Quantum Chromodynamics

lecture II

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Plan

- Introduction to QCD
  Monday, February 24, 2025
- *QCD at work: infrared safety and jets*
  Tuesday, February 25, 2025
- QCD at work: factorization and evolution
  Wednesday, February 26, 2025
- Deep structure of proton
  Thursday, February 27, 2025

Perturbative QCD at Work

- QCD – the gauge theory of the strong interactions
- QCD covers dynamics in a large range of scales
  - asymptotically free theory of quarks and gluons at short distances
  - confining theory of hadrons at long distances
- Essential and established part of toolkit for discovering new physics
  - Tevatron and LHC
  - we no longer "test" QCD

Basic concepts of perturbative QCD

- Theoretical framework for QCD predictions at high energies relies on few basic concepts
  - infrared safety
  - factorization
  - evolution

Infrared safety

- Small class of cross sections at high energies and decay rates directly calculable in perturbation theory
- Infrared safe quantities
  - free of long range dependencies at leading power in large momentum scale Q Kinoshita ‘62; Lee, Nauenberg ‘64
- General structure of cross section
  - large momentum scale Q, renormalization scale μ

$$Q^2\,\hat{\sigma}\left(Q^2, \mu^2, \alpha_s(\mu^2)\right) = \sum_n \alpha_s^n\, c^{(n)}(Q^2/\mu^2)$$

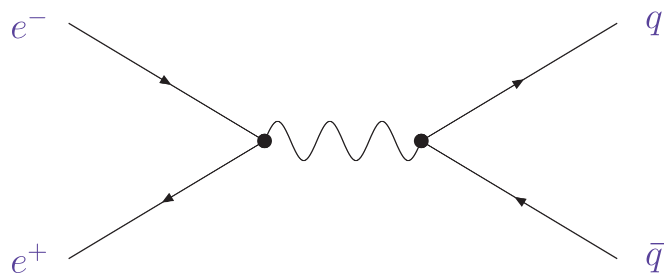
- Examples
  - total cross section in $e^+\,e^-$-annihilation

$$R^{\rm had}(s) = \frac{\sigma(e^+\,e^- \to \text{hadrons})}{\sigma(e^+\,e^- \to \mu^+\,\mu^-)}$$

  - jet cross sections in $e^+\,e^-$-annihilation
  - total width of Z-boson

Soft and collinear singularities

- e^+e^--annihilation (massless quarks)
  - Born cross section $\sigma^{(0)} = \dfrac{4\pi\alpha^2}{3s}$



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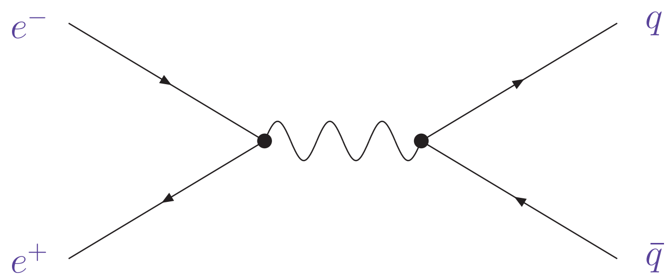
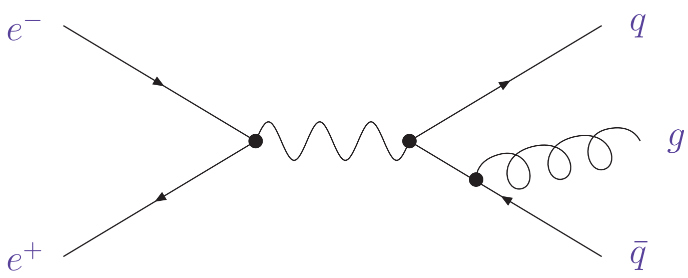
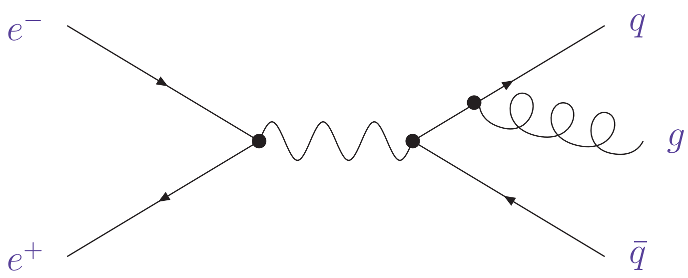


- Study QCD corrections (real emissions)



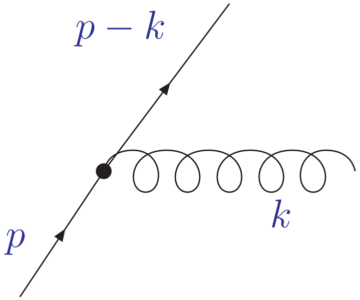


- Cross section
  - dimensional regularization $D = 4 - 2\epsilon$ (with $f(\epsilon) = 1 + \mathcal{O}(\epsilon^2)$)

$$\sigma^{q\bar{q}g} \quad = \quad \sigma^{(0)}\, 3 \sum_q e_q^2\, f(\epsilon)\, C_F\, \frac{\alpha_s}{2\pi} \int dx_1 dx_2\, \frac{x_1^2 + x_2^2 - \epsilon(2 - x_1 - x_2)}{(1-x_1)^{1+\epsilon}\,(1-x_2)^{1+\epsilon}}$$

- scaled energies $x_1 = 2\dfrac{E_q}{\sqrt{s}}$ and $x_2 = 2\dfrac{E_{\bar{q}}}{\sqrt{s}}$

- Soft and collinear divergencies ($0 \leq x_1, x_2 \leq 1$ and $x_1 + x_2 \geq 1$)



$$1 - x_1 = x_2 \frac{E_g}{\sqrt{s}} (1 - \cos\theta_{2g}) \text{ and}$$

$$1 - x_2 = x_1 \frac{E_g}{\sqrt{s}} (1 - \cos\theta_{1g})$$

- Integrate over phase space for real emission contributions

$$\sigma^{q\bar{q}g} = \sigma^{(0)}\, 3 \sum_q e_q^2\, f(\epsilon)\, C_F\, \frac{\alpha_s}{2\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} + \mathcal{O}(\epsilon) \right)$$

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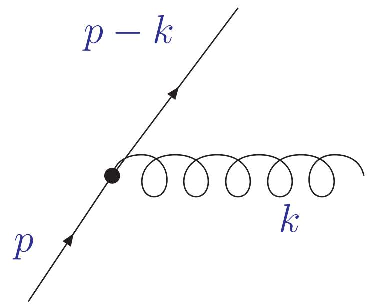

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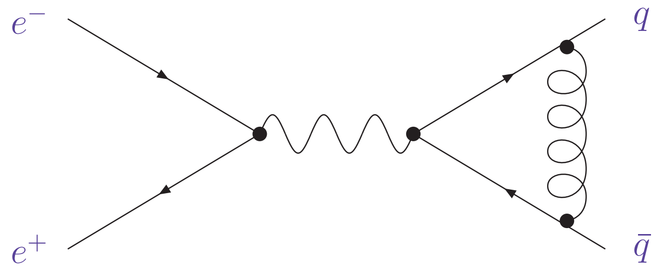
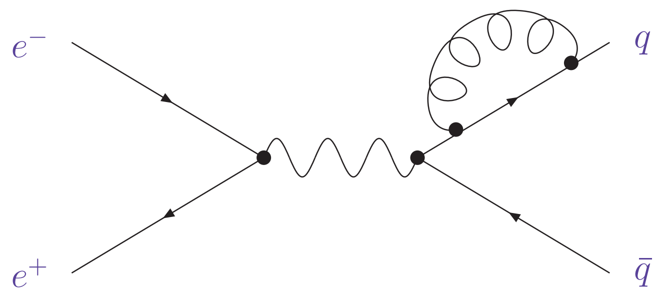
- Divergencies cancel against virtual contributions





$\times 2$

$$\sigma^{q\bar{q}(g)} = \sigma^{(0)}\, 3 \sum_q e_q^2\, f(\epsilon)\, C_F\, \frac{\alpha_s}{2\pi} \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \mathcal{O}(\epsilon) \right)$$

Infrared safety (I)

Summing up

- Total cross section $\sigma^{(1)}$ at NLO
  - sum of real emission and virtual contributions

$$\begin{aligned}\sigma^{(1)} &= \sigma^{(0)}\, 3 \sum_q e_q^2\, f(\epsilon)\, C_F\, \frac{\alpha_s}{2\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} + \mathcal{O}(\epsilon) \right) \\ & \quad + \sigma^{(0)}\, 3 \sum_q e_q^2\, f(\epsilon)\, C_F\, \frac{\alpha_s}{2\pi} \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \mathcal{O}(\epsilon) \right) \\ &= \sigma^{(0)}\, 3 \sum_q e_q^2\, f(\epsilon)\, C_F\, \frac{\alpha_s}{2\pi} \left(\frac{3}{2} + \mathcal{O}(\epsilon) \right)\end{aligned}$$

Infrared safety

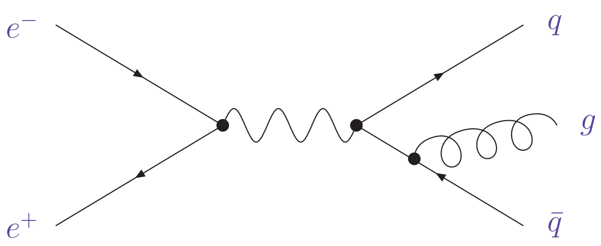
- Total cross section and $R(s)$ are finite
  - directly calculable in perturbation theory
  - use $f(\epsilon) \simeq 1 + \mathcal{O}(\epsilon)$

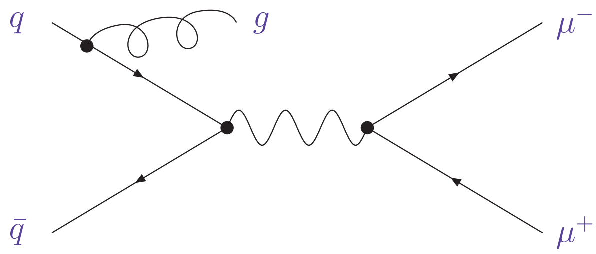
$$R(s) = 3 \sum_q e_q^2 \left\{ 1 + \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2) \right\}$$

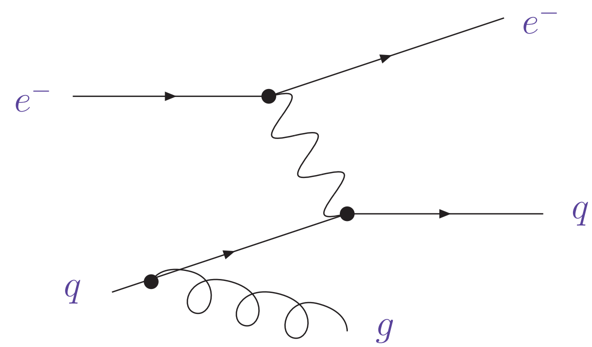
Infrared safety (II)

Collinear singularities

- Collinear divergencies remain for hadronic observables
  $\longrightarrow$ factorization





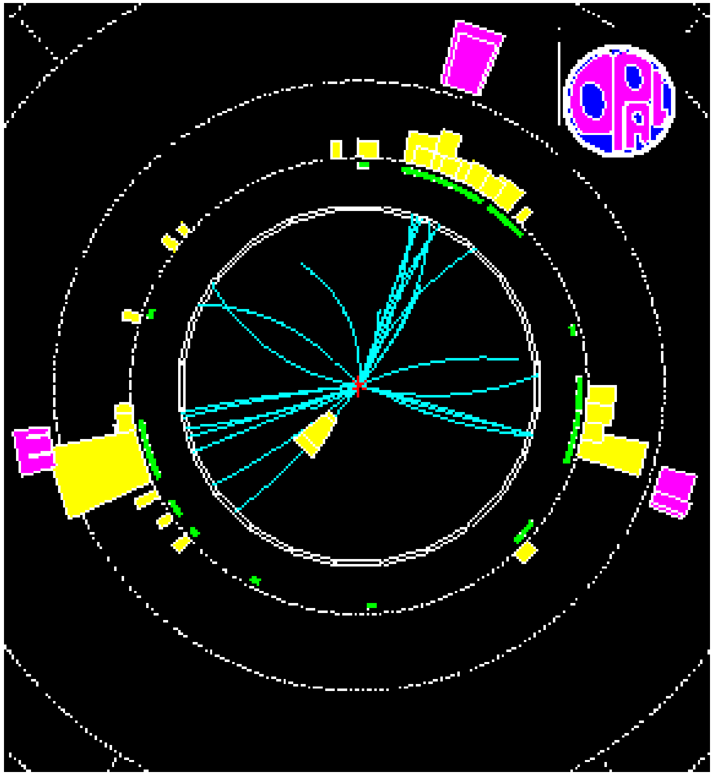


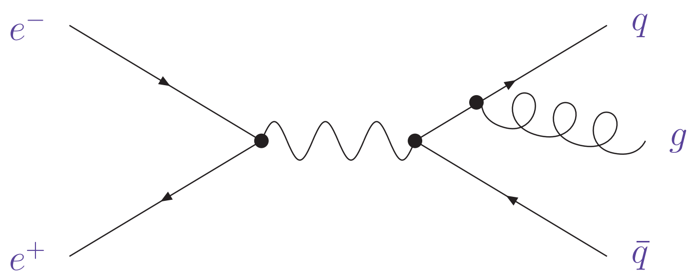
- Left: single-hadron inclusive e^+e^--annihilation (time-like kinematics)
- Center: Drell-Yan process in pp-scattering (space-like kinematics)
- Right: Deep-inelastic e^-p-scattering (space-like kinematics)

Jets in QCD

Notion of a jet

- High energy event with collimated bunch of hadrons flying roughly in same direction is called a jet
(hundreds of hadrons; contains a lot of information)





- Jets related to underlying QCD dynamics (quarks and gluons)

Jet algorithms

- Reduce complexity of final state
  (combine many hadrons to simpler objects)
- Connects parton picture to experimental signature
  (precise and quantitative)
- Mapping of particle 4-momenta $\{p_i\}$ to set of jets $\{j_k\}$

$$\Big\{ p_i \Big\} \quad \longrightarrow \quad \Big\{ j_k \Big\}$$

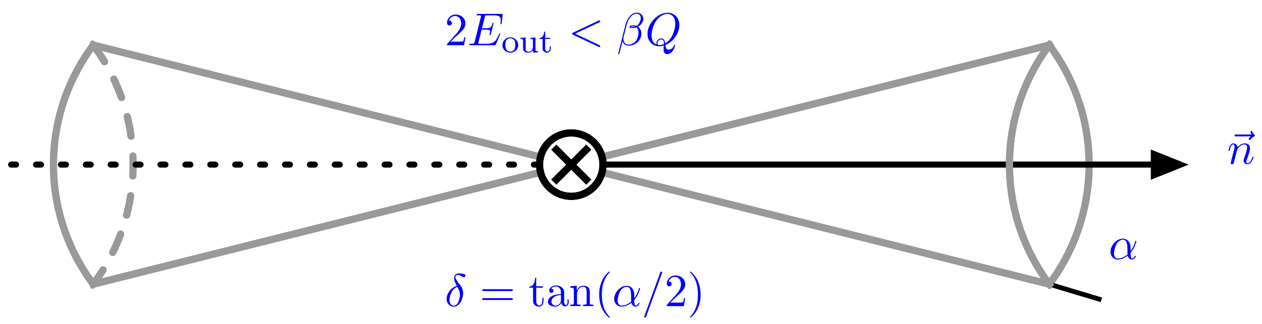
Properties of jet definitions

" Toward a standardization of jet definitions" FERMILAB-CONF-90-249-E

1. Simple to implement in an experimental analysis;
2. Simple to implement in a theoretical calculation;
3. Defined at any order of perturbation theory;
4. Yields finite cross section at any order in perturbation theory;
5. Yields a cross section that is relatively insensitive to hadronization.

Historical definitions

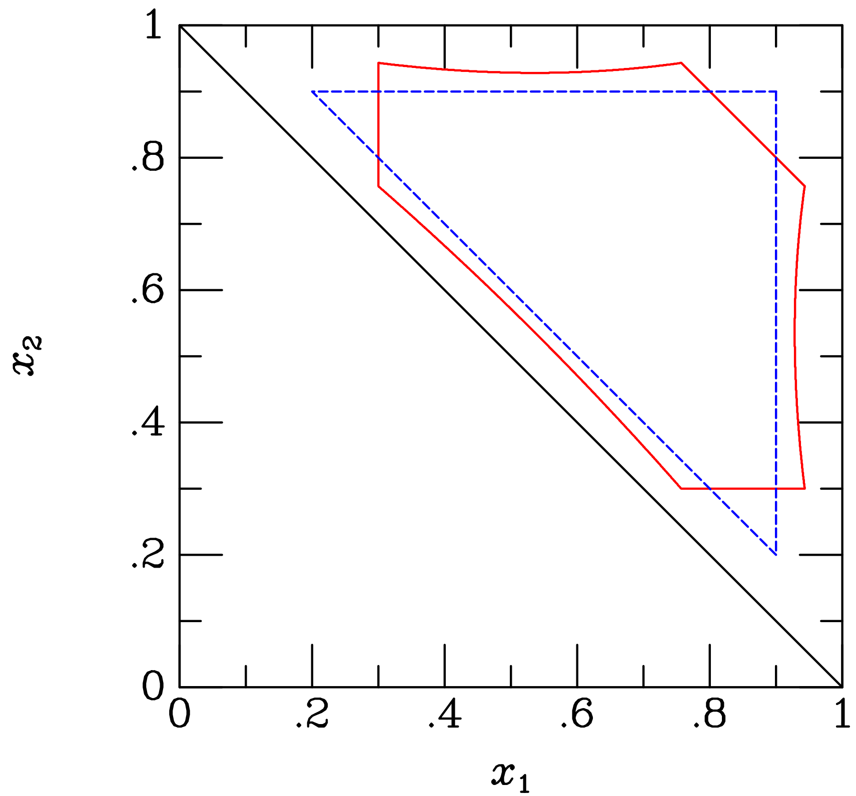
- Historically: Sterman-Weinberg criterium for two-jet event
  - small fraction $\beta/2$ of energy Q in cone of half angle δ



  - not practical for multi-particle events

- JADE algorithm: $\min\,(p_i + p_j)^2 = \min\,2E_iE_j\,(1 - \cos\theta_{ij}) > y_{\text{cut}}\,s$
  - disadvantage: combines also soft gluons at large relative k_t
    e.g. potential three-jet event

Di-jet phase space in e^+e^- annihilation

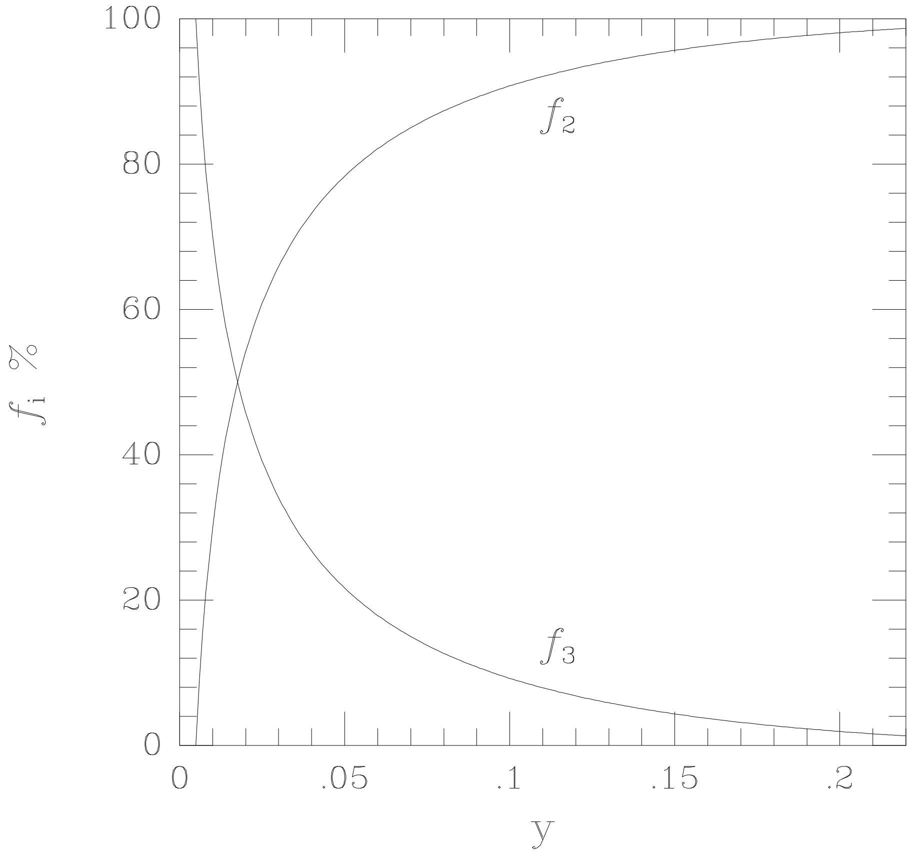


- phase space boundaries for region with two and three jets
  - Sterman-Weinberg with $(\beta, \delta) = (0.3, 30)$ (solid lines)
  - JADE algorithm with $y_{\rm cut} = 0.1$ (dashed lines)

Upshot

- Cross section varies with jet definition
- Algorithms alter energy and momentum clustering and differences affect jet multiplicities observed

Jet rates in e^+e^- annihilation



- Ratio of rates
  $f_i = \dfrac{\sigma_{i-\mathrm{jet}}}{\sigma}$
  for two and three jets
  - JADE algorithm with $y_{\mathrm{cut}} \leq 0.3$

- Recall: three-jet cross section $\sigma^{e^+e^- \to 3\mathrm{jets}}$

$$\frac{d^2\sigma^{e^+e^- \to 3\mathrm{jets}}}{dx_1 dx_2} = \sigma^{(0)}\, 3 \sum_q e_q^2\, C_F\, \frac{\alpha_s}{2\pi}\, \frac{x_1^2 + x_2^2}{(1-x_1)\,(1-x_2)}$$

Small angles θ_{qg}

- Differential expression in scaled energies $x_1 = 2\frac{E_q}{\sqrt{s}}$ and $x_2 = 2\frac{E_{\bar{q}}}{\sqrt{s}}$

$$\frac{d^2\sigma^{e^+e^-\to 3\text{jets}}}{dx_1 dx_2} = \sigma^{(0)}\,\frac{\alpha_s}{2\pi}\,C_F\,\frac{x_1^2+x_2^2}{(1-x_1)\,(1-x_2)}$$

- Transformation of variables to $x_3 = 2\frac{E_g}{\sqrt{s}}$ and $\cos\theta_{qg}$

$$\frac{d^2\sigma^{e^+e^-\to 3\text{jets}}}{d\cos\theta_{qg}dx_3} = \sigma^{(0)}\,\frac{\alpha_s}{2\pi}\,C_F\left(\frac{2}{\sin^2\theta_{qg}}\,\frac{1+(1-x_3)^2}{x_3} - x_3\right)$$

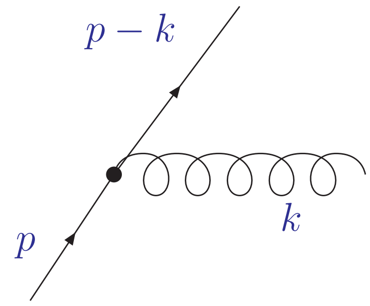
  - small angle approximation

$$\frac{2d\cos\theta_{qg}}{\sin^2\theta_{qg}} \simeq \frac{d\theta_{qg}^2}{\theta_{qg}^2} + \frac{d\theta_{\bar{q}g}^2}{\theta_{\bar{q}g}^2}$$

- Splitting function $P_{qq}^{(0)}(z) = C_F\,\frac{1+z^2}{1-z}$ associated to each jet

$$d\sigma^{e^+e^-\to 3\text{jets}} \simeq \sigma^{(0)}\sum_j \frac{\alpha_s}{2\pi}\,\frac{d\theta_{qg}^2}{\theta_{qg}^2}\,P_{qq}^{(0)}(1-z)$$

Sudakov form factor



- Splitting function $P_{qq}^{(0)}(z) = C_F \dfrac{1+z^2}{1-z}$ captures universal dynamics of collinear emissions

- Splitting function are process independent
- Independent evolution of two jets with splitting function
- Small energy fractions $z \to 1$ leads to Sudakov form factor

$$d\sigma^{e^+e^- \to 3\text{jets}} \quad \simeq \quad \sigma^{(0)} \sum_j \frac{\alpha_s}{2\pi} \frac{d\theta_{qg}^2}{\theta_{qg}^2} \frac{dz}{z}$$

  - Sudakov form factor at leading logarithmic accuracy
  - suitable for parton shower Monte Carlos

Sterman-Weinberg jet cross section

- Phase space integration (up to energy fraction $\beta/2$) leads to di-jet cross section as function of β, δ

$$\frac{\sigma^{e^+e^- \to 2\mathrm{jets}}}{\sigma^{(0)}} = 1 + \frac{\alpha_s}{2\pi} C_F \left(-8 \ln \delta \ln \beta - 6 \ln \delta + c_0\right)$$

  - collinear divergence $\ln \delta$
  - soft divergence $\ln \beta$

All-order resummation

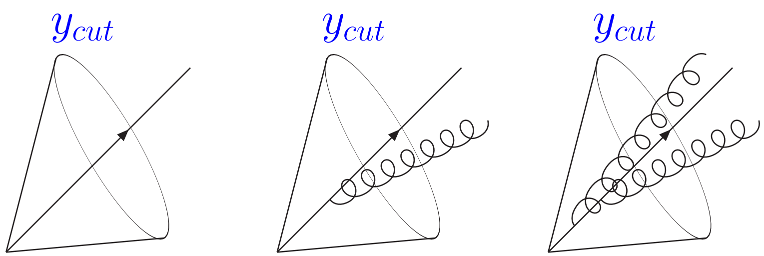
- Soft and collinear divergences exponentiate
- Resummation to all orders in perturbation theory
- NNLO QCD correction to Sterman-Weinberg jet cross section

Becher, Neubert, Rothen, Shao '15

$$\frac{\sigma^{e^+e^- \to 2\mathrm{jets}}}{\sigma^{(0)}} = 1 + \frac{\alpha_s}{2\pi} C_F \left(-8 \ln \delta \ln \beta - 6 \ln \delta + c_0\right)$$
$$+ \left(\frac{\alpha_s}{2\pi}\right)^2 \left\{ C_F^2 \left(32 \ln^2 \delta \ln^2 \beta + 48 \ln \beta \ln^2 \delta + \dots\right)\right.$$
$$\left. + C_F \beta_0 \left(4 \ln \beta \ln^2 \delta\right) + \dots \right\}$$

Modern jet definitions

- Two main classes of jet algorithms
- Sequential recombination algorithms (bottom-up approach)
  - combine particles starting from closest ones
  - choose distance measure
  - iterate recombination until few objects left, call them jets
  - e.g. k_t-clustering algorithm: $2\min(E_i^2, E_j^2)(1 - \cos\theta_{ij}) > y_{\text{cut}}\, s$

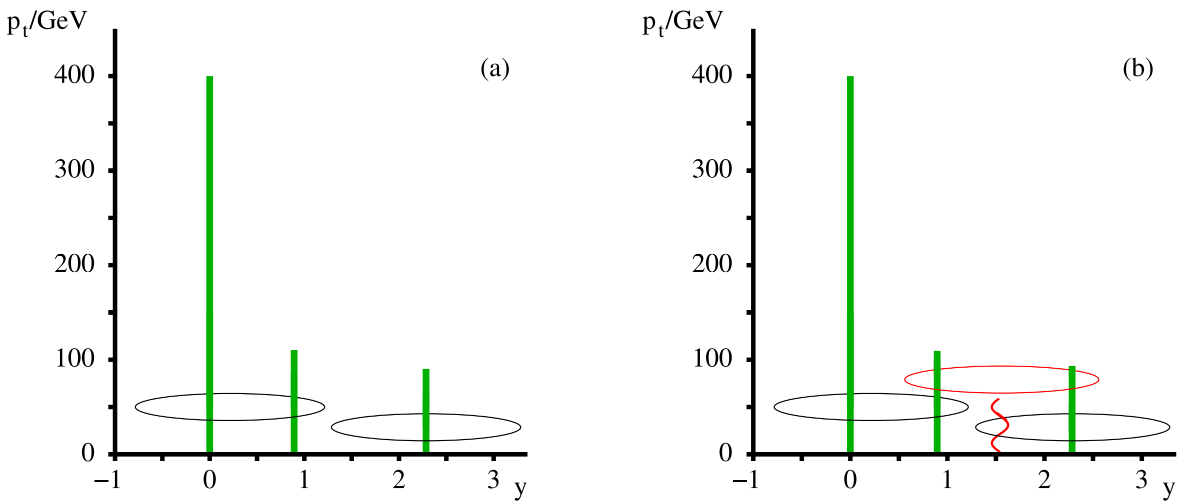


Jets in hadronic collisions

- Metric of η, ϕ
  - define cone of radius R in η, ϕ for $R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2}$

Cone algorithm

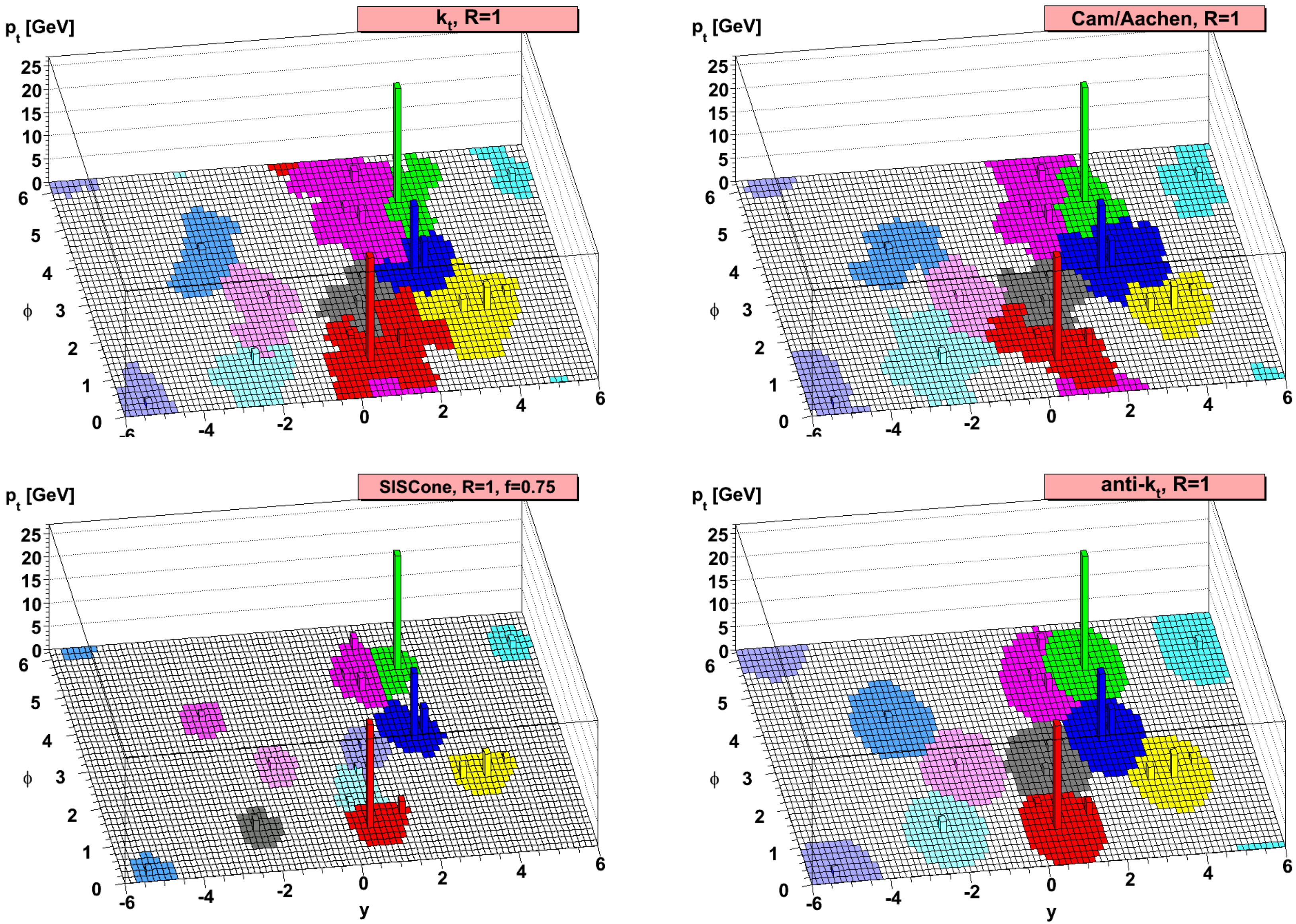
- Top-down approach: find coarse regions of energy flow
  - find stable cones
    (i.e. their axis coincides with sum of momenta of particles in it)
  - e.g. JetClu, MidPoint, ATLAS cone, CMS cone, ...
- Problem
  - infrared unsafe beyond NLO in QCD
  - e.g. midpoint cone-algorithm:
    soft seed gives rise to extra hard jet (fixed for Tevatron run II)



Cone algorithms (II)

- Clustering of parton-level event from `Herwig` and random soft radiation with different jets algorithms
    - k_t algorithm
    - Cambridge/Aachen
    - SISCone
    - anti-k_t algorithm
- Illustration of “active” catchment areas of resulting hard jets

Cone algorithms (II)



(Some) uses of hadronic jets

- Hadronic di-jets: large statistics even with high-p_t cuts
  - experimental calibration (HCAL uniformity, establish missing E_t)
  - gluon jets constrain gluon PDF at medium/large x
  - searches for quark sub-structure (di-jet angular correlations)
- Hadronic di- and three-jets: α_s determination

Summary (part II)

Perturbative QCD at work

- Basics concepts of QCD
- Infrared safety
  - cancellation of soft and collinear singularities in inclusive observables
  - example $e^+e^- \to$ hadrons at NLO
- Resummation
  - large logarithms near threshold
  - radiative corrections (higher orders) important
  - essential to control theory uncertainties

Jets

- Jet algortihms
  - infrared saftey to all orders crucial
- Jets at the LHC
  - searches for new physics at high E_T
  - constrains on gluon PDF and $\alpha_s(M_Z)$