Quantum Chromodynamics

lecture III

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IMSc Spring School on High Energy Physics Chennai, Feb 26, 2025

Plan

- Introduction to QCD
  Monday, February 24, 2025

- QCD at work: infrared safety and jets
  Tuesday, February 25, 2025

- *QCD at work: factorization and evolution*
  Wednesday, February 26, 2025

- Deep structure of proton
  Thursday, February 27, 2025

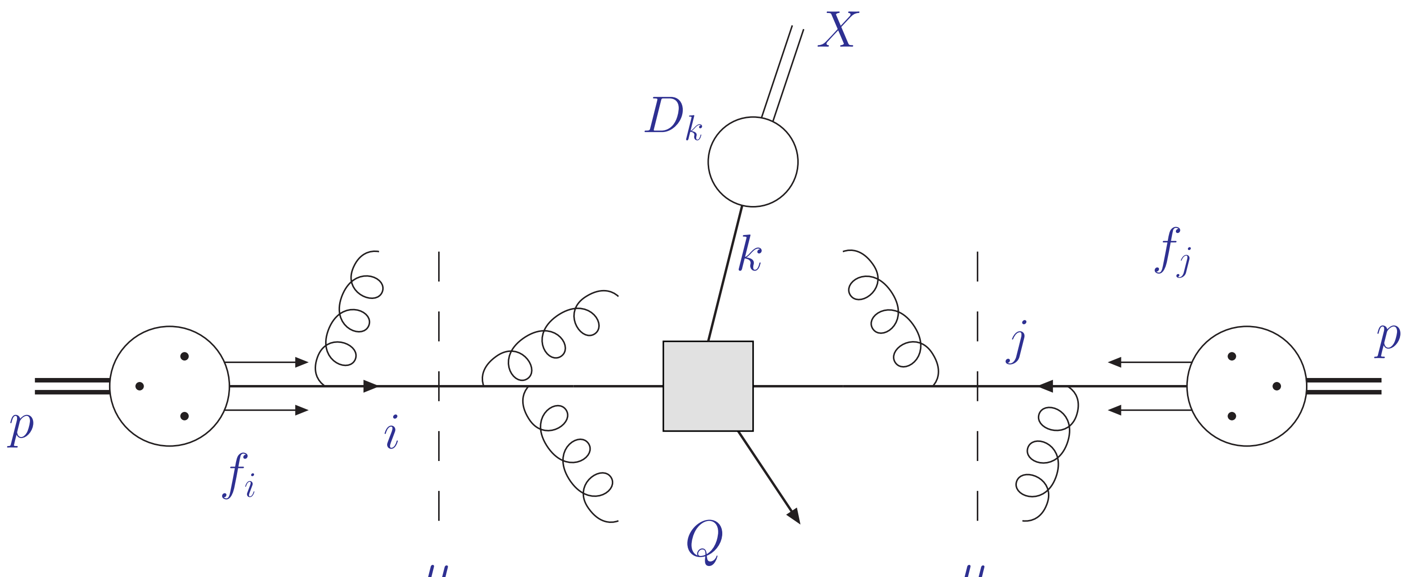
Factorization

- Large class of hard-scattering reactions with initial state hadrons
  - cross section not infrared safe
  - dependent on quark and gluon degrees of freedom in hadron
  - sensitive to nonperturbative processes at long distances
- Factorization of cross section
  - infrared safe hard part $\hat{\sigma}_{\rm pt}$ calculable in perturbative QCD
  - nonperturbative function f determined from data
  - f parametrizes hadron structure
- General structure of cross section
  - large momentum scale Q, factorization scale μ

$$Q^2 \, \sigma_{\rm phys}\,(Q) = \hat{\sigma}_{\rm pt}\,(Q/\mu, \alpha_s(\mu)) \; \otimes \; f\,(\mu)$$

  - convolution $\otimes$ in suitable kinematical variables
- Factorization
  - generalization of operator product expansion

QCD factorization

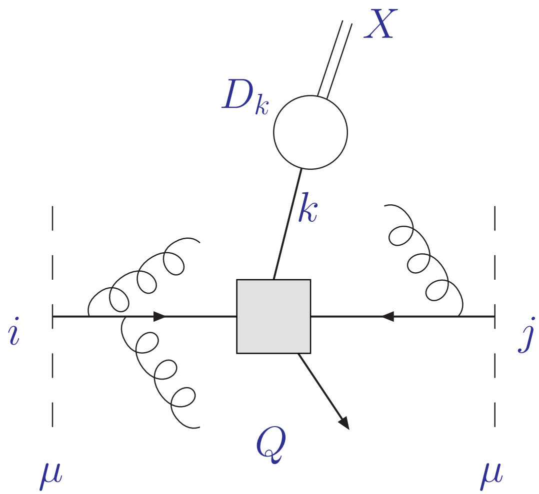


$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to X}\left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)$$

- Factorization at scale μ
  - separation of sensitivity to dynamics from long and short distances
- Hard parton cross section $\hat{\sigma}_{ij\to X}$ calculable in perturbation theory
  - cross section $\hat{\sigma}_{ij\to k}$ for parton types i, j and hadronic final state X
- Non-perturbative parameters: parton distribution functions f_i, strong coupling α_s, particle masses m_X
  - known from global fits to exp. data, lattice computations, ...

Hard scattering cross section

- Parton cross section $\hat{\sigma}_{ij\to k}$ calculable pertubatively in powers of α_s
  - known to NLO, NNLO, . . . ($\mathcal{O}(\text{few}\%)$ theory uncertainty)



- Accuracy of perturbative predictions
  - LO (leading order) ($\mathcal{O}(50 - 100\%)$ unc.)
  - NLO (next-to-leading order) ($\mathcal{O}(10 - 30\%)$ unc.)
  - NNLO (next-to-next-to-leading order) ($\lesssim$ $\mathcal{O}(10\%)$ unc.)
  - N^3LO (next-to-next-to-next-to-leading order)
  - . . .

Parton luminosity

- Long distance dynamics due to proton structure

- Cross section depends on parton distributions f_i

$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \left[\ldots \right]$$

- Parton distributions known from global fits to exp. data
  - available fits accurate to NNLO
  - information on proton structure depends on kinematic coverage

Deep-inelastic scattering

Classic example

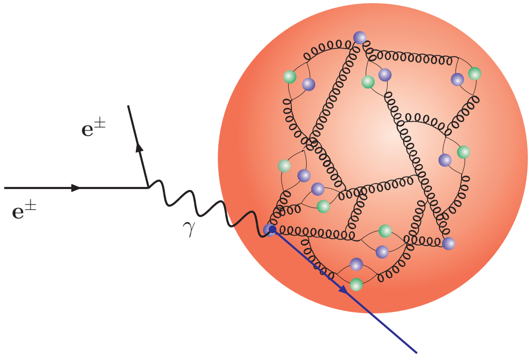
- Deep-inelastic scattering
  - test parton dynamics at factorization scale μ

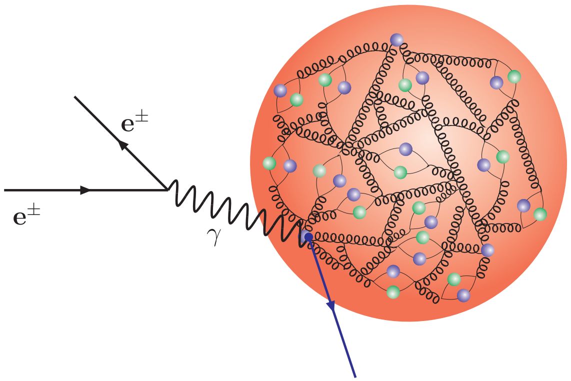
$$\sigma_{\gamma p \to X} = \sum_i f_i(\mu^2) \otimes \hat{\sigma}_{\gamma i \to X}\left(\alpha_s(\mu^2), Q^2, \mu^2\right)$$

Physics picture

- QCD factorization
  - constituent partons from proton interact at short distance
  - photon momentum $Q^2 = -q^2$, Bjorken's $x = Q^2/(2p \cdot q)$
  - low resolution

high resolution



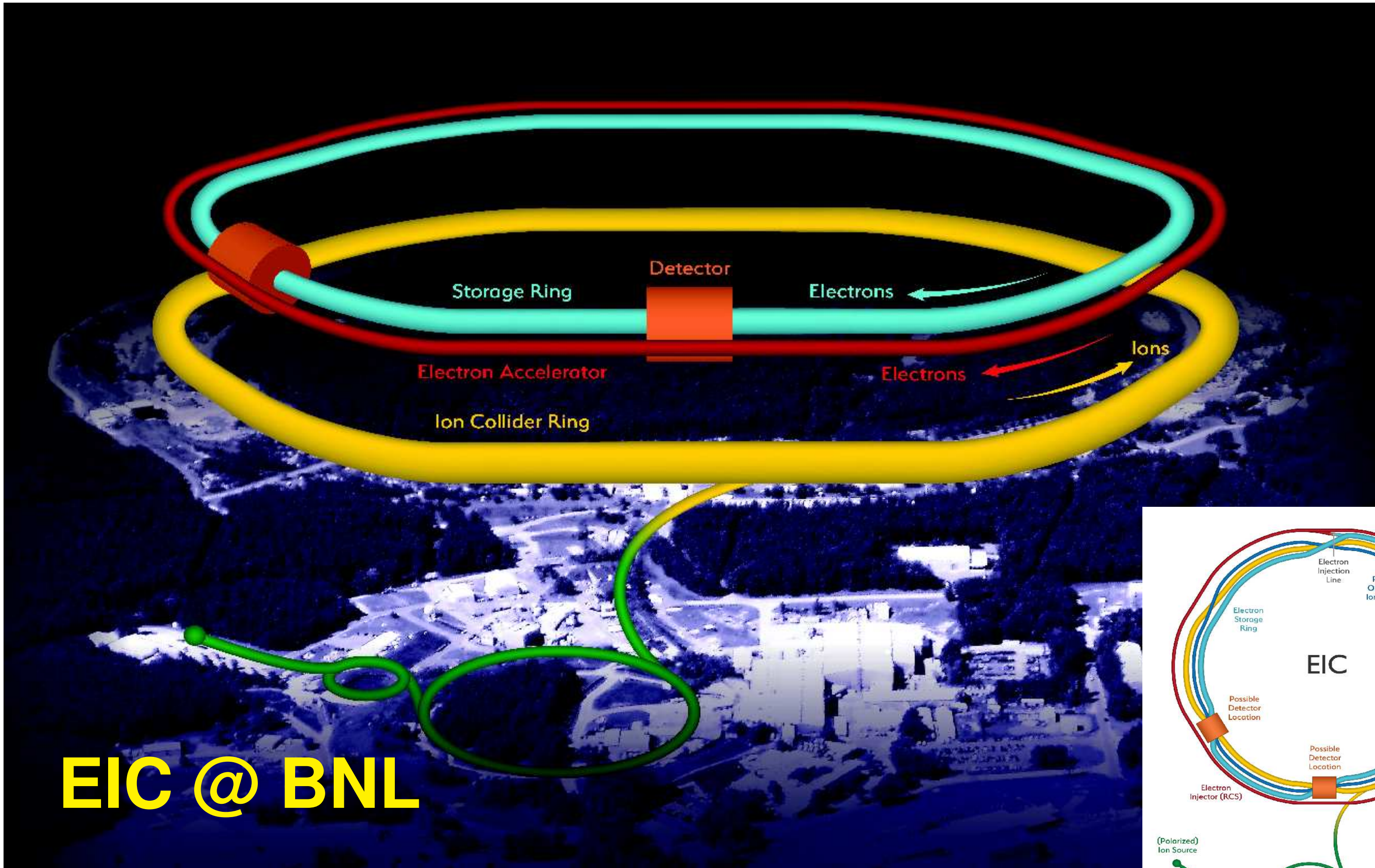


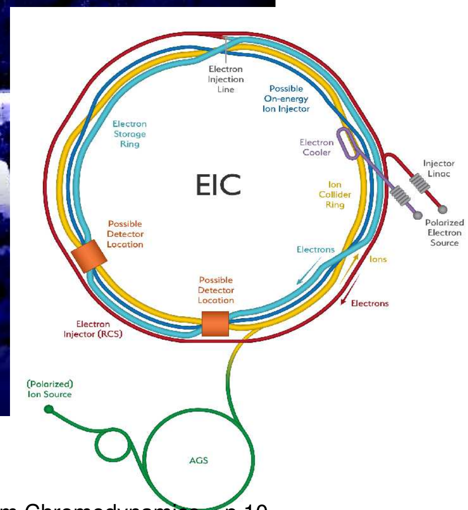
Once upon a time in the north . . .

- HERA: deep structure of proton at highest Q^2 and smallest x

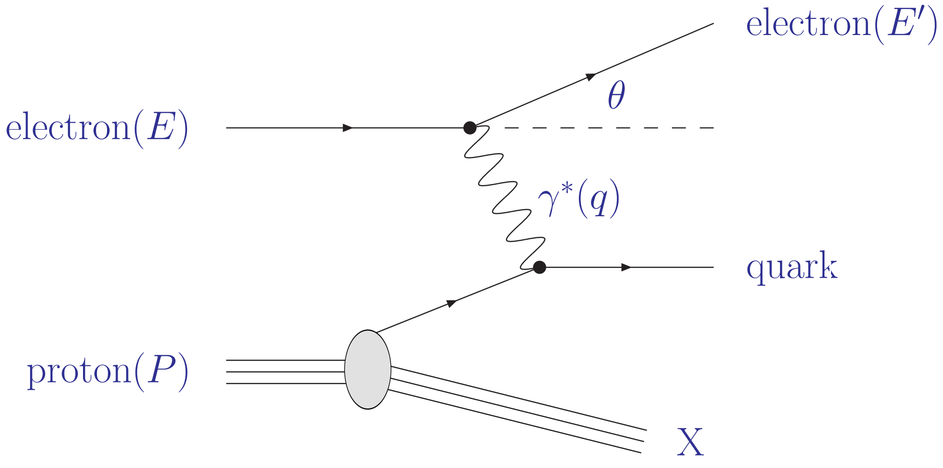
Bright future for precision hadron physics

- Electron-Ion Collider
  A machine that will unlock the secrets of the strongest force in Nature





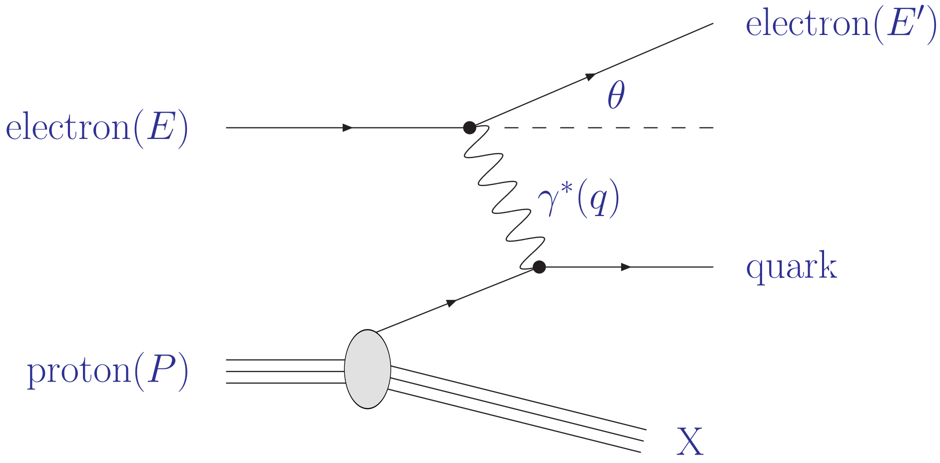
Inelastic electron-proton scattering



- Virtuality of photon: resolution
  $$Q^2 \equiv -q^2 = 4EE' \sin^2(\theta/2)$$
- Bjorken variable: inelasticity
  $$x = \frac{Q^2}{2P \cdot q} < 1$$
- Cross section (X inclusive): proton structure function F_i^p

$$(E - E')\frac{d\sigma}{d\Omega\, dE'} \stackrel{\text{lab}}{=} \underbrace{\frac{\alpha^2 \cos^2 \frac{\theta}{2}}{4E^2 \sin^4 \frac{\theta}{2}}}_{\text{Mott-scattering (point-like)}} \left\{ F_2^p(x, Q^2) + \tan^2 \frac{\theta}{2}\, F_1^p(x, Q^2) \right\}$$

Inelastic electron-proton scattering



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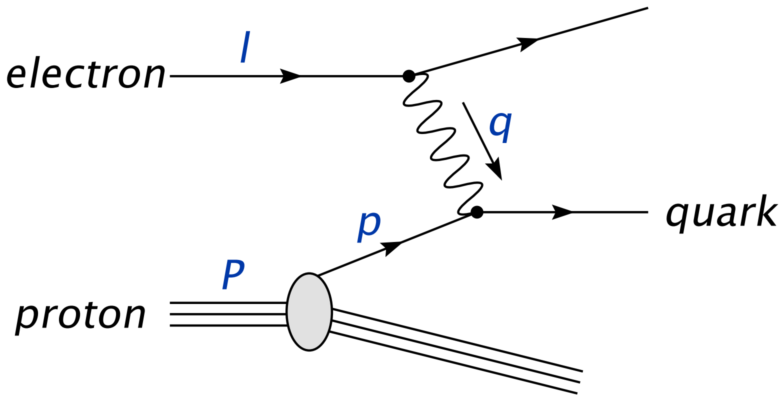
- Deep-inelastic scattering (Bjorken limit: $Q^2 \to \infty$ and x fixed)
  Parton modell (quasi-free point-like constituents, incoherence)
  $$F_2(x,Q^2) \simeq F_2(x) = \sum_i e_i^2\, x f_i(x)$$
  - $x f_i(x)$ distribution for momentum fraction x of parton i

Deep-inelastic scattering



Kinematic variables

- momentum transfer $Q^2 = -q^2$
- Bjorken variable $x = Q^2/(2p \cdot q)$

- Structure function F_2^p (up to order $\mathcal{O}(1/Q^2)$)

$$x^{-1} F_2^p(x, Q^2) \;=\; \sum_i \int_x^1 \frac{d\xi}{\xi}\, C_{2,i}\left(\frac{x}{\xi}, \alpha_s(\mu^2), \frac{\mu^2}{Q^2}\right) f_i^p(\xi, \mu^2)$$

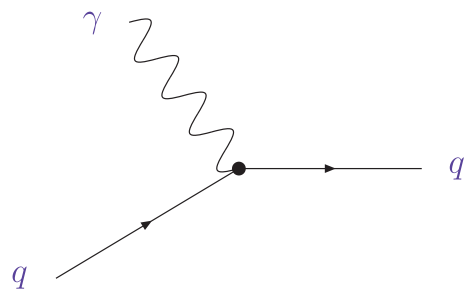
- Coefficient functions

$$C_{a,i} = \alpha_s^n \left(c_{a,i}^{(0)} + \alpha_s c_{a,i}^{(1)} + \alpha_s^2 c_{a,i}^{(2)} + \alpha_s^3 c_{a,i}^{(3)} + \alpha_s^4 c_{a,i}^{(4)} + \ldots \right)$$

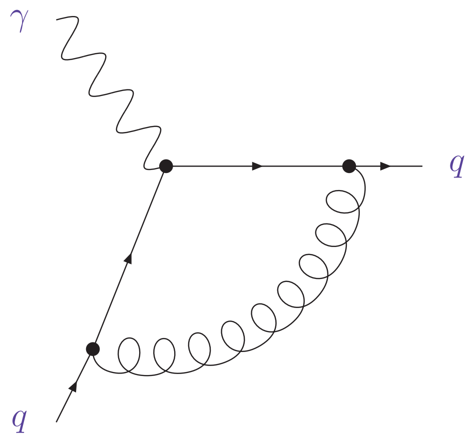
  - current frontier in perturbation theory N^{4}LO (work in progress)

Radiative corrections in a nutshell

- Leading order
  - partonic structure function
    $\hat{F}^{(0)}_{2,q} \;=\; e_q^2 \delta(1-x)$

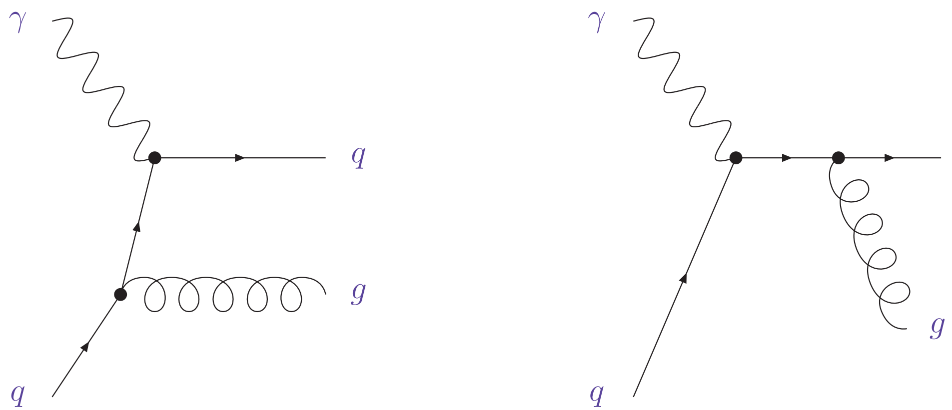


- Next-to-leading order
  - virtual correction
    (infrared divergent; proportional to Born)
  - dimensional regularization $D = 4 - 2\epsilon$



$$\hat{F}^{(1),v}_{2,q} \quad = \quad e_q^2\, C_F\, \frac{\alpha_s}{4\pi}\, \delta(1-x) \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \zeta_2 + \mathcal{O}(\epsilon)\right)$$

- Next-to-leading order



  - add real and virtual corrections $\hat{F}^{(1)}_{2,q} = \hat{F}^{(1),r}_{2,q} + \hat{F}^{(1),v}_{2,q}$
  - collinear divergence remains splitting functions $P^{(0)}_{qq}$

$$
\begin{aligned}
\hat{F}^{(1)}_{2,q} \quad = \quad & e_q^2\, C_F\, \frac{\alpha_s}{4\pi} \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} \Bigg\{ \frac{1}{\epsilon} \left(\frac{4}{1-x} - 2 - 2x + 3\delta(1-x) \right) \\
& +4\frac{\ln(1-x)}{1-x} - 3\frac{1}{1-x} - (9+4\zeta_2)\delta(1-x) \\
& \quad -2(1+x)(\ln(1-x) - \ln(x)) - 4\frac{1}{1-x}\ln(x) + 6 + 4x \\
& +\mathcal{O}(\epsilon) \Bigg\}
\end{aligned}
$$

- Structure of NLO correction
  - absorb collinear divergence $P_{qq}^{(0)}$ in renormalized parton distributions

$$\hat{F}_{2,q}^{(1),\mathrm{bare}} = e_q^2 \frac{\alpha_s}{4\pi} \left(\frac{\mu^2}{Q^2} \right)^{\epsilon} \left\{ \frac{1}{\epsilon} P_{qq}^{(0)}(x) + c_{2,q}^{(1)}(x) + \mathcal{O}(\epsilon) \right\}$$

$$f_q^{p,\mathrm{ren}}(\mu_F^2) = f_q^{p,\mathrm{bare}} - \frac{\alpha_s}{4\pi} \frac{1}{\epsilon} P_{qq}^{(0)}(x) \left(\frac{\mu^2}{\mu_F^2} \right)^{\epsilon}$$

  - partonic (physical) structure function at factorization scale μ_F

$$\hat{F}_{2,q} = e_q^2 \left(\delta(1-x) + \frac{\alpha_s}{4\pi} \left\{ c_{2,q}^{(1)}(x) - \ln\left(\frac{Q^2}{\mu_F^2} \right) P_{qq}^{(0)}(x) \right\} \right)$$

- Details in chapt. 4.8 of
  The Theory of Quark and Gluon Interactions
  F.J. Yndurain

Wilson coefficients

- Recall QCD corrections to Wilson coefficients $c^{(1)}_{2,q}$ in $\hat{F}^{(1)}_{2,q}$ (cross section of hard parton scattering)

$$\hat{F}^{(1)}_{2,q} \quad = \quad e_q^2\, C_F\, \frac{\alpha_s}{4\pi} \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} \left\{ \frac{1}{\epsilon} \left(\frac{4}{1-x} - 2 - 2x + 3\delta(1-x) \right) \right.$$
$$+4\frac{\ln(1-x)}{1-x} - 3\frac{1}{1-x} - (9 + 4\zeta_2)\delta(1-x)$$
$$\left. -2(1+x)(\ln(1-x) - \ln(x)) - 4\frac{1}{1-x}\ln(x) + 6 + 4x + \mathcal{O}(\epsilon) \right\}$$

  - large radiative corrections as $x \to 1$

- Mellin transform with moments N (integral ransform $x \to N$)

$$f(N) \;=\; \int\limits_0^1 dx\, x^{N-1} f(x)$$

  - dictionary:

$$\begin{aligned} [\ln(1-x)/(1-x)]_+ \quad &\to \quad \ln^2 N \\ [1/(1-x)]_+ \quad &\to \quad \ln N \\ \delta(1-x) \quad &\to \quad \mathrm{const}_N \end{aligned}$$

Threshold resummation

- Coefficient function in large x-limit have large logarithms at n$^{\rm th}$-order

$$\alpha_s^n \frac{\ln^{2n-1}(1-x)}{(1-x)_+} \longleftrightarrow \alpha_s^n \ln^{2n}(N)$$

- Threshold resummation in Mellin space

$$C^N = (1 + \alpha_s\, g_{01} + \alpha_s^2\, g_{02} + \ldots) \cdot \exp\left(G^N\right) + \mathcal{O}(N^{-1} \ln^n N)$$

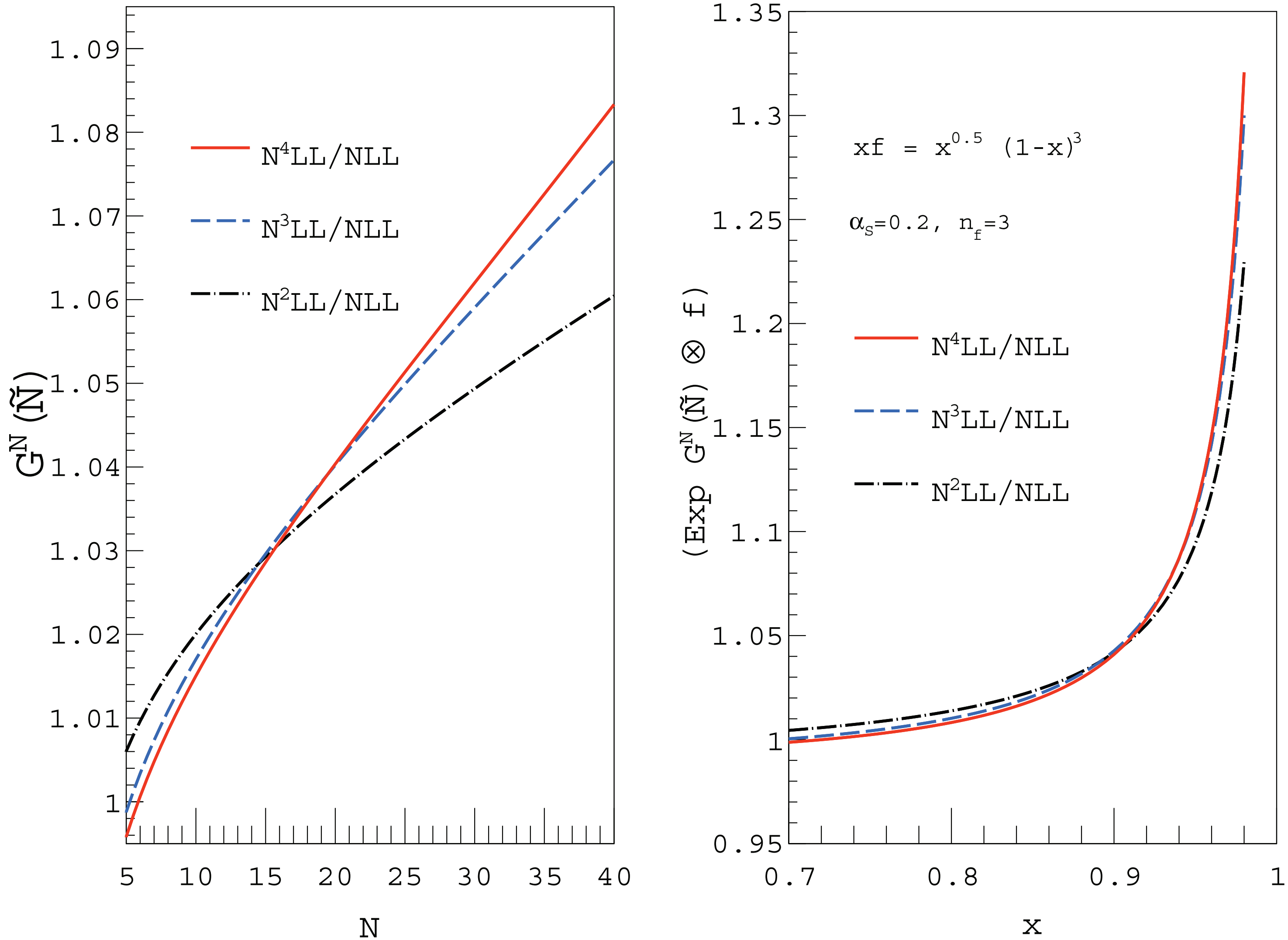
- Control over logarithms $\ln(N)$ with $\lambda = \beta_0 \alpha_s \ln(N)$ to N^{k}LL accuracy

$$G^N = \ln(N) g_1(\lambda) + g_2(\lambda) + \alpha_s g_3(\lambda) + \alpha_s^2 g_4(\lambda) + \alpha_s^3 g_5(\lambda) + \ldots$$

  - $g_1(\lambda)$: LL Sterman '87; Appell, Mackenzie, Sterman '88
  - $g_2(\lambda)$: NLL Catani Trenatdue '89
  - $g_3(\lambda)$: NNLL or N^{2}LL Vogt '00; Catani, Grazzini, de Florian, Nason '03
  - $g_4(\lambda)$: N^{3}LL S.M., Vermaseren, Vogt '05
  - $g_5(\lambda)$: N^{4}LL Das, S.M., Vogt '19

- Resummed G^N predicts fixed orders in perturbation theory
  - generating functional for towers of large logarithms

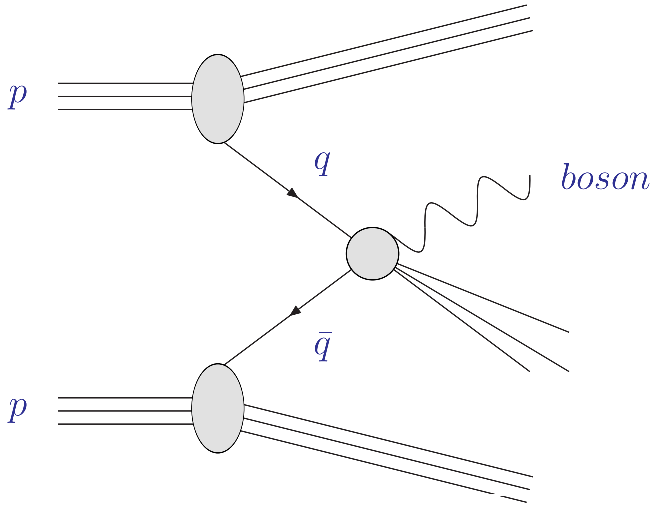
Numerical results for DIS



- Left: Resummed exponent G^N normalized to NLL for DIS plotted successively up to N^{4}LL for $\alpha_s = 0.2$ and $n_f = 3$
- Right: Resummed series convoluted with typical shape for a quark distribution $xf = x^{0.5}(1-x)^3$ up to N^{4}LL

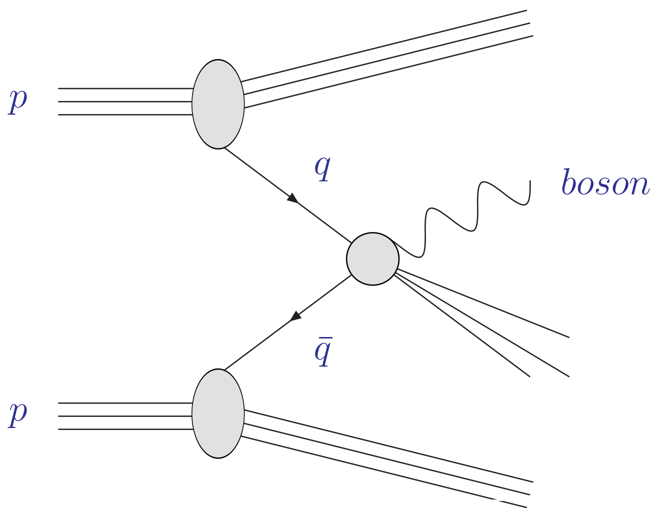
Drell-Yan process

Vector boson production



- Kinematical variables (inclusive)
  - energy (cms) $s = Q^2$ (time-like)
  - scaling variable $x = M^2_{W^\pm/Z}/s$

Vector boson production



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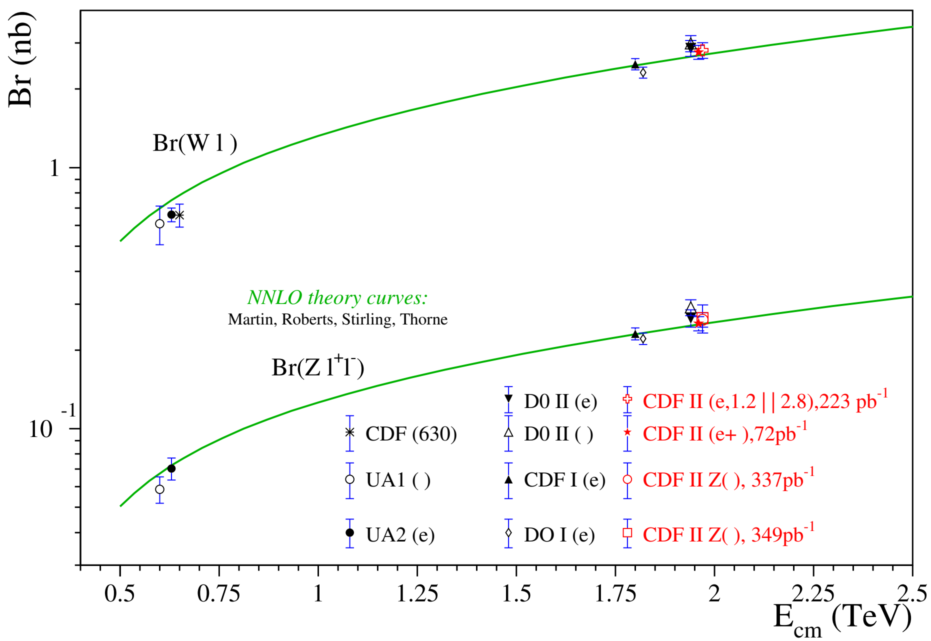
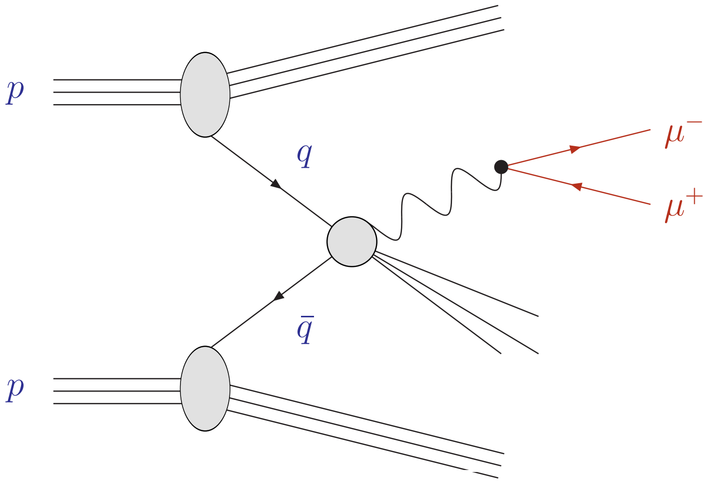


- Early measurements of $W^\pm$ and Z cross sections at hadron colliders

  CERN's SppS; Fermilab's Tevatron

QCD corrections to W/Z production



- Hadronic cross section $\sigma_{pp\to V}$ with $\tau = M_V^2/s$ and $V = \gamma^*/W^\pm/Z$
  - renormalization/factorization (hard) scale $\mu = \mathcal{O}(M_V)$

$$\sigma_{pp\to V} = \sum_{ij} \int_\tau^1 \frac{dx_1}{x_1} \int_{x_1}^1 \frac{dx_2}{x_2} f_i\left(\frac{x_1}{x_2}, \mu^2\right) f_j\left(x_2, \mu^2\right) \hat{\sigma}_{ij\to V}\left(\frac{\tau}{x_1}, \frac{\mu^2}{M_V^2}, \alpha_s(\mu^2)\right)$$

- Partonic cross section $\hat{\sigma}_{ij\to V}$

$$\hat{\sigma}_{ij\to V} \quad = \quad \underbrace{\alpha_s^2 \Big[\hat{\sigma}^{(0)}_{ij\to V} + \alpha_s\, \hat{\sigma}^{(1)}_{ij\to V}}_{} \; + \; \alpha_s^2\, \hat{\sigma}^{(2)}_{ij\to V} \; + \; \dots \Big]$$

NLO: standard approximation (large uncertainties)

Kinematics (differential)

- Proton-proton scattering (two broad-band beams of incoming partons)
  - cms of parton-parton scattering boosted wrt incoming protons
- Final state variables (simple transformations under longitud. boosts)

$$p^\mu = (E, p_x, p_y, p_z) = (m_t \cosh y, p_t \sin \phi, p_t \cos \phi, m_t \sinh y)$$

  - rapidity $y = \frac{1}{2} \ln \left(\frac{E + p_z}{E - p_z} \right)$
  - transverse momentum p_t and mass $m_t = \sqrt{p_t^2 + m^2}$
  - azimuthal angle ϕ

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  - transverse momentum p_t and mass $m_t = \sqrt{p_t^2 + m^2}$
  - azimuthal angle ϕ
- Differences in rapidity Δy and azimuthal angle $\Delta \phi$ invariant under boosts
- In practice (for $E \gg m_p$)
  - pseudo-rapidity $\eta = -\ln \tan \left(\frac{\theta}{2} \right)$ with angle from beam axis

Differential distributions (I)

- Cross section for $\hat{\sigma}_{q\bar{q}\to e^+e^-}$
  - Born cross section

$$\hat{\sigma}_{q\bar{q}\to e^+e^-} = \frac{4\pi\alpha^2}{3s}\,\frac{e_q^2}{N_c} = \sigma^{(0)}\,\frac{e_q^2}{N_c}$$

- Born result for invariant mass distribution $\dfrac{d\hat{\sigma}}{dM^2}$
  - use of $s = M_V^2$ implies $\delta(s - M^2)$

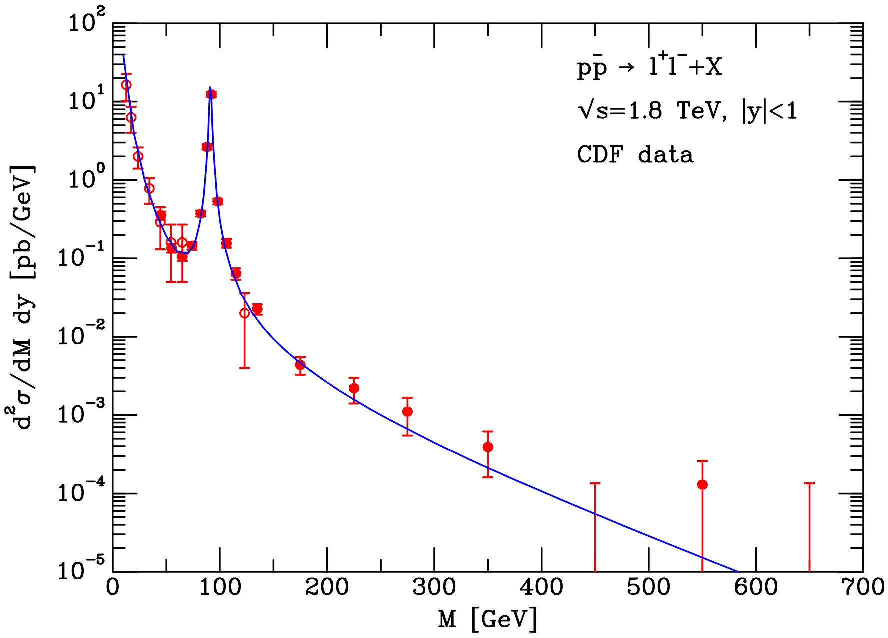
$$M^2\frac{d\hat{\sigma}}{dM^2} = \sigma^{(0)}\,\frac{e_q^2}{N_c}\,\delta(s - M^2)$$

- Hadronic cross section from convolution with parton distributions
  - Born result

$$M^4\frac{d\sigma}{dM^2} = \sigma^{(0)}\,\frac{1}{N_c}\,\frac{M^2}{s}\,\times$$

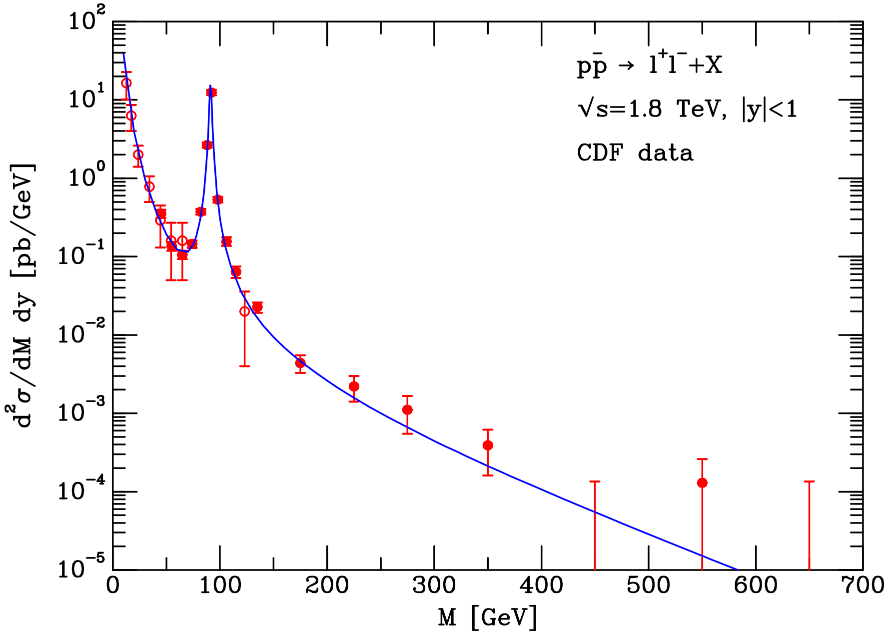
$$\int\limits_0^1 dx_1\,dx_2\,\delta\left(x_1x_2 - \frac{M^2}{s}\right)\sum_q e_q^2\,\left\{f_q(x_1)\,f_{\bar{q}}(x_2) + f_{\bar{q}}(x_1)\,f_q(x_2)\right\}$$

Differential distributions (II)



- Invariant mass distribution $\frac{d\sigma}{dM^2}$ of lepton pair for Z-production in $p\bar{p}$-collisions
  - CDF data at $\sqrt{s} = 1.8$ TeV and NLO QCD prediction

Differential distributions (II)



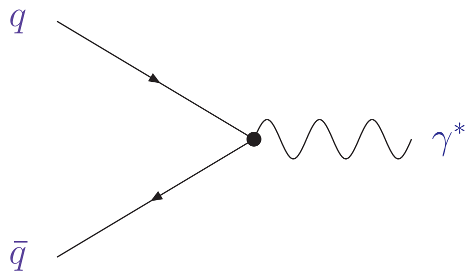
- Invariant mass distribution $\frac{d\sigma}{dM^2}$ of lepton pair for Z-production in $p\bar{p}$-collisions
  - CDF data at $\sqrt{s} = 1.8$ TeV and NLO QCD prediction

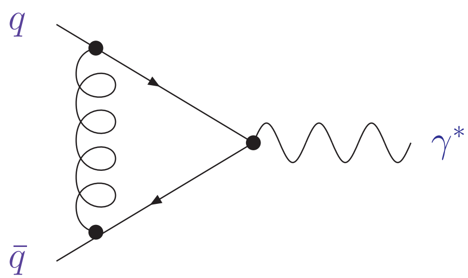
- Double-differential cross section $\frac{d\sigma}{dM^2 dy}$ local in PDFs
  - $y = \frac{1}{2} \ln \left(\frac{x_1}{x_2} \right)$ lepton-pair rapidity

Radiative corrections in a nutshell

- Leading order
  - partonic cross section $x = \tau / x_1$

    $\hat{\sigma}^{(0)}_{q\bar{q} \to V} \; = \; \delta(1-x)$

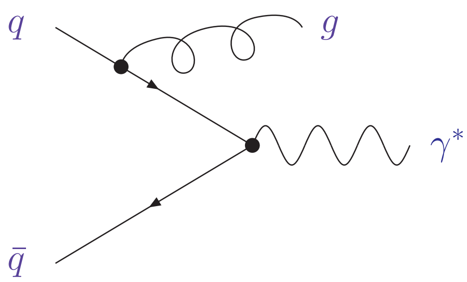




- Next-to-leading order
  - virtual correction
    (infrared divergent; proportional to Born)
  - dimensional regularization $D = 4 - 2\epsilon$

$$\hat{\sigma}^{(1),v}_{q\bar{q} \to V} \quad = \quad C_F \, \frac{\alpha_s}{4\pi} \, \delta(1-x) \left(\frac{\mu^2}{M_V^2} \right)^{\epsilon} \left(- \frac{4}{\epsilon^2} - \frac{6}{\epsilon} - 16 + 2\zeta_2 + \mathcal{O}(\epsilon) \right)$$

- Next-to-leading order



  - add real and virtual corrections $\hat{\sigma}^{(1)}_{q\bar{q}\to V} = \hat{\sigma}^{(1),r}_{q\bar{q}\to V} + \hat{\sigma}^{(1),v}_{q\bar{q}\to V}$
  - collinear divergence remains in splitting function $P^{(0)}_{qq}$

$$\hat{\sigma}^{(1)}_{q\bar{q}\to V} = \frac{\alpha_s}{4\pi} C_F \left(\frac{\mu^2}{M_V^2}\right)^{\epsilon} \left\{ \frac{1}{\epsilon} \left(\frac{8}{1-x} - 4 - 4x + 6\delta(1-x) \right) \right.$$
$$+ \left(16 \frac{\ln(1-x)}{1-x} + (-16 + 8\zeta_2)\,\delta(1-x) \right.$$
$$\left. -4 \frac{1+x^2}{1-x} \ln(x) + 8 \frac{1+x^2}{1-x} \ln(1-x) \right)$$
$$\left. + \mathcal{O}(\epsilon) \right\}$$

- Structure of NLO correction
  - absorb collinear divergence $P^{(0)}_{qq}$ in renormalized parton distributions

$$\hat{\sigma}^{(1),\mathrm{bare}}_{q\bar{q}\to V} = \frac{\alpha_s}{4\pi}\left(\frac{\mu^2}{M_V^2}\right)^{\epsilon}\left\{\frac{1}{\epsilon}\,2\,P^{(0)}_{qq}(x) + \hat{\sigma}^{(1)}_{q\bar{q}\to V}(x) + \mathcal{O}(\epsilon)\right\}$$

$$q^{\mathrm{ren}}(\mu_F^2) = q^{\mathrm{bare}} - \frac{\alpha_s}{4\pi}\,\frac{1}{\epsilon}\,P^{(0)}_{qq}(x)\left(\frac{\mu^2}{\mu_F^2}\right)^{\epsilon}$$

  - partonic (physical) structure function at factorization scale μ_F

$$\hat{\sigma}_{q\bar{q}\to V} = \delta(1-x) + \frac{\alpha_s}{4\pi}\left\{\hat{\sigma}^{(1)}_{q\bar{q}\to V}(x) - \ln\left(\frac{M_V^2}{\mu_F^2}\right)2\,P^{(0)}_{qq}(x)\right\}$$

Coefficient functions

- QCD corrections to coefficient function of hard scattering in $\hat{\sigma}^{(1)}_{q\bar{q}\to V}$

$$\hat{\sigma}^{(1)}_{q\bar{q}\to V} = \frac{\alpha_s}{4\pi} C_F \left(\frac{\mu^2}{M_V^2}\right)^{\epsilon} \left\{ \frac{1}{\epsilon} \left(\frac{8}{1-x} - 4 - 4x + 6\delta(1-x) \right) \right.$$

$$+ \left(16 \frac{\ln(1-x)}{1-x} + (-16 + 8\zeta_2)\, \delta(1-x) \right.$$

$$\left. -4 \frac{1+x^2}{1-x} \ln(x) + 8 \frac{1+x^2}{1-x} \ln(1-x) \right)$$

$$\left. + \mathcal{O}(\epsilon) \right\}$$

  - large radiative corrections as $x \to 1$

Threshold resummation

- Improved predictions for Drell-Yan process near production threshold by resumming large logarithms
- Enhanced cross-section accuracy by accounting for soft and collinear radiation effects

Summary (part III)

Perturbative QCD at work

- Factorization and evolution
  - scattering with initial state hadrons requires collinear factorization
  - separation of long and short distance physics
  - factorization induces evolution equations via renormalization group
  - parton distribution function

Inclusive DIS

- NLO QCD corrections
  - illustration of factorization, infrared safety and evolution for $ep \to X$
  - intuitive picture for parton evolution and proton structure

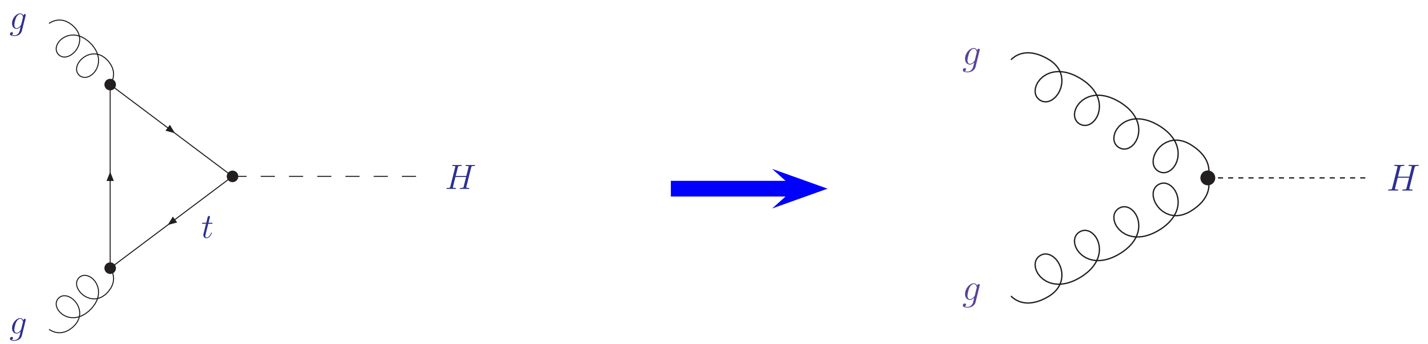
$W^{\pm}/Z$*-boson production*

- $W^{\pm}/Z$-boson production at hadron colliders
- NLO QCD corrections
  - illustration of factorization, infrared safety and evolution for $q\bar{q} \to V$

Back-up

Higgs boson production in gg-fusion

Effective theory

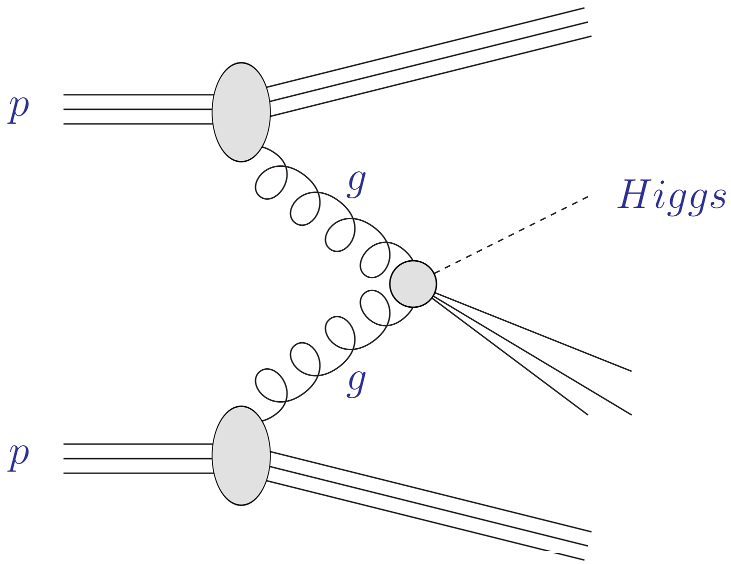


- Integration of top-quark loop (finite result)
  - decay width $H \to gg$ ($m_q = 0$ for light quarks, m_t heavy)

$$\Gamma_{H\to gg} = \frac{G_\mu\, m_H^3}{64\,\sqrt{2}\,\pi^3}\, \alpha_s^2\; f\left(\frac{m_H^2}{4m_t^2}\right)$$

- Effective theory in limit $m_t \to \infty$; Lagrangian $\mathcal{L} = -\frac{1}{4}\,\frac{H}{v}\,C_H\,G^{\mu\nu\,a}\,G^a_{\mu\nu}$
  - operator $HG^{\mu\nu\,a}\,G^a_{\mu\nu}$ relates to stress-energy tensor
  - additional renormalization proportional to QCD β-function required
    Kluberg-Stern, Zuber ‘75; Collins, Duncan, Joglekar ‘77

QCD corrections to ggF



- Hadronic cross section $\sigma_{pp\to H}$ with $\tau = m_H^2/S$
  - renormalization/factorization (hard) scale $\mu = \mathcal{O}(m_H)$

$$\sigma_{pp\to H} = \sum_{ij} \int_\tau^1 \frac{dx_1}{x_1} \int_{x_1}^1 \frac{dx_2}{x_2} f_i\left(\frac{x_1}{x_2}, \mu^2\right) f_j\left(x_2, \mu^2\right) \hat{\sigma}_{ij\to H}\left(\frac{\tau}{x_1}, \frac{\mu^2}{m_H^2}, \alpha_s(\mu^2)\right)$$

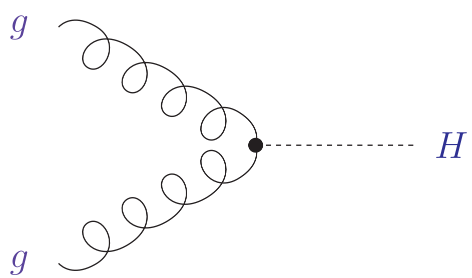
- Partonic cross section $\hat{\sigma}_{ij\to H}$

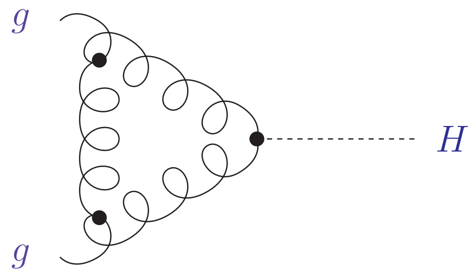
$$\hat{\sigma}_{ij\to H} \quad = \quad \underbrace{\alpha_s^2 \Big[\, \hat{\sigma}^{(0)}_{ij\to H} + \alpha_s\, \hat{\sigma}^{(1)}_{ij\to H}}_{\text{NLO: standard approximation (large uncertainties)}} \; + \; \alpha_s^2\, \hat{\sigma}^{(2)}_{ij\to H} \; + \; \ldots \Big]$$

Radiative corrections in a nutshell

- Leading order
  - partonic cross section $x = \tau/x_1$

    $\hat{\sigma}^{(0)}_{gg\to H} \;=\; \delta(1-x)$





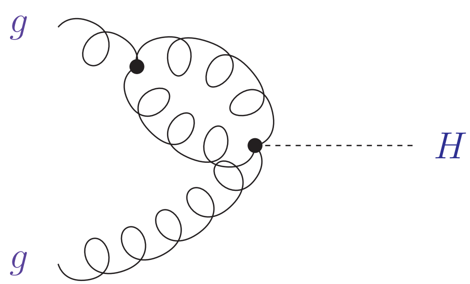
- Next-to-leading order
  - virtual correction (time-like kinematics)
    (infrared divergent; proportional to Born)
  - dimensional regularization $D = 4 - 2\epsilon$

$$\hat{\sigma}^{(1),v}_{gg\to H} \;=\; C_A\,\frac{\alpha_s}{4\pi}\,\delta(1-x)\left(\frac{\mu^2}{m_H^2}\right)^{\epsilon}\left(-\frac{2}{\epsilon^2}+7\,\zeta_2+\mathcal{O}(\epsilon)\right)$$

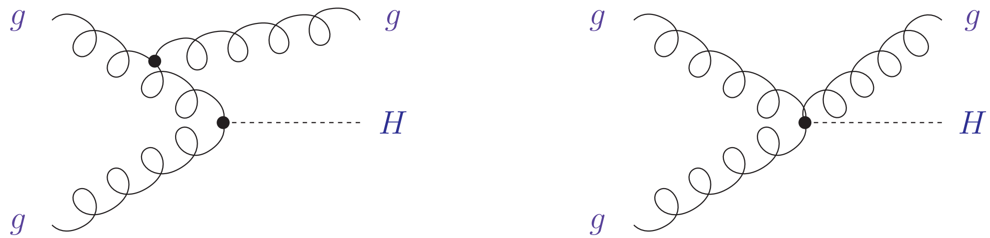
  - additional contribution from renormalization of effective operator

$$\alpha_s^{\rm bare} \;=\; \alpha_s^{\rm ren}\left\{1-\frac{\beta_0}{\epsilon}\,\frac{\alpha_s^{\rm ren}}{4\pi}+\mathcal{O}(\alpha_s^2)\right\}$$

  - massless tadpoles vanish
    in dimensional regularization



- Next-to-leading order



- add real and virtual corrections $\hat{\sigma}^{(1)}_{gg\to H} = \hat{\sigma}^{(1),r}_{gg\to H} + \hat{\sigma}^{(1),v}_{gg\to H}$
- collinear divergence remains in splitting function $P^{(0)}_{gg}$

$$\hat{\sigma}^{(1)}_{gg\to H} = \frac{\alpha_s}{4\pi}\left(\frac{\mu^2}{m_H^2}\right)^{\epsilon}\Bigg\{$$

$$\frac{1}{\epsilon}C_A\left(\frac{8}{1-x}+\frac{8}{x}-8(2-x+x^2)+\frac{22}{3}\delta(1-x)\right)-\frac{1}{\epsilon}n_f\,\frac{4}{3}\delta(1-x)$$

$$+C_A\left(16\frac{\ln(1-x)}{1-x}+\left(\frac{22}{3}+8\zeta_2\right)\delta(1-x)-16x(2-x+x^2)\ln(1-x)\right.$$

$$\left.-8\frac{(1-x+x^2)^2}{1-x}\ln(x)-\frac{22}{3}(1-x)^3\right)$$

$$+\mathcal{O}(\epsilon)\Bigg\}$$

- Structure of NLO correction
  - absorb collinear divergence $P^{(0)}_{gg}$ in renormalized parton distributions

$$\hat{\sigma}^{(1),\mathrm{bare}}_{gg\to H} = \frac{\alpha_s}{4\pi}\left(\frac{\mu^2}{m_H^2}\right)^{\epsilon}\left\{\frac{1}{\epsilon}\,2\,P^{(0)}_{gg}(x) + \hat{\sigma}^{(1)}_{gg\to H}(x) + \mathcal{O}(\epsilon)\right\}$$

$$g^{\mathrm{ren}}(\mu_F^2) = g^{\mathrm{bare}} - \frac{\alpha_s}{4\pi}\,\frac{1}{\epsilon}\,P^{(0)}_{gg}(x)\left(\frac{\mu^2}{\mu_F^2}\right)^{\epsilon}$$

  - partonic (physical) structure function at factorization scale μ_F

$$\hat{\sigma}_{gg\to H} = \delta(1-x) + \frac{\alpha_s}{4\pi}\left\{\hat{\sigma}^{(1)}_{gg\to H}(x) - \ln\left(\frac{m_H^2}{\mu_F^2}\right)\,2\,P^{(0)}_{gg}(x)\right\}$$