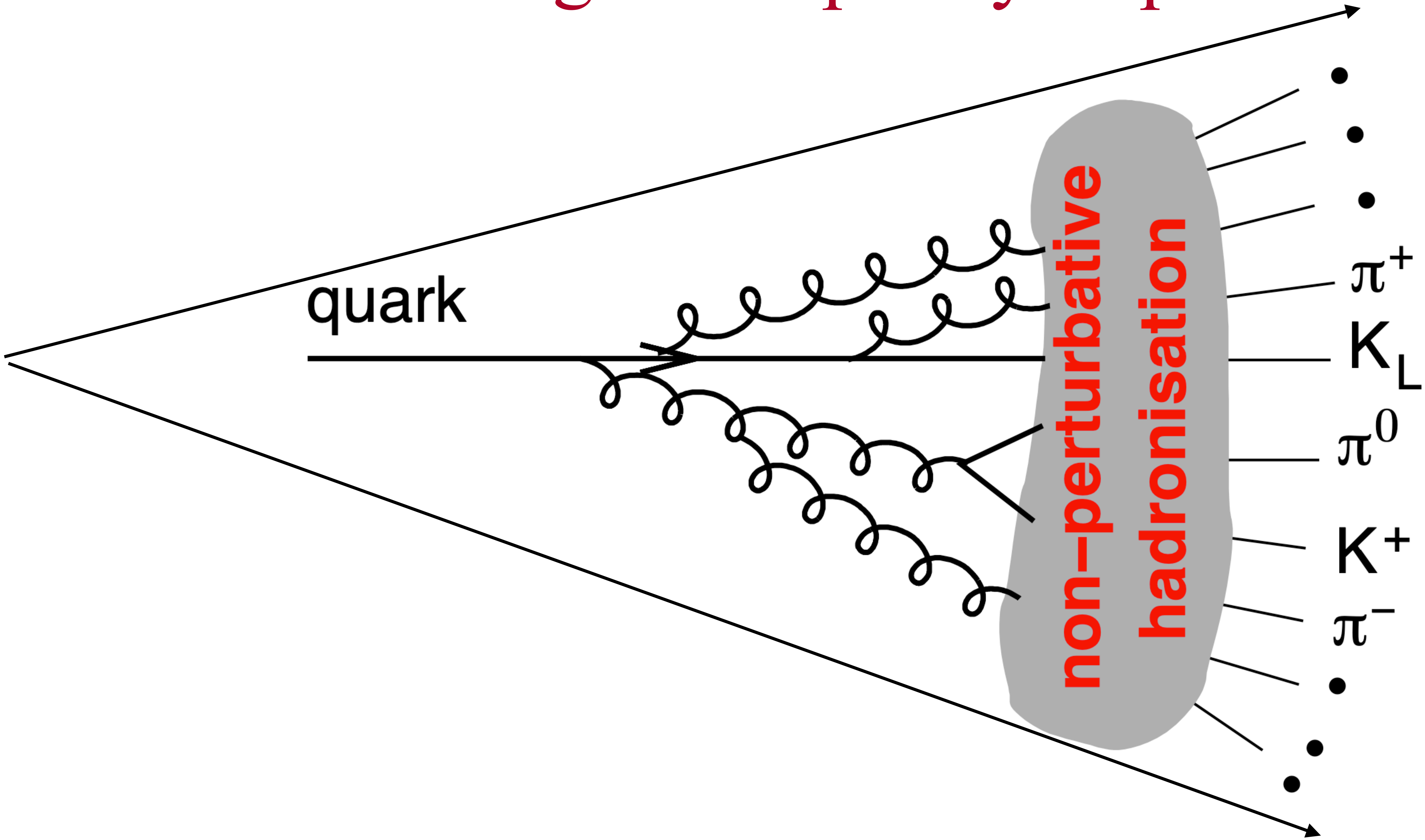
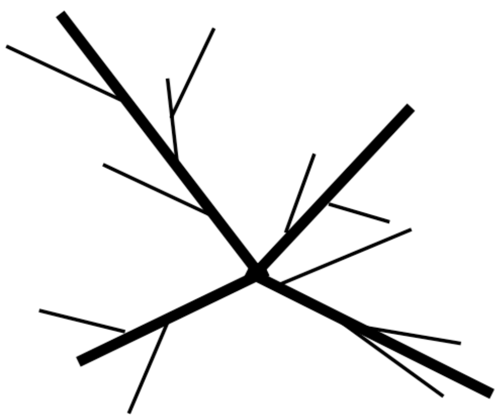


A spray of particles gets produced along the line of flight of the original parton, governed by the factor : $dS = \approx \frac{2\alpha_S C_{F/A}}{\pi} \frac{dE}{E} \frac{d\theta}{\theta}$.

Particles to Parton mapping

Multileg + PS



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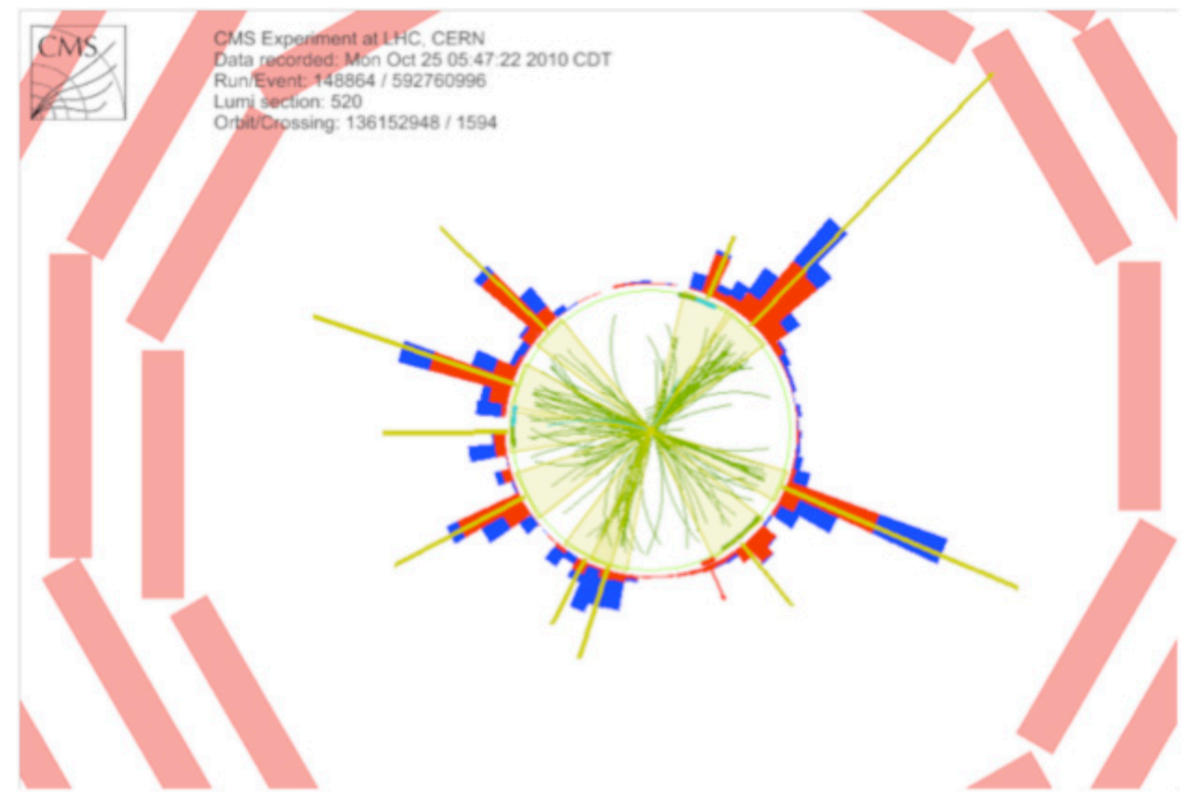
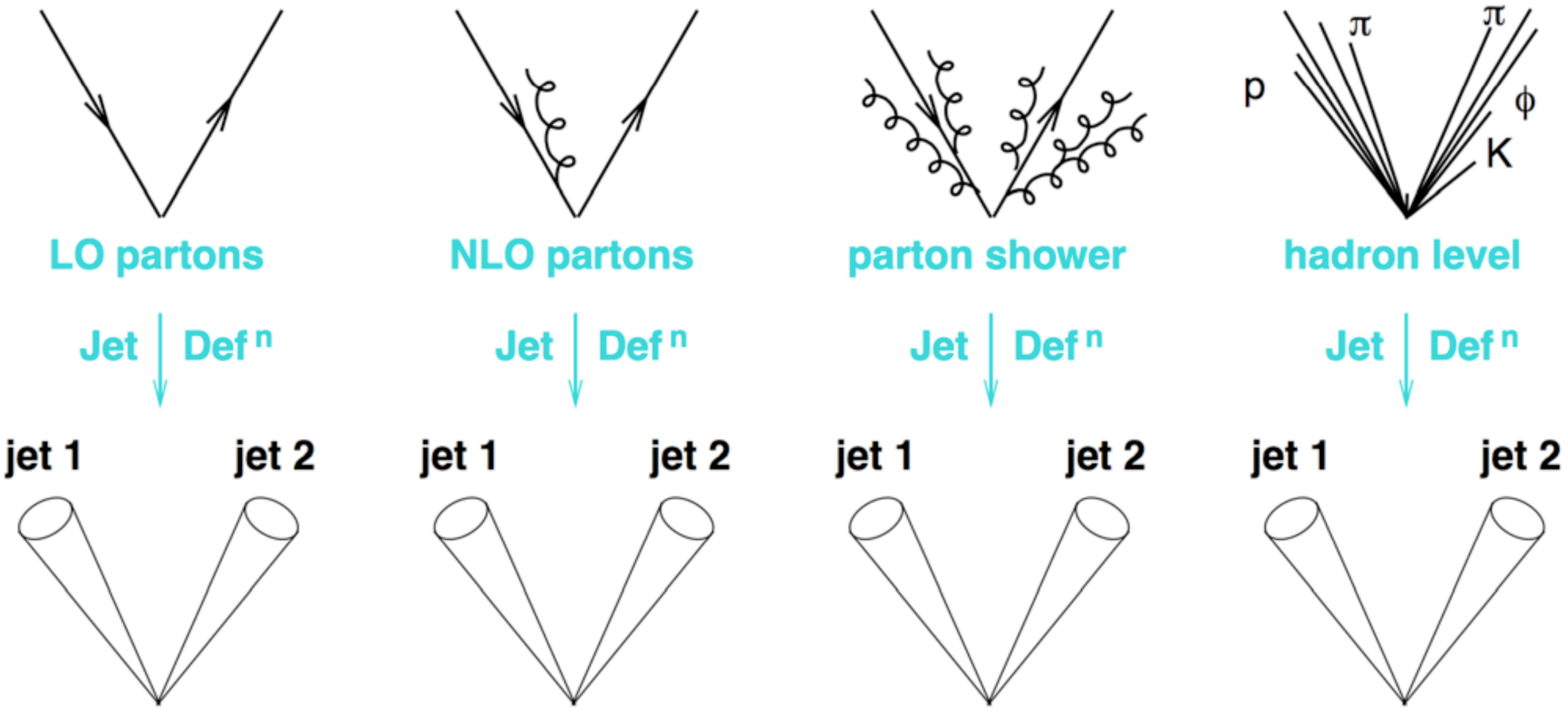


QCD predictions

Real data

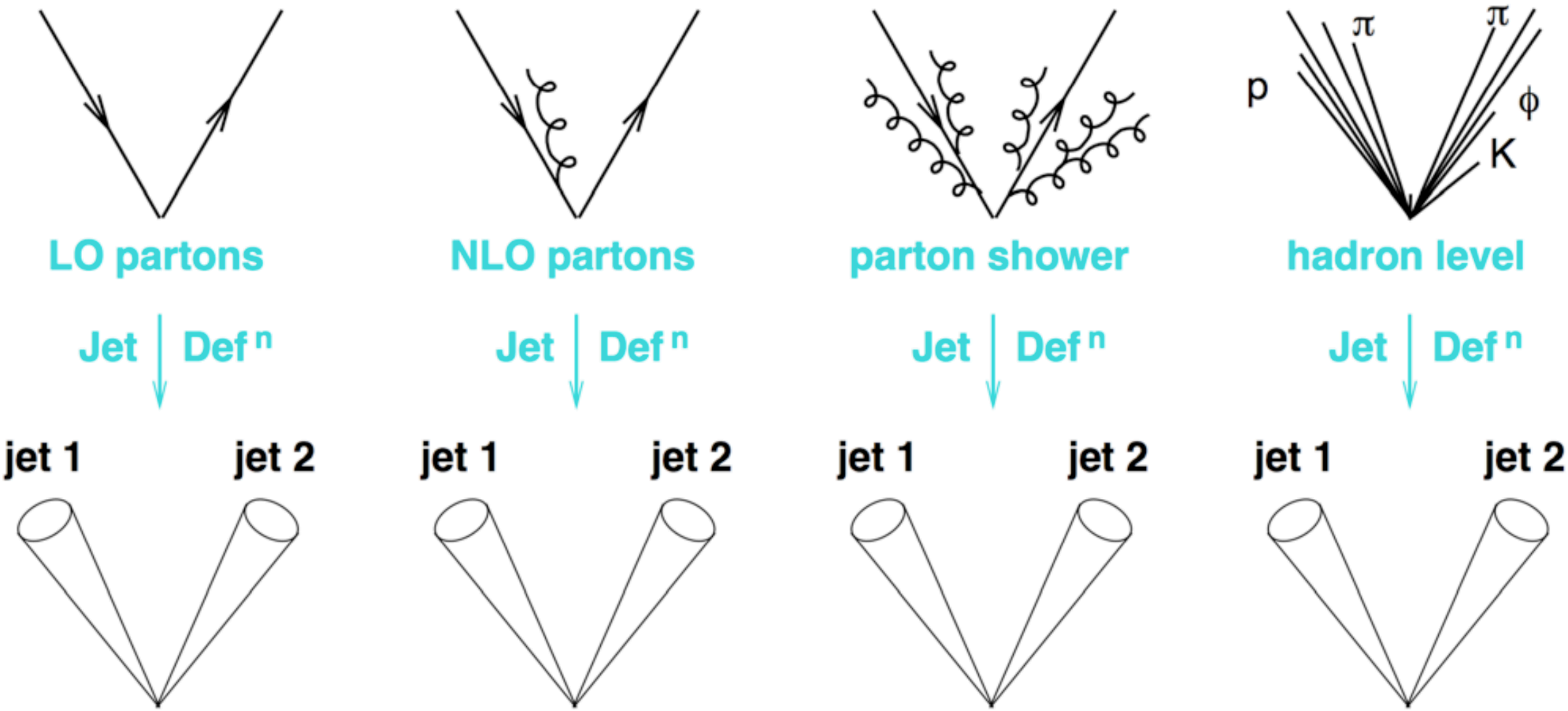
Jets

The requirement of a Jet algorithm



Projection to jets should be resilient to QCD effects

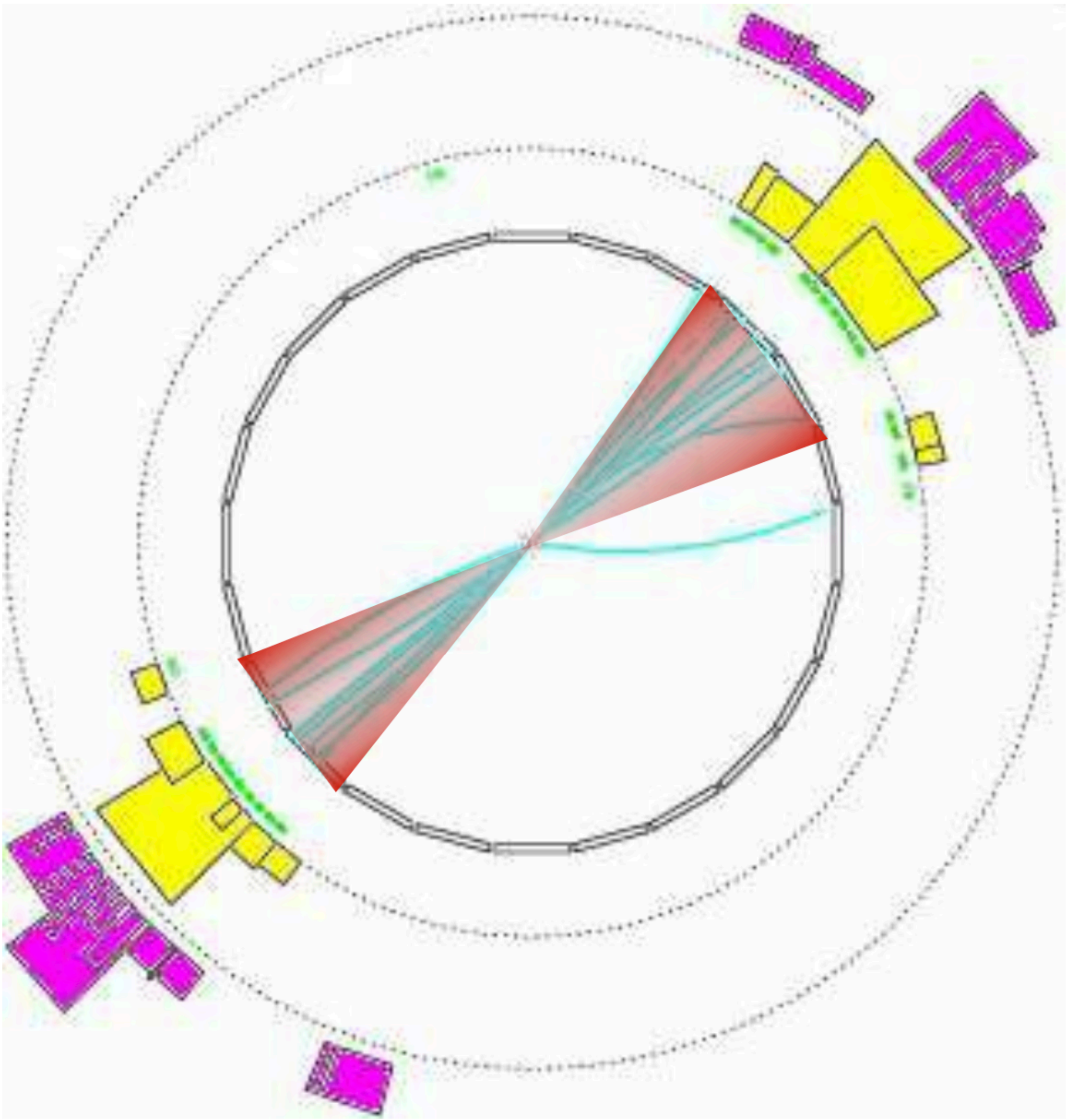
The requirement of a Jet algorithm



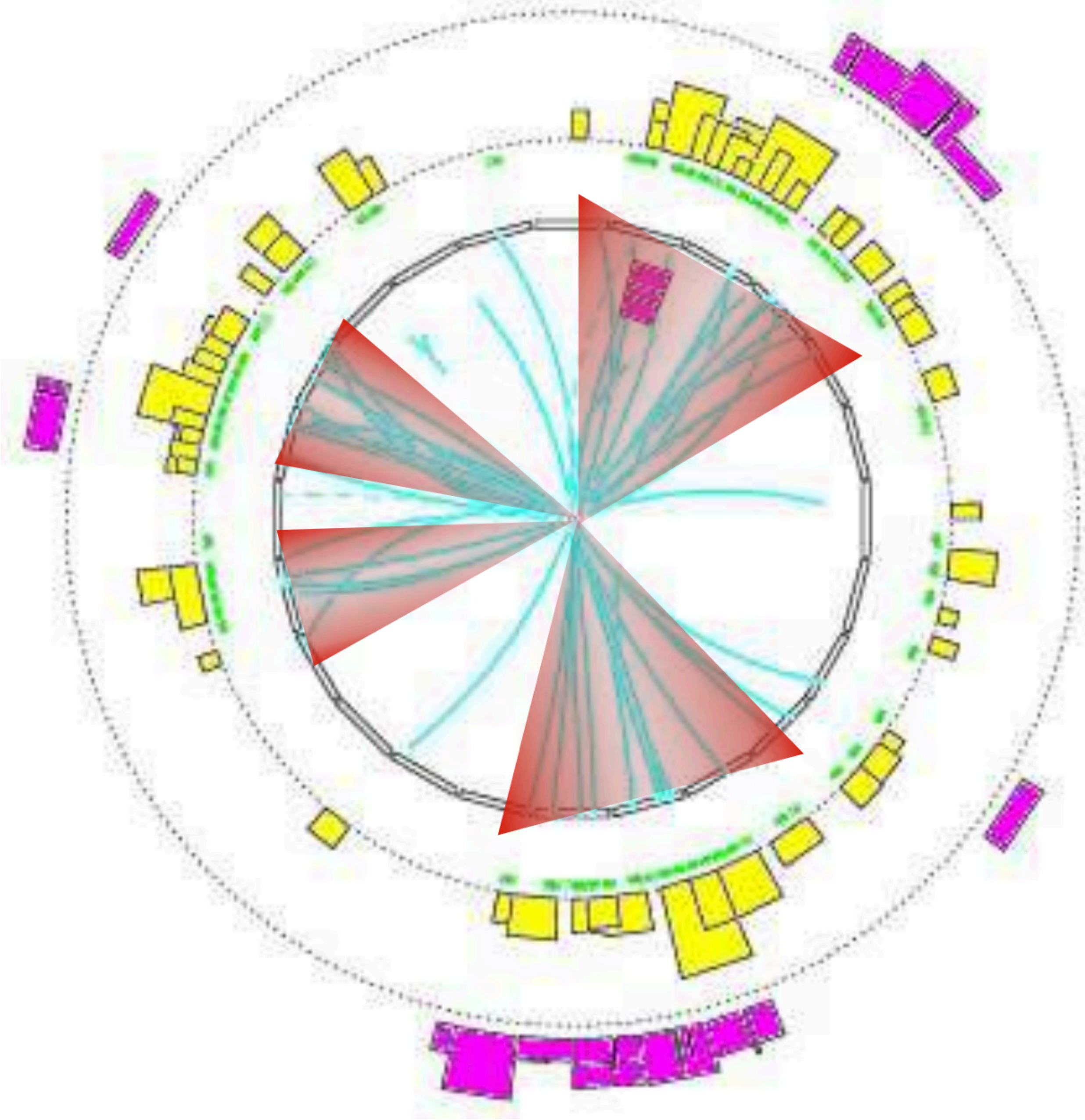
Projection to jets should be resilient to QCD effects

Statuary warning : A JET IS NOT SYNONYMOUS TO PARTON.

Jet reconstruction is a combinatorial puzzle



2 clear jets



3 jets?

or 4 jets?

So what should be a jet definition?

$\{p_i\}$ —— jet definition ——→ $\{j_k\}$

particles,
4-momenta,
calorimeter towers, ...

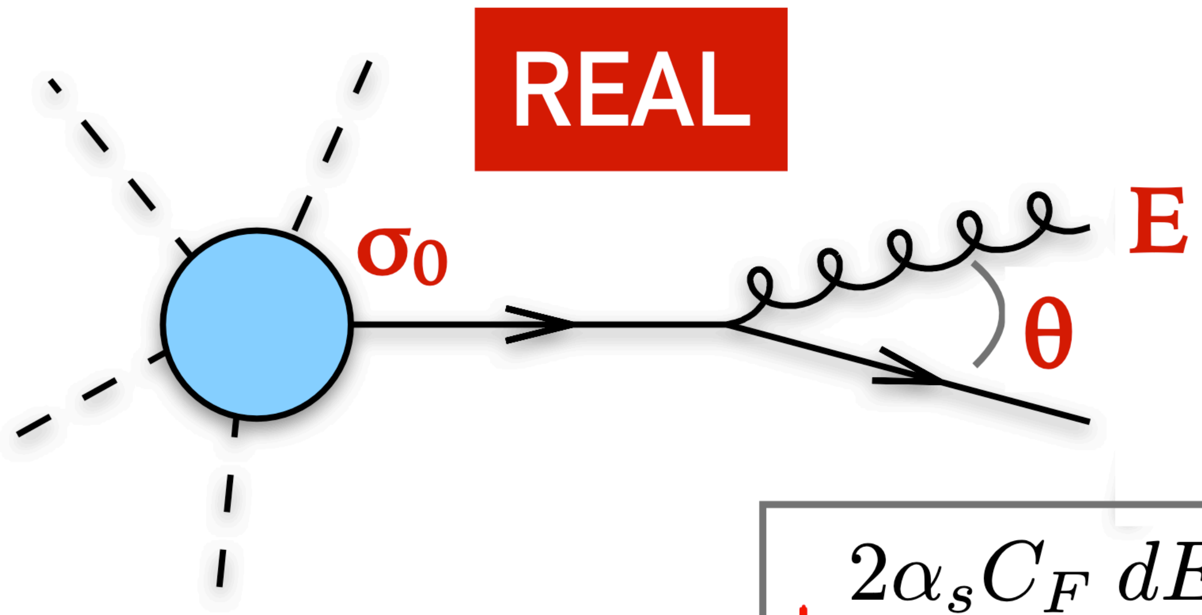
jets

- Which particles do you put together into a same jet?
- How do you recombine their momenta
(4-momentum sum is the obvious choice, right?)

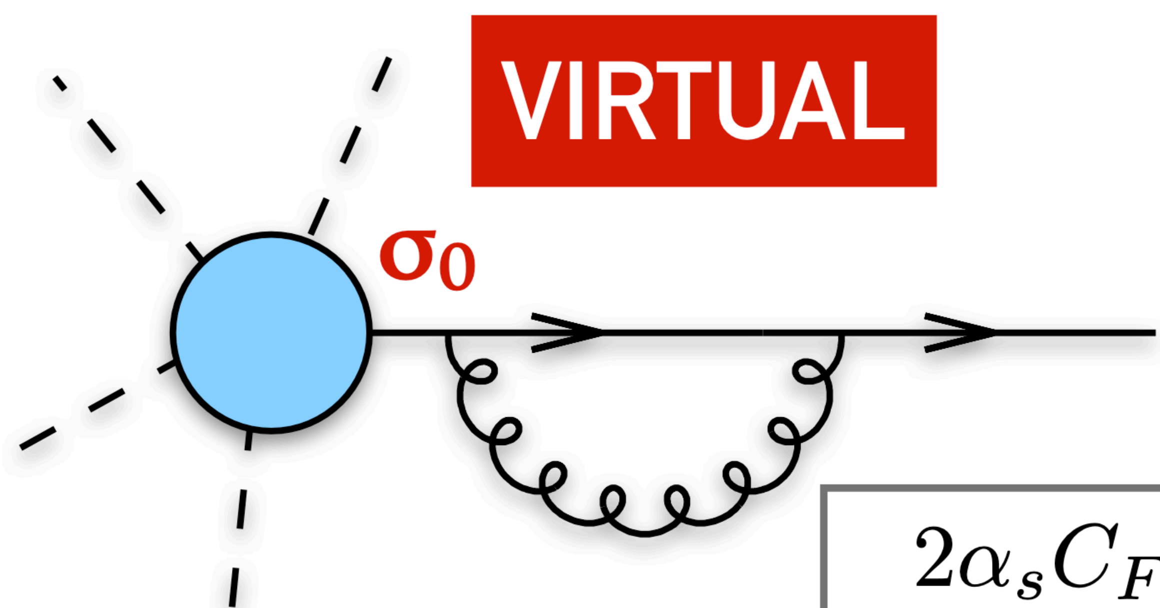
"Jet [definitions] are legal contracts between theorists and experimentalists"
-- MJ Tannenbaum

They're also a way of organising the information in an event
1000's of particles per events, up to 40.000,000 events per second

A jet definition better be IRC safe



$$+\frac{2\alpha_s C_F}{\pi}\frac{dE}{E}\frac{d\theta}{\theta}$$



$$-\frac{2\alpha_s C_F}{\pi}\frac{dE}{E}\frac{d\theta}{\theta}$$

The IRC divergences from real emissions are cancelled by soft modes of loop integral.

If you are "**infrared and collinear safe**", i.e. your measurement doesn't care whether a soft/collinear gluon has been emitted then the **real and virtual divergences cancel.**

What's the point of jet construction

Jets can serve two purposes

- They can be ***observables***, that one can measure and calculate
- They can be ***tools***, that one can employ to extract specific properties of the final state

Different clustering algorithms have different properties and characteristics that can make them more or less appropriate for each of these tasks

IRC safety

An observable is **infrared and collinear safe** if, in the limit of a **collinear splitting**, or the **emission of an infinitely soft** particle, the observable remains **unchanged:**

$$O(X; p_1, \ldots, p_n, p_{n+1} \to 0) \to O(X; p_1, \ldots, p_n)$$

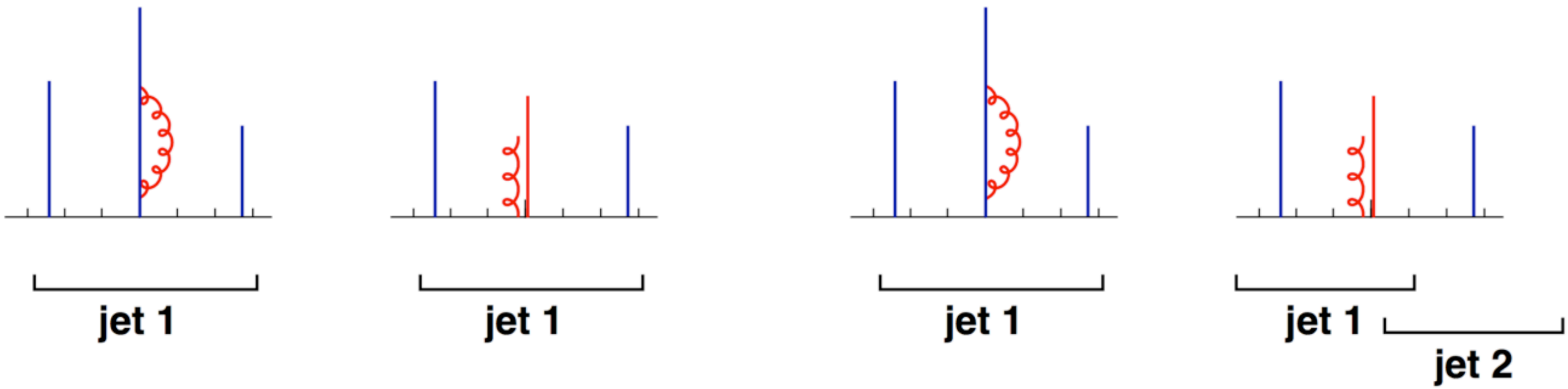
$$O(X; p_1, \ldots, p_n \parallel p_{n+1}) \to O(X; p_1, \ldots, p_n + p_{n+1})$$

This property ensures cancellation of **real** and **virtual** divergences in higher order calculations

If we wish to be able to calculate a jet rate in perturbative QCD the jet algorithm that we use must be IRC safe:
soft emissions and collinear splittings must not change the hard jets

Jet defs : which one is legal?

Collinear Safe | **Collinear Unsafe**



$\alpha_s^n \times (-\infty)$ $\alpha_s^n \times (+\infty)$ $\alpha_s^n \times (-\infty)$ $\alpha_s^n \times (+\infty)$

Infinities cancel | **Infinities do not cancel**

Perturbative calculations of jet observable will only be possible with collinear (and infrared) safe jet definitions

First pointed out by Sterman & Weinberg

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VOLUME 39, NUMBER 23 PHYSICAL REVIEW LETTERS 5 DECEMBER 1977

Jets from Quantum Chromodynamics

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and

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$$\sigma_a = (d\sigma/d\Omega)_0\, \Omega(g_E{}^2/3\pi^2)[-3\ln(E\delta/\mu) - 2\ln^2 2\epsilon - 4\ln(E\delta/\mu)\ln(2\epsilon) + \tfrac{17}{4} - \pi^2/3], \quad (2)$$

$$\sigma_b = (d\sigma/d\Omega)_0\, \Omega(g_E{}^2/3\pi^2)[2\ln^2(2\epsilon E/\mu) - \pi^2/6], \quad (3)$$

$$\sigma_c = (d\sigma/d\Omega)_0\, \Omega\{1 + (g_E{}^2/3\pi^2)[-2\ln^2(E/\mu) + 3\ln(E/\mu) - \tfrac{7}{4} + \pi^2/6]\}, \quad (4)$$

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$$\sigma_a = (d\sigma/d\Omega)_0\, \Omega (g_E{}^2/3\pi^2)[-3\ln(E\delta/\mu) - 2\ln^2 2\epsilon - 4\ln(E\delta/\mu)\ln(2\epsilon) + \tfrac{17}{4} - \pi^2/3], \quad (2)$$

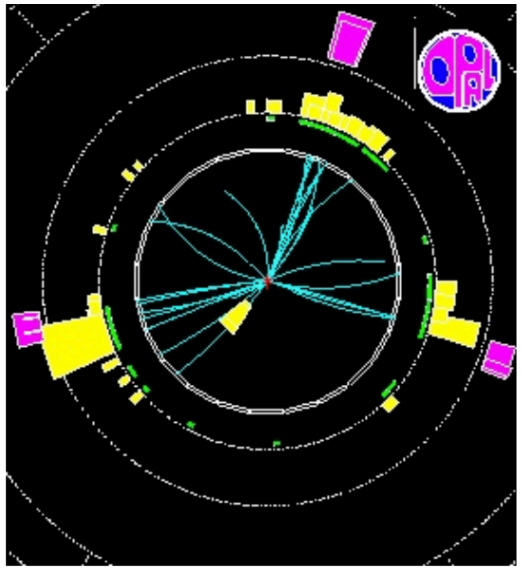
$$\sigma_b = (d\sigma/d\Omega)_0\, \Omega (g_E{}^2/3\pi^2)[2\ln^2(2\epsilon E/\mu) - \pi^2/6], \quad (3)$$

$$\sigma_c = (d\sigma/d\Omega)_0\, \Omega \{1 + (g_E{}^2/3\pi^2)[-2\ln^2(E/\mu) + 3\ln(E/\mu) - \tfrac{7}{4} + \pi^2/6]\}, \quad (4)$$

As expected, each separate contribution is singular for $\mu \to 0$, but cancellations[8] occur in the sum, and the final result is free of mass singularities:

$$\sigma(E, \theta, \Omega, \epsilon, \delta) = (d\sigma/d\Omega)_0\, \Omega[1 - (g_E{}^2/3\pi^2)(3\ln\delta + 4\ln\delta\ln 2\epsilon + \pi^2/3 - \tfrac{5}{2})]. \quad (6)$$

Main approaches of jet clustering



1. Find regions where a lot of energy flows

or

2. Decide which particles are "close", aggregate them

In HEP these are usually called **cone** and **sequential recombination** algorithms respectively

(in other fields they are often called partitional-type clustering and agglomerative hierarchical clustering)

Two main classes of algorithms

- **Sequential recombination algorithms**

  Bottom-up approach: combine particles starting from **closest ones**

  How? Choose a **distance measure**, iterate recombination until few objects left, call them jets

  Works because of mapping closeness ⇔ QCD divergence

  Examples: Jade, k_t, Cambridge/Aachen, anti-k_t, …..

  Usually trivially made IRC safe, but their algorithmic complexity scales like N^3

- **Cone algorithms**

  Top-down approach: find coarse regions of energy flow.

  How? Find **stable cones** (i.e. their axis coincides with sum of momenta of particles in it)

  Works because QCD only modifies energy flow on small scales

  Examples: JetClu, MidPoint, ATLAS cone, CMS cone, SISCone…...

  Can be programmed to be fairly fast, at the price of being complex and IRC unsafe