

Tutorial I

IMSc Spring School on High Energy Physics 2025

In Quantum Chromodynamics one defines the renormalized coupling as follows

$$g_0 \rightarrow Z_g \mu^\epsilon g(\mu), \quad (1)$$

where g_0 denotes the bare (unrenormalized) coupling, g is the renormalized coupling, Z_g the renormalization constant and μ an arbitrary scale.

We apply dimensional regularization and ϵ is the parameter for the shift from 4 dimensions:

$$D = 4 - 2\epsilon. \quad (2)$$

- Calculate the QCD β -function (recall $\alpha_s(\mu) = \frac{g(\mu)^2}{4\pi}$)

$$\beta(\alpha_s) = \mu^2 \frac{d\alpha_s(\mu)}{d\mu^2} \quad (3)$$

by using the renormalization constant

$$Z_g = 1 - \frac{1}{2} \frac{\alpha_s(\mu)}{4\pi} \left(\frac{1}{\epsilon} - \gamma_E + \ln(4\pi) \right) \left\{ \frac{11N_c - 2n_f}{3} \right\} \quad (4)$$

where N_c is the number of colors and n_f the numbers of quark flavors. γ_E denotes Euler's constant.

- Solve the QCD β -function at leading order. What choices do you have for the initial conditions? How do you fix them?
- What are the limits on the matter content of QCD in order to remain an asymptotically free gauge theory? What is the value of n_f for the conformal fix-point?
- Compute $\alpha_s^{LO}(M_Z)$ (leading order) and $\alpha_s^{NLO}(M_Z)$ (next-to-leading order) from input $\alpha_s^{NNLO}(M_Z)$ (next-to-next-to-leading order). Choose some order of perturbation theory and compute $\alpha_s^{(n_f=4)}(M_Z)$ and $\alpha_s^{(n_f=5)}(M_Z)$ from input $\alpha_s^{(n_f=3)}(M_Z)$.

Hint: This task can be easily solved with the Mathematica package `RunDec.m` or with the C-version `CRunDec`, cf. <https://arxiv.org/abs/1703.03751v1> .

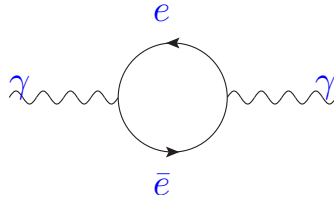


Figure 1: Photon self-energy diagram at one loop.

- Compute in Quantum Electrodynamics the divergent part of the fermionic contribution to the photon self-energy at one loop.

Hint:

- Feynman rules in QED:

$$\text{photon propagator} \quad \frac{-g_{\mu\nu}}{p^2 + i0}$$

$$\text{fermion propagator} \quad \frac{i}{\not{p} - m + i0}$$

$$\text{fermion-photon vertex} \quad ie\gamma_\mu$$

- one-loop integral:

$$\int \frac{d^D \ell}{(2\pi)^D} \frac{1}{(\ell^2 + 2q\ell + a + i\varepsilon)^n} = (-1)^n \frac{i}{(4\pi)^{2-\epsilon}} \frac{\Gamma(n-2+\epsilon)}{\Gamma(n)} (q^2 - a - i\varepsilon)^{2-\epsilon-n}$$

- Feynman parametrization:

$$\frac{1}{AB} = \int_0^1 \frac{1}{[xA + (1-x)B]^2} dx$$

- Gamma function:

$$\Gamma(\epsilon) = \frac{1}{\epsilon} + \dots$$