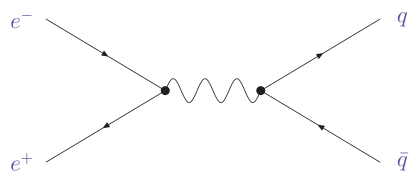
Sven-Olaf Moch
University of Hamburg

IMSc, Chennai, Feb 25, 2025

Tutorial II

IMSc Spring School on High Energy Physics 2025

The amplitude for electron-positron annihilation and subsequent quark-anti-quark production is given to leading order by the following diagram:



The photon has momentum $q, +q^2 = Q^2 > 0$ and the quark and anti-quark carry the momenta $p, p^2 = 0$ and $p_1, p_1^2 = 0$.

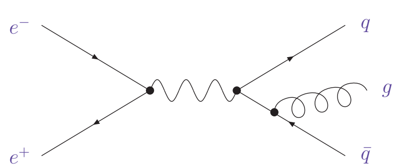
- Calculate the total cross section for this reaction.

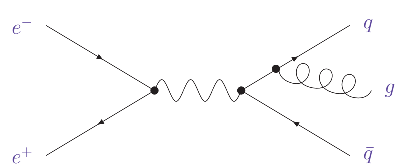
$$e^- + e^+ \;\rightarrow\; \gamma(q) \;\rightarrow\; \mathrm{q}(p) + \bar{\mathrm{q}}(p_1)\,. \tag{1}$$

Next, we want to consider the radiative corrections in Quantum Chromodynamics to order α_s. The amplitude for electron-positron annihilation and production of a quark-anti-quark pair and additional radiation of a gluon with momentum k,

$$e^- + e^+ \;\rightarrow\; \gamma(q) \;\rightarrow\; \mathrm{q}(p) + \bar{\mathrm{q}}(p_1) + \mathrm{g}(p_2)\,, \tag{2}$$

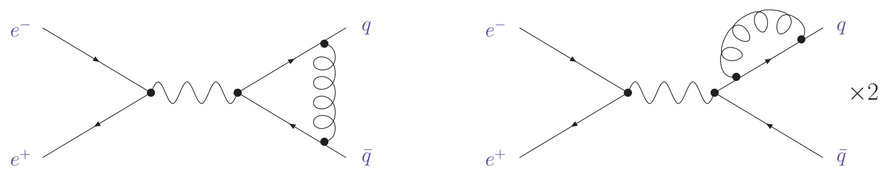
is given by the following diagrams:





- This process can be decribed with the help of the scaling variable $x = (2p \cdot q)/Q^2$. What is the physical interpretation of x?
- Calculate the cross section $d\sigma_T$ for this reaction for transversely polarized photons as a function of the scaling variable $x = (2p\cdot q)/Q^2$ and the photon momentum $q^2 = Q^2$.
- Why is the result divergent in the infrared?

- What is the role of the following set of diagrams?



Hint: The squared amplitude $\mathcal{A}$ for the reaction $\gamma(q,\mu) \to \mathrm{q}(p) + \bar{\mathrm{q}}(p_1) + \mathrm{g}(p_2,\mu_1)$ is given by

$$\sum_{spins} |\mathcal{A}|^2 = (e^2 g_s^2 C_F)\, g^{\mu\nu} g^{\mu_1\mu_2} \tag{3}$$
$$\Big\{ \mathbf{tr}(\not{p}_1 \gamma_\mu (\not{p} + \not{p}_2) \gamma_{\mu_1} \not{p} \gamma_{\mu_2} (\not{p} + \not{p}_2) \gamma_\nu) \frac{1}{(p+p_2)^4}$$
$$+ \mathbf{tr}(\not{p}_1 \gamma_{\mu_1} (\not{p}_1 + \not{p}_2) \gamma_\mu \not{p} \gamma_\nu (\not{p}_1 + \not{p}_2) \gamma_{\mu_2}) \frac{1}{(p_1+p_2)^4}$$
$$- \mathbf{tr}(\not{p}_1 \gamma_{\mu_1} (\not{p}_1 + \not{p}_2) \gamma_\mu \not{p} \gamma_{\mu_2} (\not{p} + \not{p}_2) \gamma_\nu) \frac{1}{(p+p_2)^2 (p_1+p_2)^2}$$
$$- \mathbf{tr}(\not{p}_1 \gamma_\mu (\not{p} + \not{p}_2) \gamma_{\mu_1} \not{p} \gamma_\nu (\not{p}_1 + \not{p}_2) \gamma_{\mu_2}) \frac{1}{(p+p_2)^2 (p_1+p_2)^2} \Big\} .$$

Here, e is the electro-magnetic charge, $C_F = (N_c^2 - 1)/(2N_c)$ arises from the quark color charge and g_s is the strong coupling constant.
The differential phase space for the reaction in Eq.(2) in x is in D-dimensions given by

$$d\mathrm{PS} = \frac{1}{2} \frac{1}{(4\pi)^{D/2-1}} \frac{(Q^2)^{D/2-2}}{\Gamma(D/2-1)} (1-x)^{D/2-2} \int_0^1 dy\, [y(1-y)]^{D/2-2} , \tag{4}$$

where the relation of the scattering angle θ in the center-of-mass system for q is related to y as $y = \frac{1}{2}(1 + \cos\theta)$.

It is advantageous to automatize the calculation, e.g. with the symbolic computer algebra system FORM, see `https://github.com/vermaseren/form` . A FORM script `nloEpEM.frm` is available on the web-page of this lecture and the output of running the FORM script is listed below.

```
FORM 4.3.0 (Nov 12 2022, v4.3.0) 64-bits         Run: Sun Feb 23 07:09:57 2025
    ***********************************************
    *
    * computation of the time-like PTqq0 splitting function in QCD
    * run with form (available from https://github.com/vermaseren/form)
    *
    * S. Moch, 2025
    *
    * Output definitions:
    * den(x) = 1/x;   pow(x,a) = x^a;  InvGamma(x?) = 1/Gamma(x);
    *
    ***********************************************
    #-

   nlorM2 =

       + delta_([1-x])*Gamma(1 - ep)*InvGamma(1 - 2*ep)*cf*ep^(-2) * (
          + 4
          )

       + delta_([1-x])*Gamma(1 - ep)*InvGamma(1 - 2*ep)*cf*ep^(-1) * (
          + 4*den(1 - 2*ep)
          - 2*den(1 - 2*ep)*den(2 - 2*ep)
          )

       + Gamma(1 - ep)*InvGamma(1 - 2*ep)*cf*ep^(-1) * (
          - 4*pow([1-x], - 1 - ep)*pow(x, - 2*ep)
          + 2*pow([1-x], - ep)*pow(x,1 - 2*ep)
          + 2*pow([1-x], - ep)*pow(x, - 2*ep)
          );

  0.05 sec out of 0.05 sec
```