Patterns in scattering amplitudes

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Trends in Astroparticle and Particle Physics
28 Sep 2024

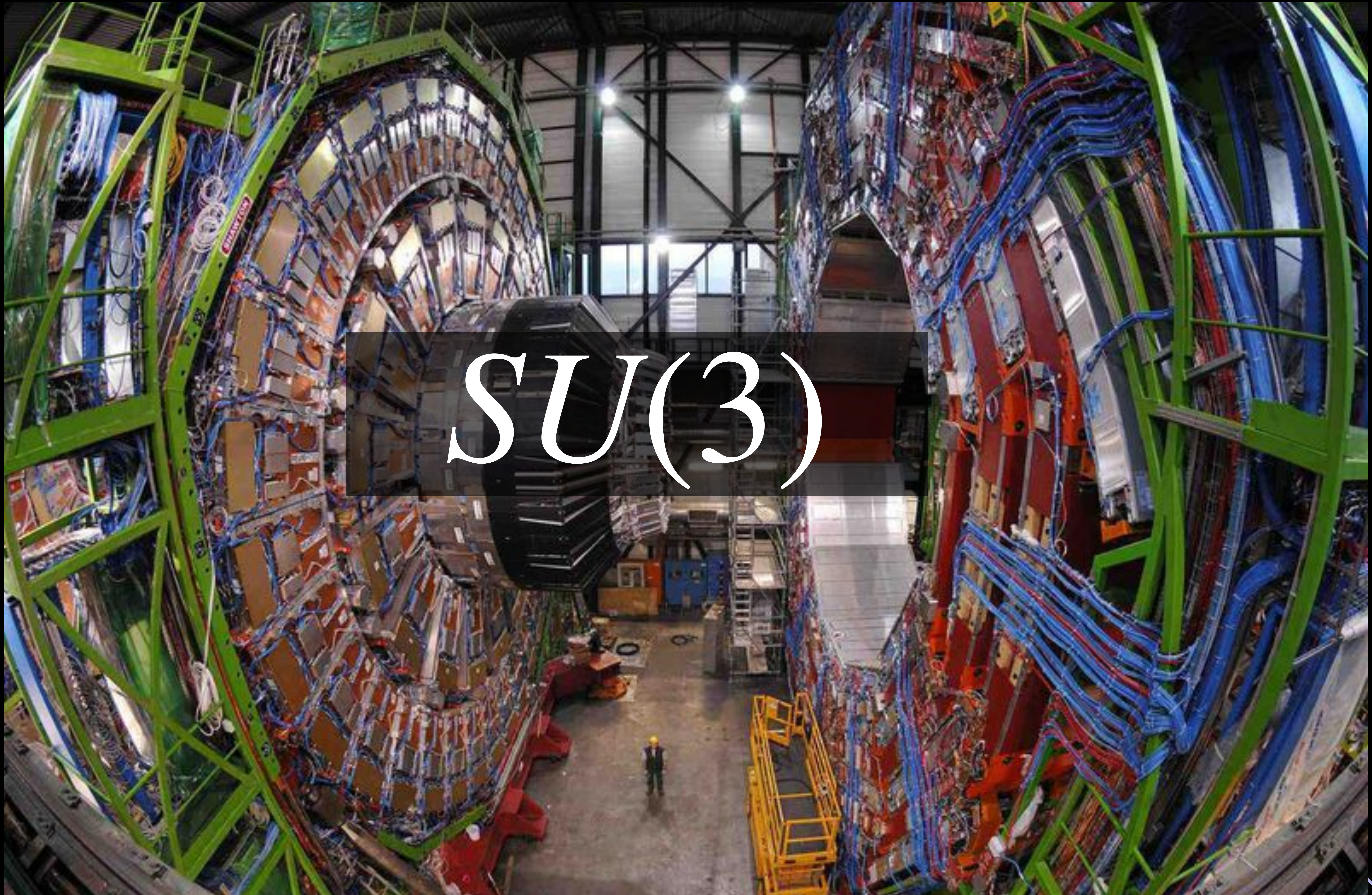
Rutherford Scattering

The first major scattering experiment

Many scatterings later…

$$SU(3) \times SU(2) \times U(1)$$

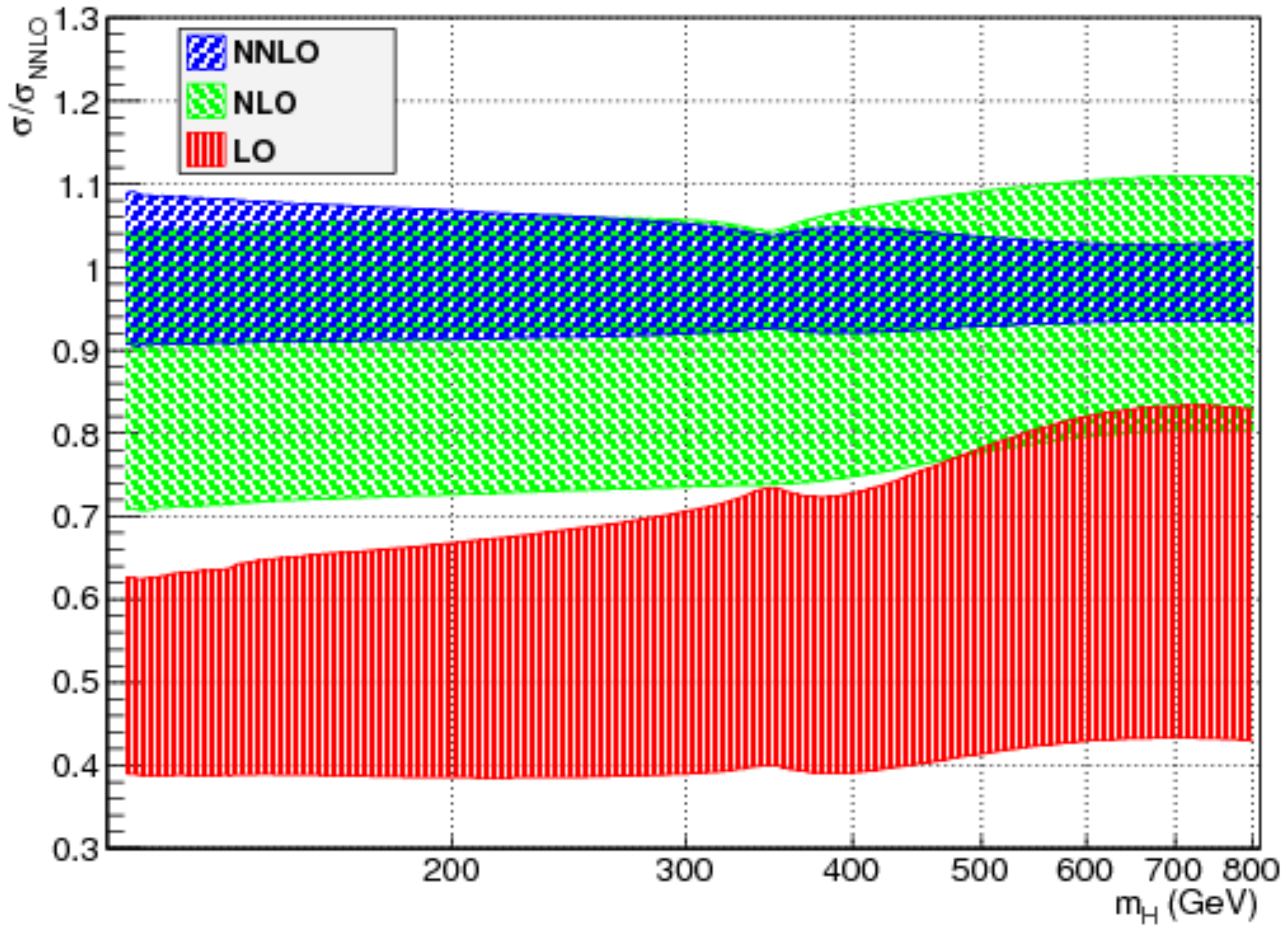
Standard Model of Particle Physics

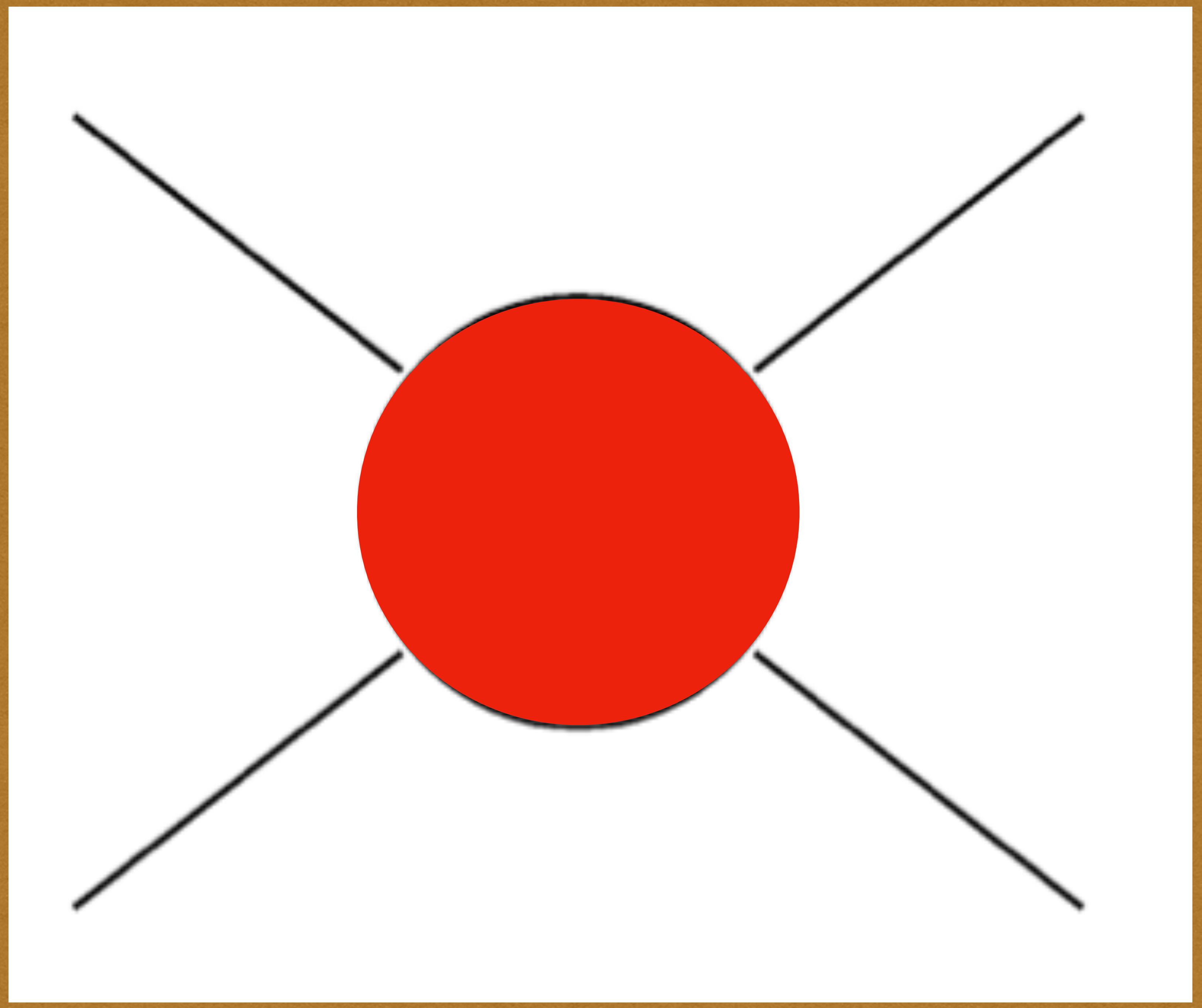

$SU(3)$

QCD $SU(3)$ is indispensable

Gluon fusion cross-section at LO, NLO, and NNLO.

NLO	Spira, Djouadi, Graudenz, Zerwas ('91, '93), Dawson ('91)
NNLO	Group-1 Harlander, Kilgore ('02),
	Group-2 Anastasiou, Melnikov, ('02),
	Group-3 Ravindran, Smith, v.Neerven ('03)





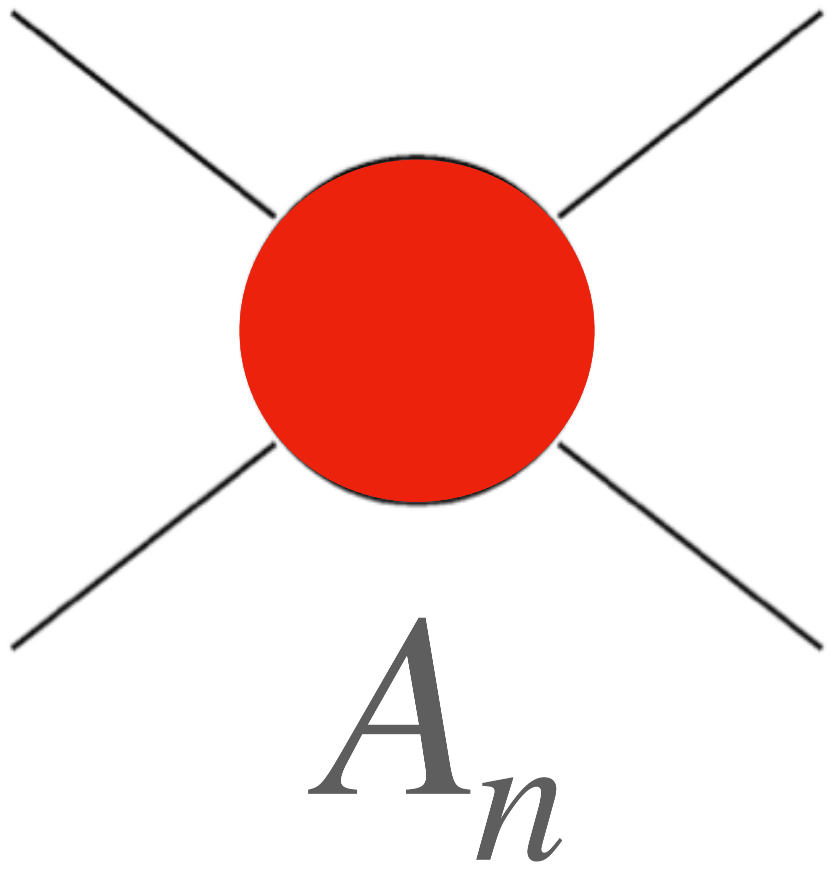
A_n

Scattering Amplitudes

Fixed # of external particles

only virtual corrections

Amplitudes in the Infrared (IR) limit



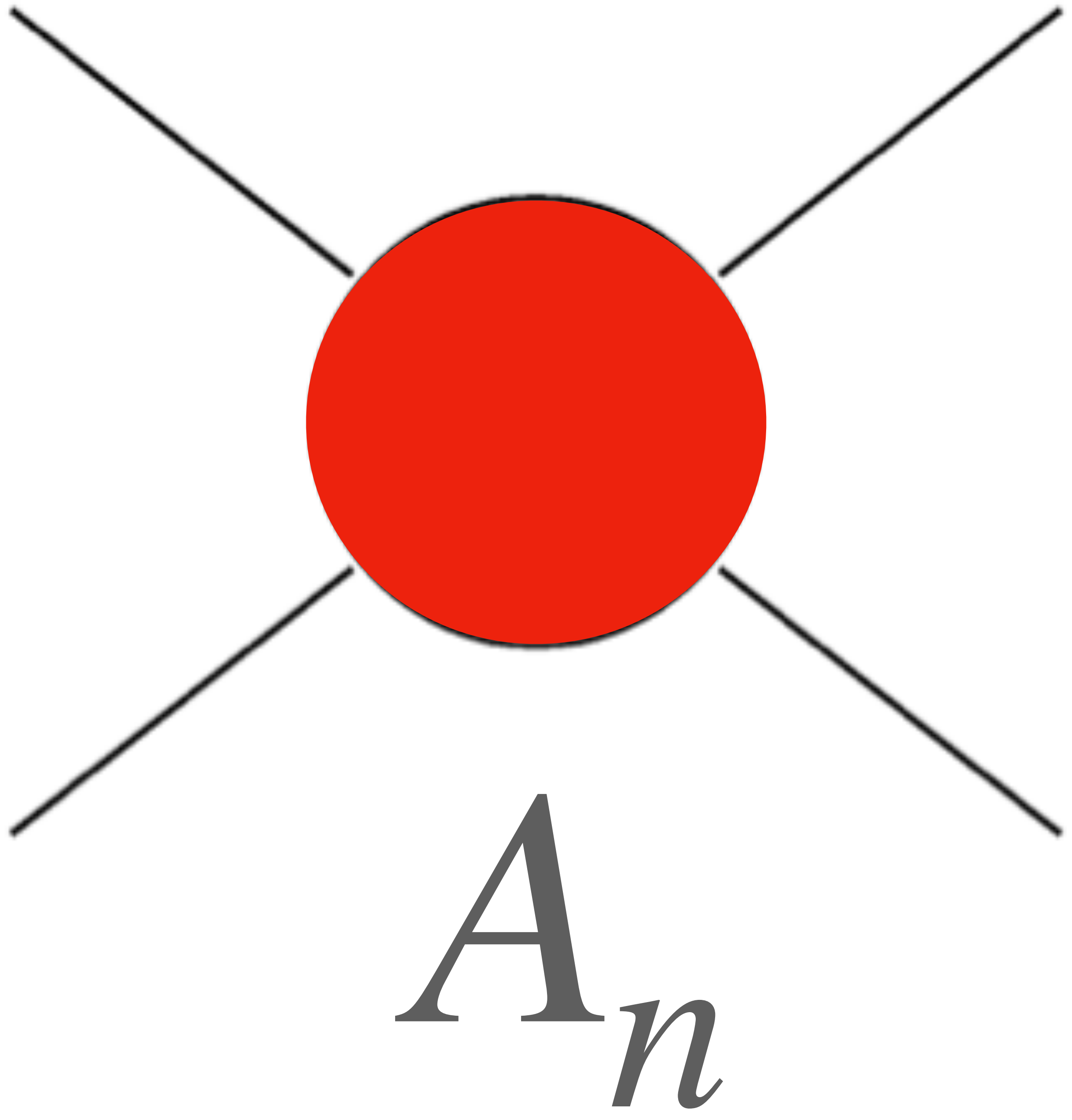
Integral over loop momenta

→ Soft gluon

→ propagator goes on-shell

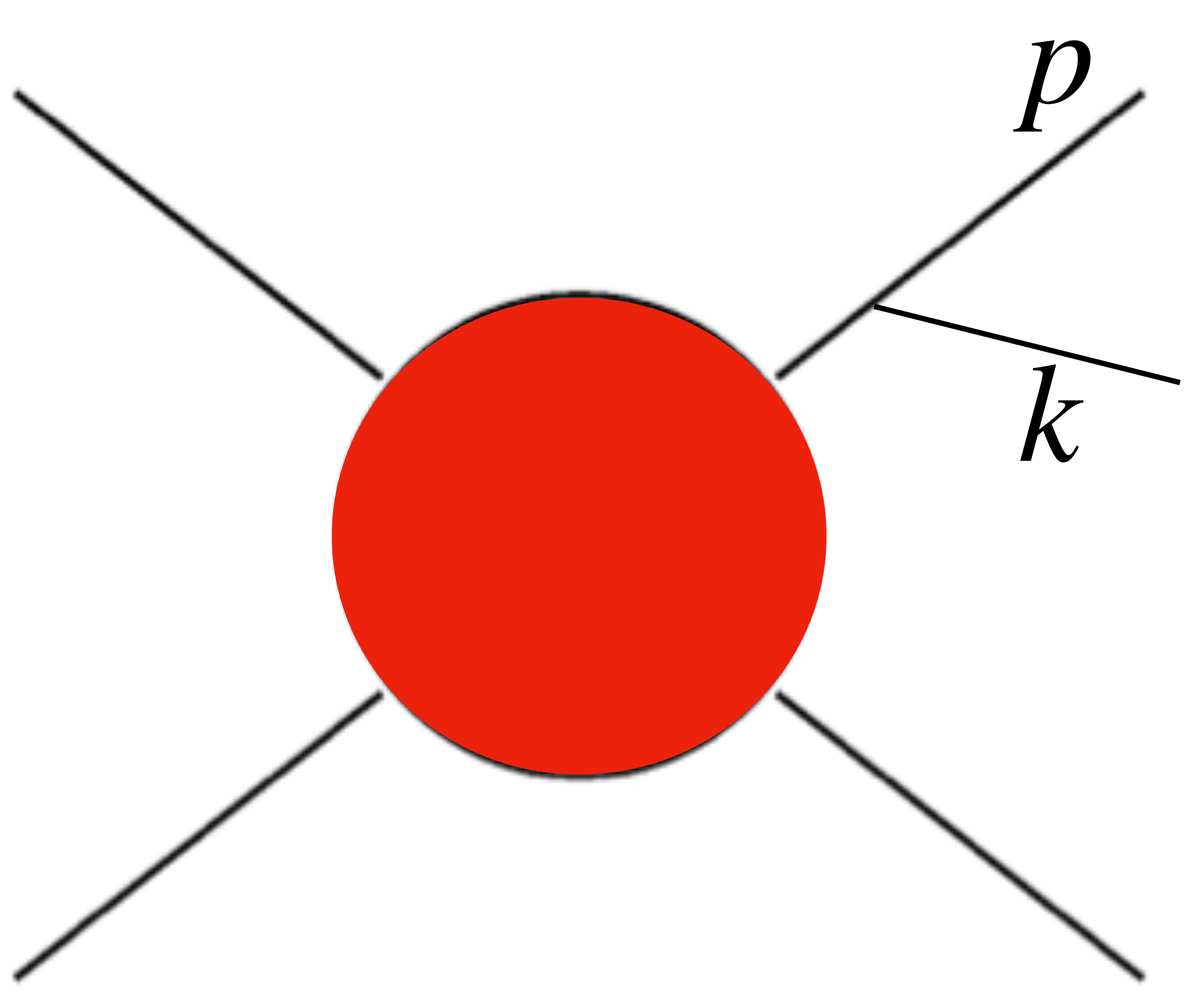
→ Divergence

Why should we care about the IR limit ?



- large contributions
- Resummation
- Subtraction of poles

Large contributions



$$\text{Soft}: \quad \mathcal{M} \sim \mathcal{M}^{(0)} \times \frac{p^\mu}{p \cdot k}$$

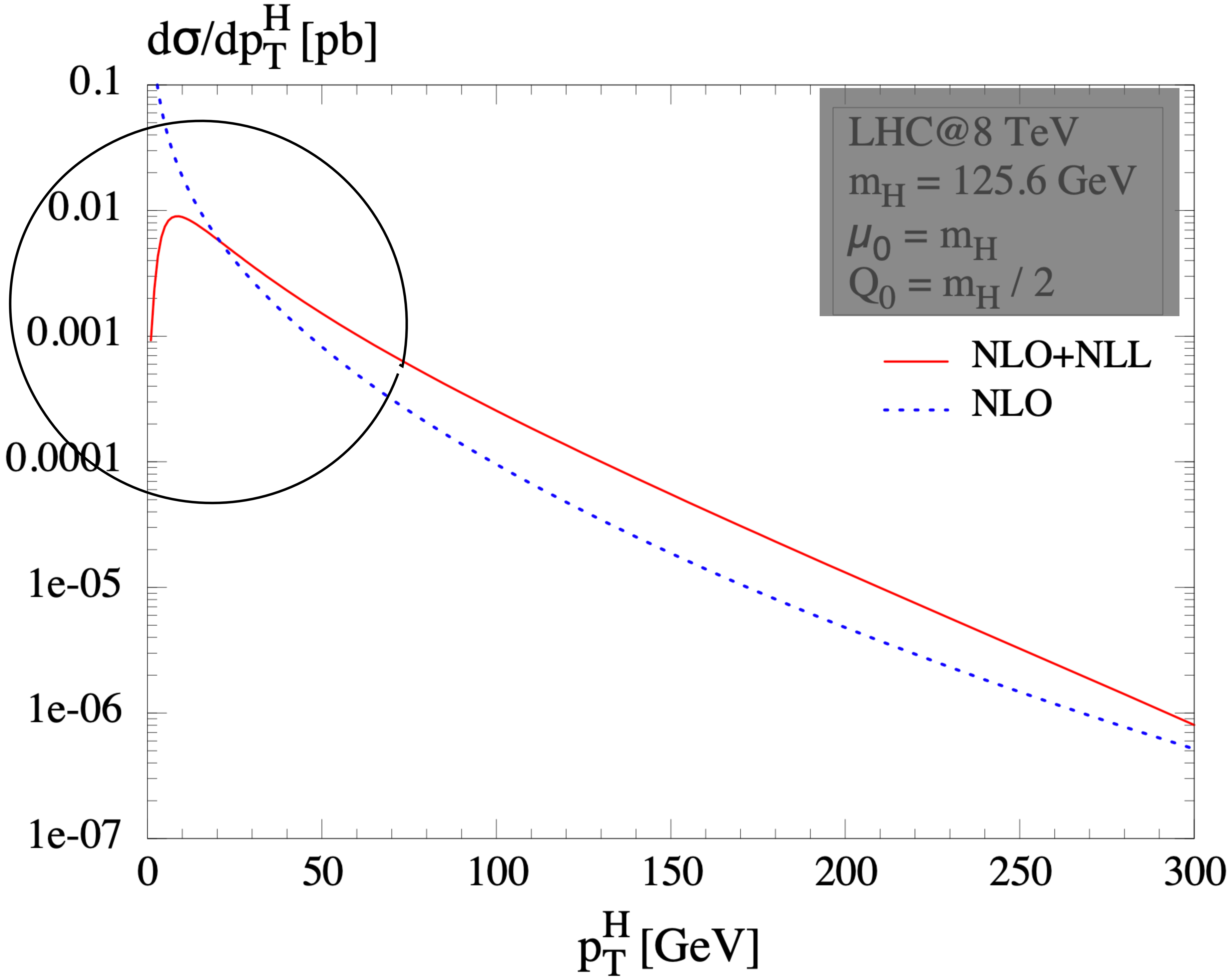
$$\text{Collinear}: \quad |\mathcal{M}|^2 \sim |\mathcal{M}^{(0)}|^2 \times \frac{1}{k_T^2}$$

Singularities Factorize!

$k \to$ gluon momentum

$p \to$ the parton emitting the gluon

Large contributions → Divergent distribution



Plan of the talk

1. Scattering Amplitudes in IR limit
2. Webs
3. Uniqueness Theorem and a new Formalism
4. Summary

Agarwal, Pal, Srivastav, AT ;
arxiv: 2307.15924

Agarwal, Pal, Srivastav, AT ;
arxiv: 2305.17452

Agarwal, Pal, Srivastav, AT ;
JHEP 02 (2023) 258

Agarwal, Pal, Srivastav, AT ;
JHEP 06 (2022) 020

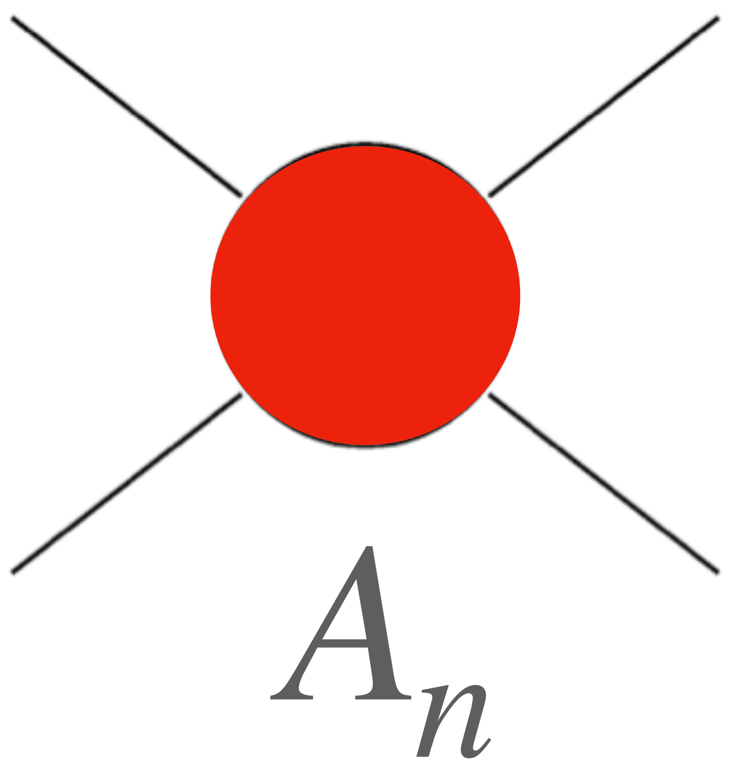
Agarwal, Magnea, Pal, AT ;
JHEP 03 (2021) 188

Agarwal, Danish, Magnea, Pal, AT ;
JHEP 05 (2020) 128

2018

Time

Fixed-angle multi-parton Scattering Amplitude
In IR limit



Jet function

$$\mathcal{A}_n\left(\frac{p_i}{\mu}, \alpha_s(\mu), \epsilon\right) = \prod_{i=1}^{n} \frac{\mathcal{J}_i\left(\frac{(p_i \cdot n_i)^2}{n_i^2 \mu^2}, \alpha_s(\mu^2), \epsilon\right)}{\mathcal{J}_{E,i}\left(\frac{(\beta_i \cdot n_i)^2}{n_i^2}, \alpha_s(\mu^2), \epsilon\right)}$$

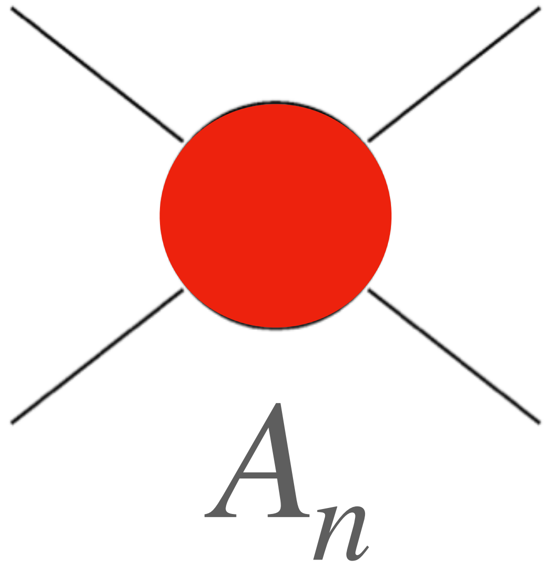
$$\times\, \mathcal{S}_n\left(\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon\right) \mathcal{H}_n\left(\frac{p_i \cdot p_j}{\mu^2}, \frac{(p_i \cdot n_i)^2}{n_i^2 \mu^2}, \alpha_s(\mu^2), \epsilon\right)$$

Soft function

Hard function

Multi-parton Scattering Amplitude
In IR limit

IR behaviour $\leftrightarrow$ Wilson line correlator



Soft matrix

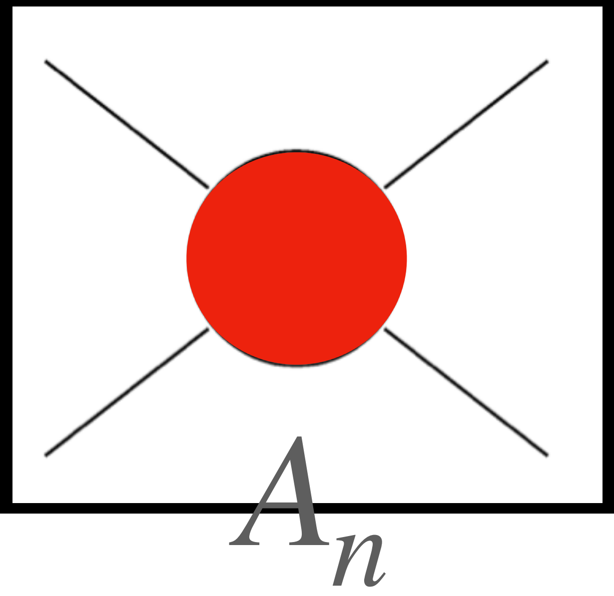
$$\mathcal{S}\left(\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon\right) \equiv \langle 0 | \, T \left[\prod_{k=1}^{n} \Phi_{\beta_k}(\infty, 0) \right] | 0 \rangle$$

Wilson line

$$\Phi_\beta\left(\infty, 0\right) \equiv P \exp\left[ig \int_0^\infty d\lambda \, \beta \cdot \mathbf{A}(\lambda\beta) \right]$$

Soft anomalous dimension

$$\mathcal{S}_n\left(\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon\right) = \mathcal{P} \exp\left[-\frac{1}{2} \int_0^{\mu^2} \frac{d\lambda^2}{\lambda^2} \mathbf{\Gamma}_n\left(\beta_i \cdot \beta_j, \alpha_s(\lambda^2), \epsilon\right) \right]$$



1-loop Soft Anomalous dimension

$$\Gamma^1 = -\sum \mathbf{T}_i \cdot \mathbf{T}_j \;\; \xi_{ij} \;\; \coth(\xi_{ij})$$

$$\xi_{ij} = \cosh^{-1}\left(-\frac{\gamma_{ij}}{2}\right)$$

$$\gamma_{ij} = 2\frac{p_i \cdot p_j}{\sqrt{p_i^2 p_j^2}}$$

Minkowskian angles

Diagrammatic Exponentiation
(A complementary approach)

Kinematic factor $K(D)$

Color factor $C(D)$

$$S_n = \sum_D K(D)\, C(D)$$

Mitov, Sterman, Sung; 2010

Gardi, Laenen, Stavenga, White; 2010

Gardi, Smillie, White; 2011

$$S_n = \exp\left[\mathscr{W}_n\right]$$

Gardi, White; 2011

Dukes, Gardi, Steingrimsson, White; 2013

Gardi, Smillie, White; 2013

Dukes, Gardi, McAslan, Scott, White; 2016

Modified colour factors $\widetilde{C}(D)$

$$\mathscr{W} = \sum_D K(D)\, \widetilde{C}(D)$$

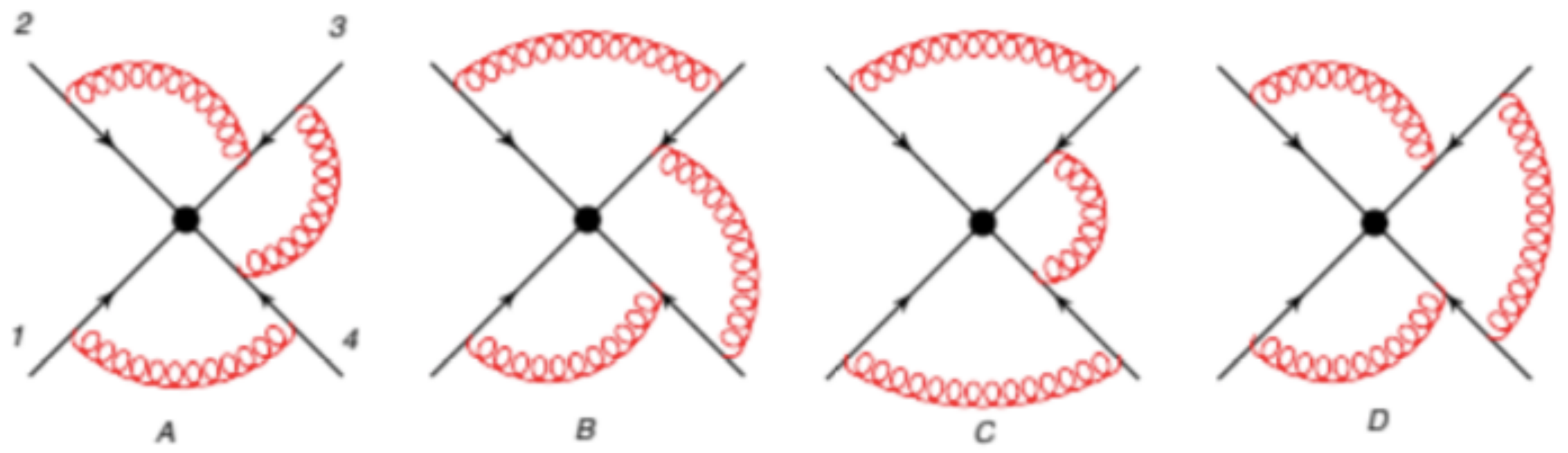
See also: Vladimirov, 2014-2017 for Alternative approach

For Eikonal Form factors these are called webs. **Gatheral; Frenkel, Taylor; Sterman**

Multi-parton Webs

Web (W) : A set of diagrams closed under permutations of the gluon attachments on the Wilson lines.

(Gardi, Smillie, White, *et al* *2010-2013*)



The exponent $W(\gamma_i)$ grouped into webs

$$S_n = \exp\left(\sum W \right)$$

$R_w(D, D')$
Web mixing matrix

$$S_n = \exp\left(\sum_W \sum_{D,D' \in W} K(D)\, R_W(D, D')\, C(D) \right)$$

Properties of web mixing matrices

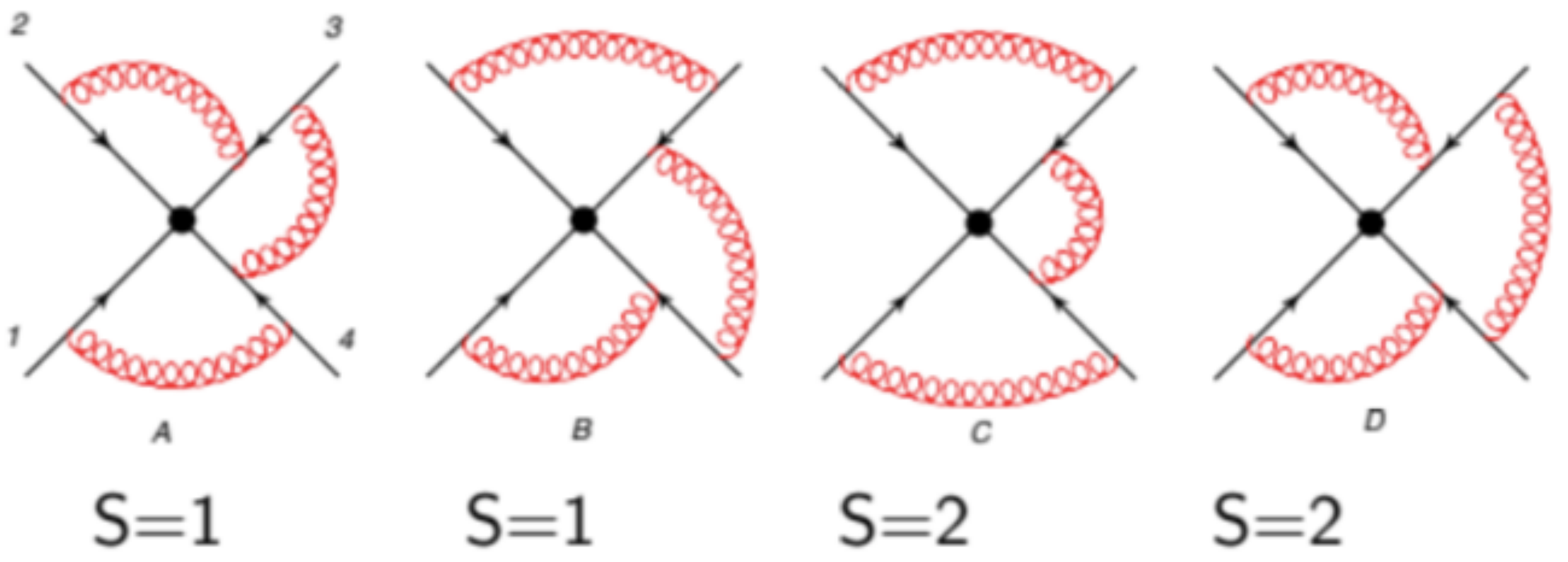
(Gardi, Smillie, White, et al 2010-2013)

Projector $R^2 = R$

Row sum rule

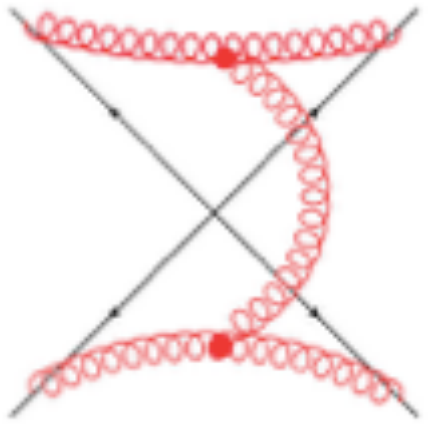
Ensures the cancellation of leading divergences in webs

$$\sum_{D'} R\,(D, D') = 0$$



Column sum rule (Conjecture)

$$\sum_{D} s(D)\,R\,(D, D') = 0$$

$\Longrightarrow$ 

Exponentiated colour factor

Connection with Mathematical structures (Posets)

(Dukes, Gardi, McAslan, Scott, White)

Indirect handle on (difficult) Kinematics?

$$S_n = \exp\left(\sum_W \sum_{D,D' \in W} K(D)\, R_W(D, D')\, C(D) \right)$$

Objects of interest

1) Rank of the mixing matrix R_W

2) The mixing matrix R_W

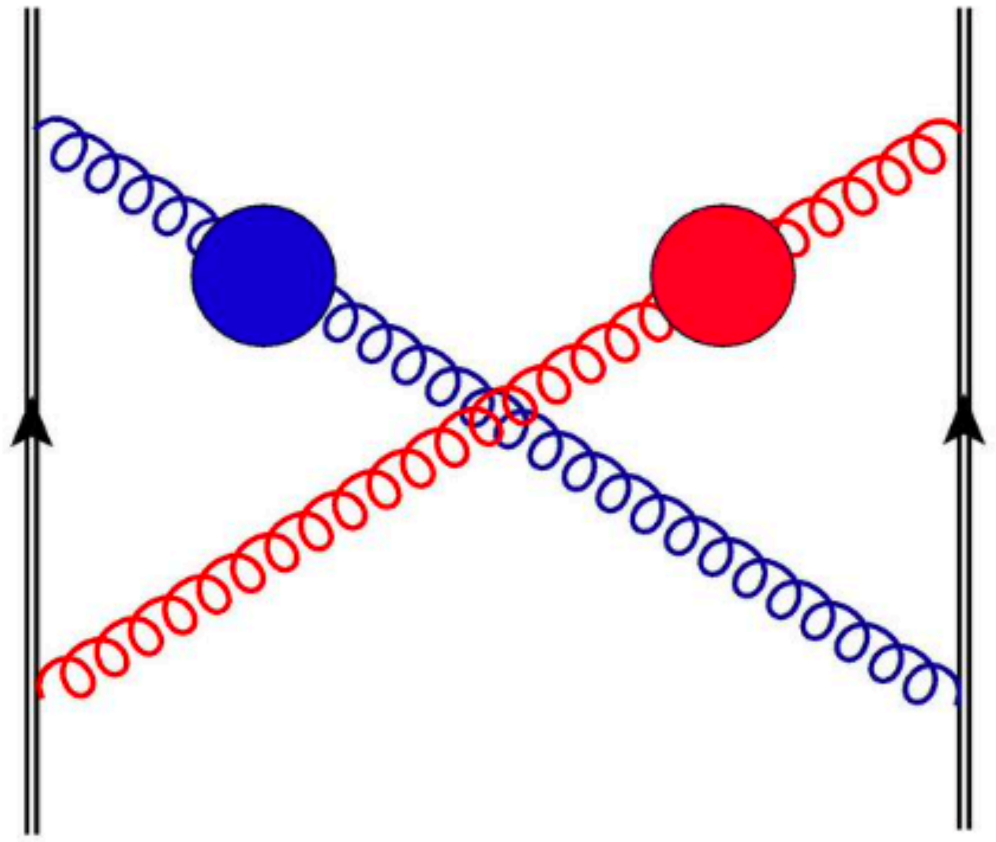
3) Exponenitated Colour factors

$$\sum_{D,D' \in W} K(D)\, R_W(D, D')\, C(D) = \sum_{i=1}^{\text{rank}(W)} (K^T Y^{-1})_i\, (YC)_i$$

Exponentiat

$$Y R_W Y^{-1} = diag(1,\ldots,1,0,\ldots,0)$$

Drawing the diagrams slightly differently
(Apologies for inconvenience!)



The tails of the Wilson lines are not visually meeting at the origin.

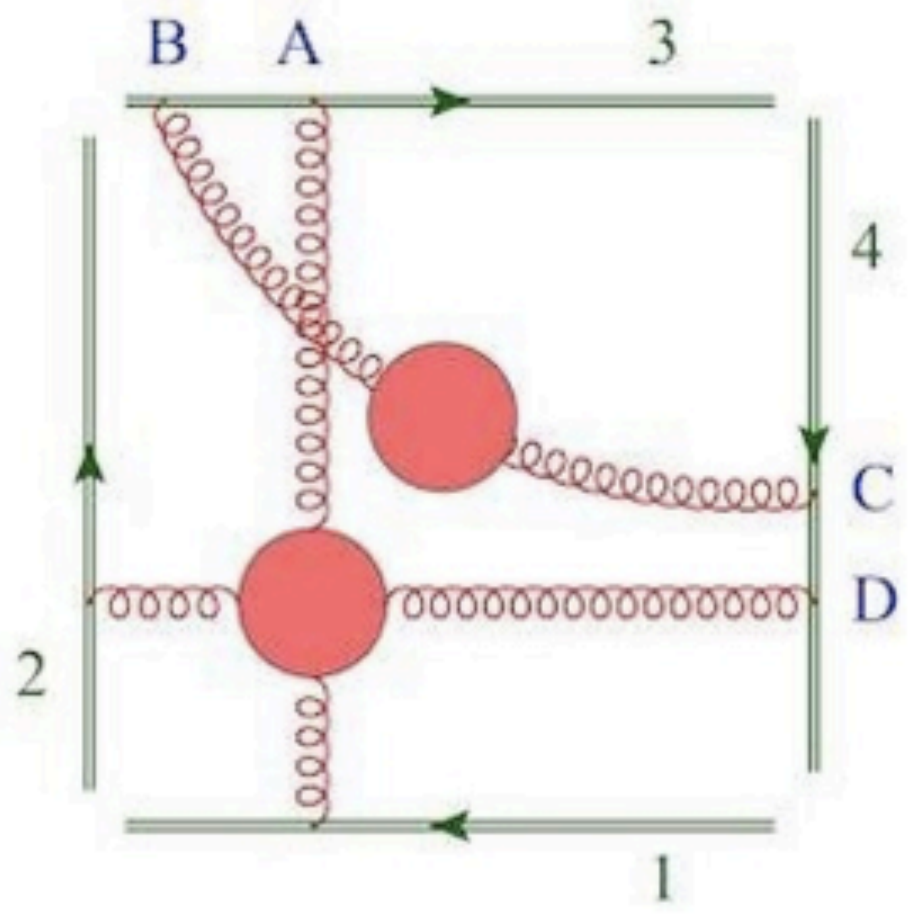
This makes drawing the diagrams easy.

Results at 4 loops

(One of the 4-leg webs)

$\mathbf{W}_{4,\mathrm{I}}^{(1,0,1)}(1,1,2,2)$

Agarwal, Danish, Magnea, Pal, AT ; 2020



Diagrams	Sequences	S-factors
C_1	$\{\{BA\},\{CD\}\}$	1
C_2	$\{\{BA\},\{DC\}\}$	0
C_3	$\{\{AB\},\{CD\}\}$	0
C_4	$\{\{AB\},\{DC\}\}$	1

$$R = \begin{pmatrix} \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad D = (\mathbf{1}_3, 0)$$

Exponentiated Color factors

$$(YC)_1 = if^{abg} f^{cdg} f^{edh} \mathbf{T}_1^a \mathbf{T}_2^b \mathbf{T}_3^e \mathbf{T}_3^c \mathbf{T}_4^h - if^{abg} f^{cdg} f^{cej} \mathbf{T}_1^a \mathbf{T}_2^b \mathbf{T}_3^j \mathbf{T}_4^d \mathbf{T}_4^e \,,$$

$$(YC)_2 = -if^{abg} f^{cdg} f^{cej} \mathbf{T}_1^a \mathbf{T}_2^b \mathbf{T}_3^j \mathbf{T}_4^d \mathbf{T}_4^e \,,$$

$$(YC)_3 = if^{abg} f^{cdg} f^{edh} \mathbf{T}_1^a \mathbf{T}_2^b \mathbf{T}_3^e \mathbf{T}_3^c \mathbf{T}_4^h - f^{abg} f^{cdg} f^{cej} f^{edh} \mathbf{T}_1^a \mathbf{T}_2^b \mathbf{T}_3^j \mathbf{T}_4^h \,.$$

Is there a pattern?

$$R = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$R = \begin{pmatrix} \frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$$

$$R = \begin{pmatrix} \frac{1}{6} & 0 & -\frac{1}{3} & 0 & \frac{1}{6} & 0 \\ -\frac{1}{3} & \frac{1}{2} & -\frac{1}{3} & 0 & \frac{2}{3} & -\frac{1}{2} \\ -\frac{1}{3} & 0 & \frac{2}{3} & 0 & -\frac{1}{3} & 0 \\ \frac{1}{6} & -\frac{1}{2} & -\frac{1}{3} & 1 & \frac{1}{6} & -\frac{1}{2} \\ \frac{1}{6} & 0 & -\frac{1}{3} & 0 & \frac{1}{6} & 0 \\ \frac{2}{3} & -\frac{1}{2} & -\frac{1}{3} & 0 & -\frac{1}{3} & \frac{1}{2} \end{pmatrix}$$

$$R = \begin{pmatrix} \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \\ -\frac{1}{6} & \frac{1}{6} & \frac{1}{6} & -\frac{1}{6} \\ -\frac{1}{6} & \frac{1}{6} & \frac{1}{6} & -\frac{1}{6} \\ \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \end{pmatrix}$$

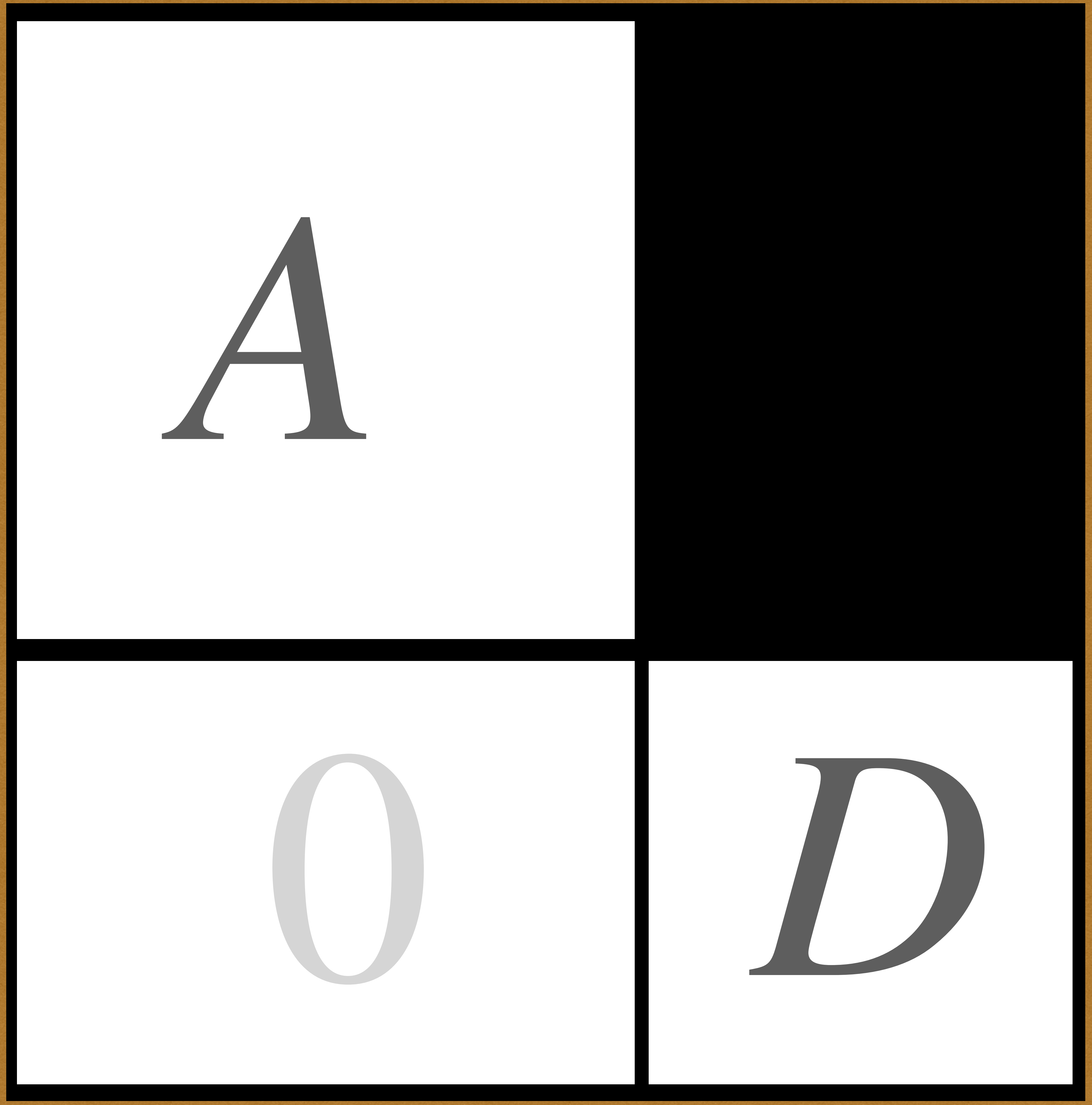
$$R = \begin{pmatrix} \frac{1}{3} & -\frac{1}{3} & 0 & 0 & -\frac{1}{3} & \frac{1}{3} \\ -\frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & -\frac{1}{6} \\ -\frac{1}{6} & \frac{1}{6} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{1}{3} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{6} & -\frac{1}{6} \\ -\frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & -\frac{1}{6} \\ \frac{1}{3} & -\frac{1}{3} & 0 & 0 & -\frac{1}{3} & \frac{1}{3} \end{pmatrix}$$

A sample of 4-loop
Cweb mixing matrices

Agarwal, Magnea, Pal, AT ;
JHEP 03 (2021) 188

Agarwal, Danish, Magnea, Pal, AT ;
JHEP 05 (2020) 128

CwebGen 2.0



Agarwal, Pal, Srivastav, AT ;
JHEP 06 (2022) 020

Uniqueness theorem

An organising idea

A web in a web!

Classification of diagrams

Irreducible

$$s(d) = 0$$

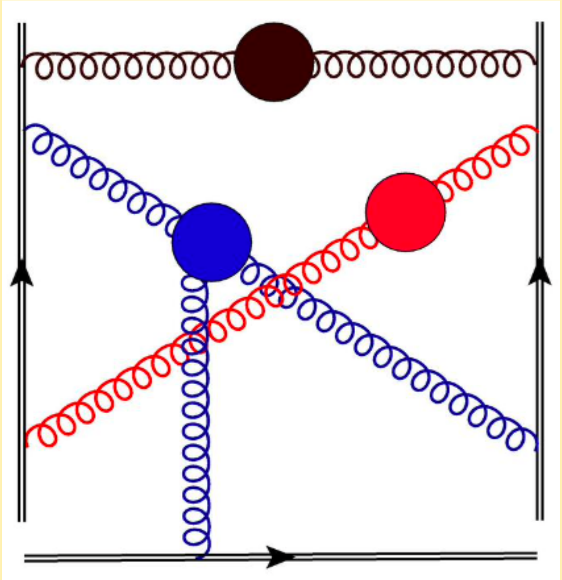
Partially Entangled

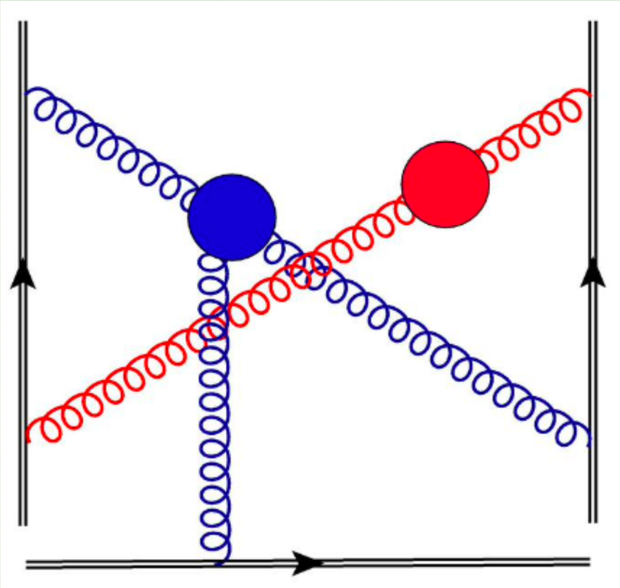
Completely Entangled

Reducible

$$s(d) \neq 0$$

Classification of diagrams





Irreducible

$$s(d) = 0$$

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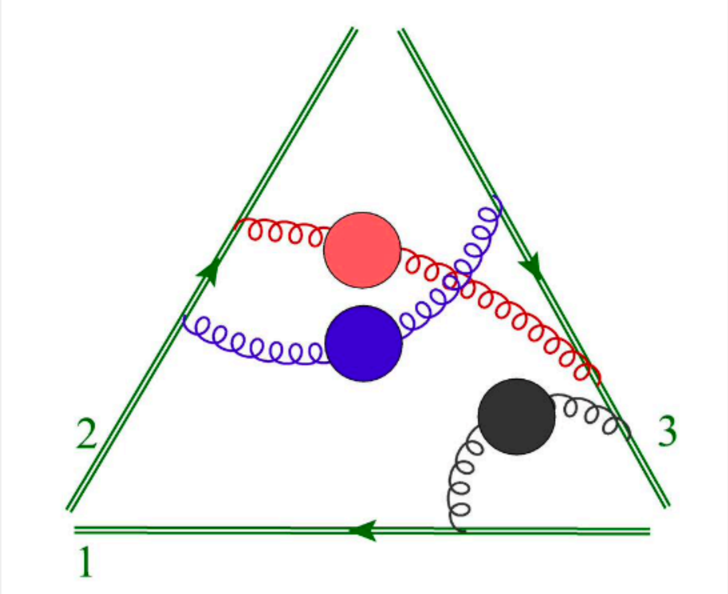

A general web

Normal Ordering

d_1	$\cdots$	d_k	d_{k+1}	$\cdots$	d_l	d_{l+1}	$\cdots$	d_{l+m}

Completely Entangled — Partially Entangled — Reducible

$$R = \left(\begin{array}{cc|c} I_{k\times k} & (A_U)_{k\times(l-k)} & \multirow{2}{*}{$B_{l\times m}$} \\ O_{(l-k)\times k} & (A_L)_{(l-k)\times(l-k)} & \\ \hline \multicolumn{2}{c|}{O_{m\times l}} & D_{m\times m} \end{array}\right)$$

Webs containing only reducible diagrams
$(s(d_i) \neq 0, \quad \forall i)$

Uniqueness Theorem:

For a given column weight vector

$S = \{s(d_1), \ldots, s(d_n)\}$

$s(d_i) \neq 0, \ \forall i$

the mixing matrix is unique.

Agarwal, Pal, Srivastav, AT ;
JHEP 06 (2022) 020

Webs containing only reducible diagrams

$$(s(d_i) \neq 0, \quad \forall i)$$

Uniqueness Theorem: For a given column weight vector

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	Web-1 at $\mathcal{O}(\alpha_s^N)$	Web-2 at $\mathcal{O}(\alpha_s^M)$
Identical	$S = \{s(d_1), \ldots, s(d_n)\}$	$S = \{s(d_1), \ldots, s(d_n)\}$
Identical	R	R

A and D diagonal blocks of mixing matrix R

$$R = \left(\begin{array}{cc|c} I_{k\times k} & (A_U)_{k\times(l-k)} & \multirow{2}{*}{$B_{l\times m}$} \\ O_{(l-k)\times k} & (A_L)_{(l-k)\times(l-k)} & \\ \hline \multicolumn{2}{c|}{O_{m\times l}} & D_{m\times m} \end{array}\right)$$

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The Block D satisfies the known properties of the mixing matrix!

$D^2 = D$

Satisfy Row Sum Rule

Satisfy Column Sum Rule

Block D

$$R = \left(\begin{array}{cc|c} I_{k\times k} & (A_U)_{k\times(l-k)} & B_{l\times m} \\ O_{(l-k)\times k} & (A_L)_{(l-k)\times(l-k)} & \\ \hline \multicolumn{2}{c|}{O_{m\times l}} & D_{m\times m} \end{array}\right)$$

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The Block D satisfies the known properties of the mixing matrix!

$D^2 = D$ Satisfy Row Sum Rule Satisfy Column Sum Rule

If $S = \{s_{l+1}, \ldots, s_{l+m}\}$ With all entries non vanishing

Using Uniqueness Theorem

D block is known if any web with same S has been calculated.

Fused web Formalism

Agarwal, Pal, Srivastav, AT ;
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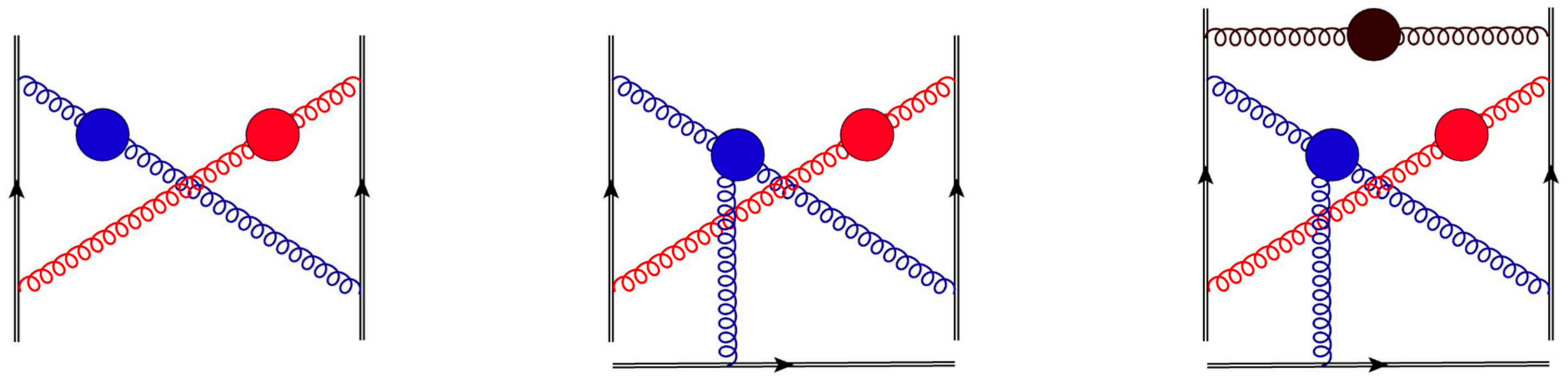
Block A

Coarse graining : The idea of Fused Webs

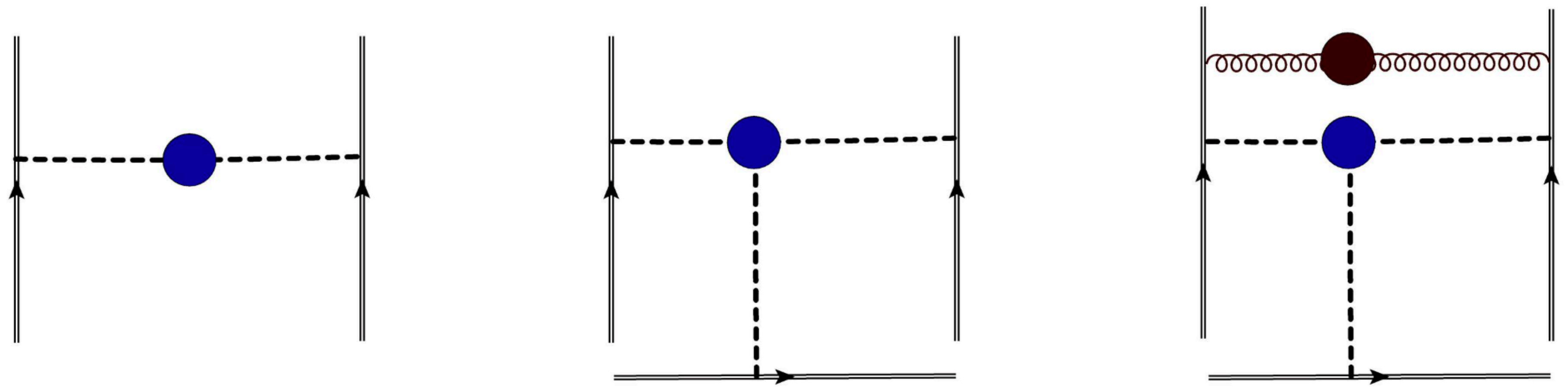
Original diagrams



(a) (b) (c)

$$R = \left(\begin{array}{cc|c} I_{k\times k} & (A_U)_{k\times(l-k)} & \multirow{2}{*}{$B_{l\times m}$} \\ O_{(l-k)\times k} & (A_L)_{(l-k)\times(l-k)} & \\ \hline \multicolumn{2}{c|}{O_{m\times l}} & D_{m\times m} \end{array}\right)$$

Fused diagrams



(d) (e) (f)

Colour factor of a Fused diagram = Colour factor of the original diagram

s-factors are defined in the usual way.

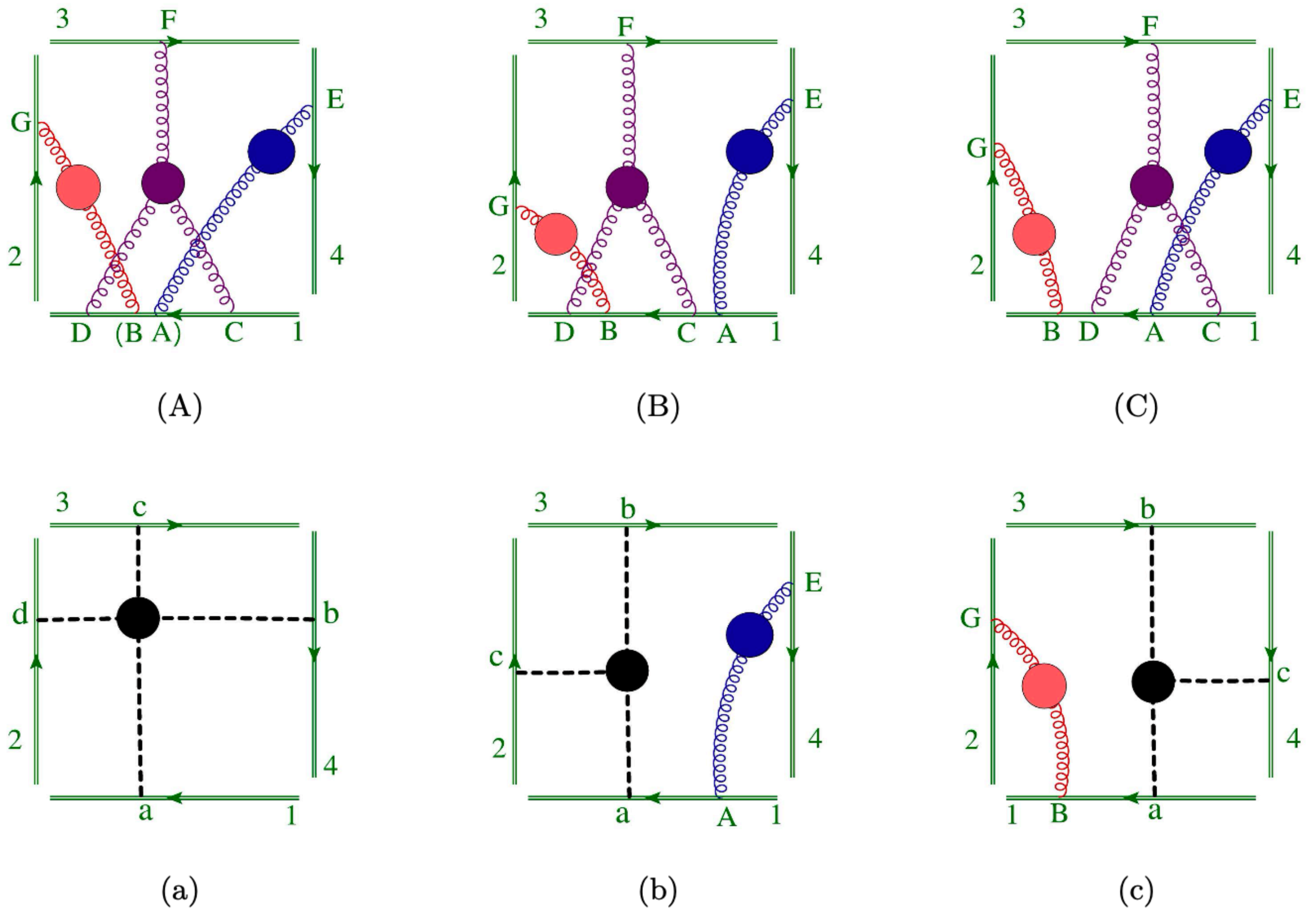
Application of fused web formalism

A sample web

12 diagrams

Completely Entangled : 2 Partially Entangled. : 4	→	Block A
Reducible. : 6	→	Block D

$$
R = \left(\begin{array}{c|cc|c}
\mathrm{I}_2 & \multicolumn{2}{c|}{A_U} & \\
\hline
\mathrm{O}_{4\times 2} & R\,(1_2) & X & B \\
 & \mathrm{O}_{2\times 2} & R\,(1_2) & \\
\hline
\multicolumn{3}{c|}{\mathrm{O}_{6\times 6}} & D
\end{array}\right)
$$



Cweb $W_4^{(2,1)}(1,1,1,4)$,

(Showing only completely and partially entangled diagrams)

Application of fused web formalism

Cweb $W_4^{(2,1)}(1,1,1,4)$

12 diagrams

Completely Entangled: 2
Partially Entangled: 4 → Block A

Reducible: 6 → Block D

$$R = \left(\begin{array}{c|cc|c} I_2 & \multicolumn{2}{c|}{A_U} & \\ \cline{1-3} O_{4\times 2} & R(1_2) & X & B \\ & O_{2\times 2} & R(1_2) & \\ \hline \multicolumn{3}{c|}{O_{6\times 6}} & D \end{array}\right)$$

$$R = \frac{1}{6}\begin{pmatrix} 6 & 0 & -3 & -3 & -3 & -3 & -1 & 2 & 2 & -1 & 2 & 2 \\ 0 & 6 & -3 & -3 & -3 & -3 & 2 & -1 & 2 & 2 & 2 & -1 \\ 0 & 0 & 3 & -3 & 0 & 0 & -1 & -1 & -1 & -1 & 2 & 2 \\ 0 & 0 & -3 & 3 & 0 & 0 & 2 & -1 & 2 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 3 & -3 & -1 & -1 & 2 & 2 & -1 & -1 \\ 0 & 0 & 0 & 0 & -3 & 3 & -1 & 2 & -1 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -1 & -1 & -1 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 2 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 2 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 2 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 2 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -1 & -1 & -1 & -1 & 2 \end{pmatrix}$$

Agarwal, Pal, Srivastav, AT ;
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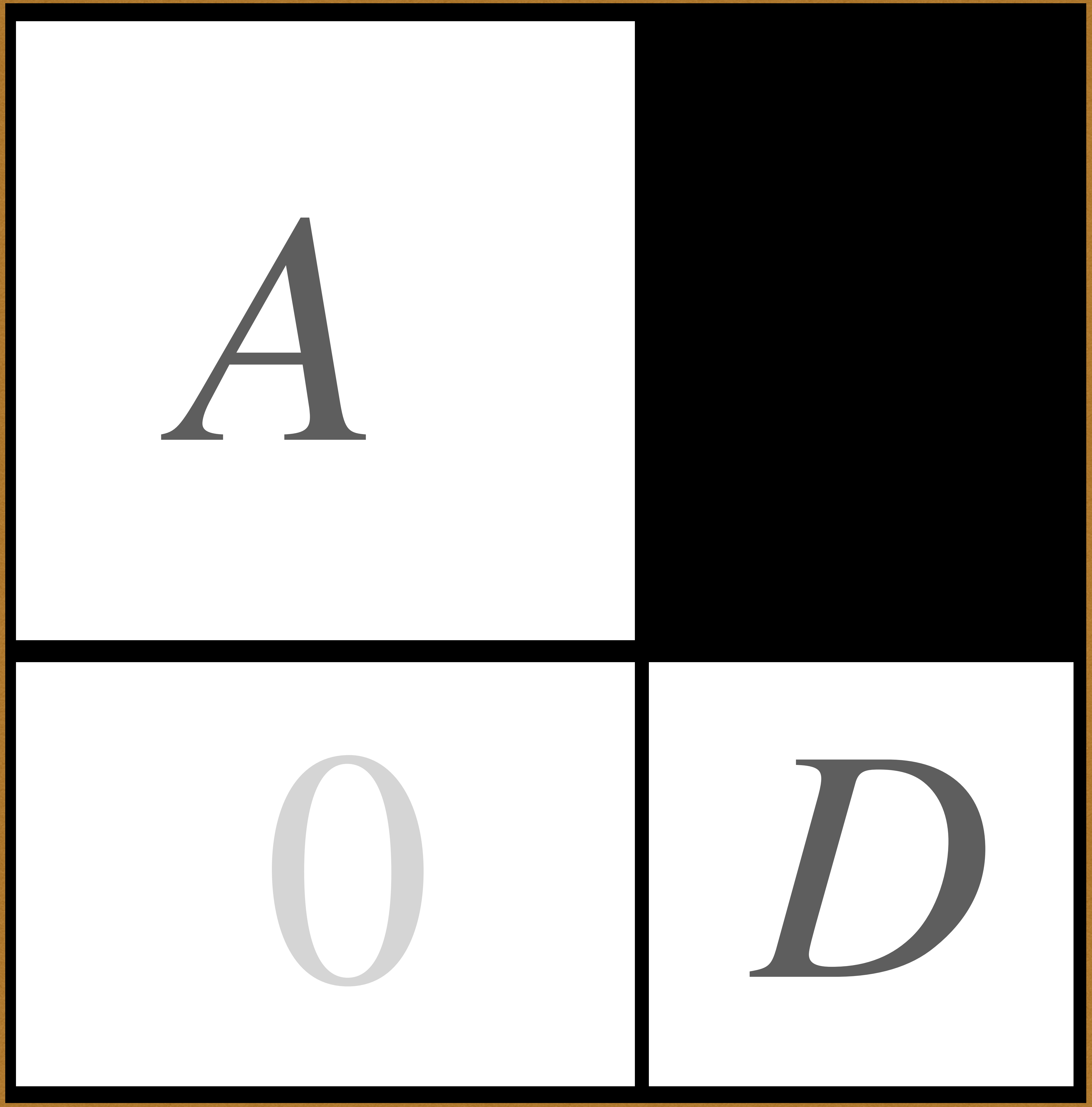
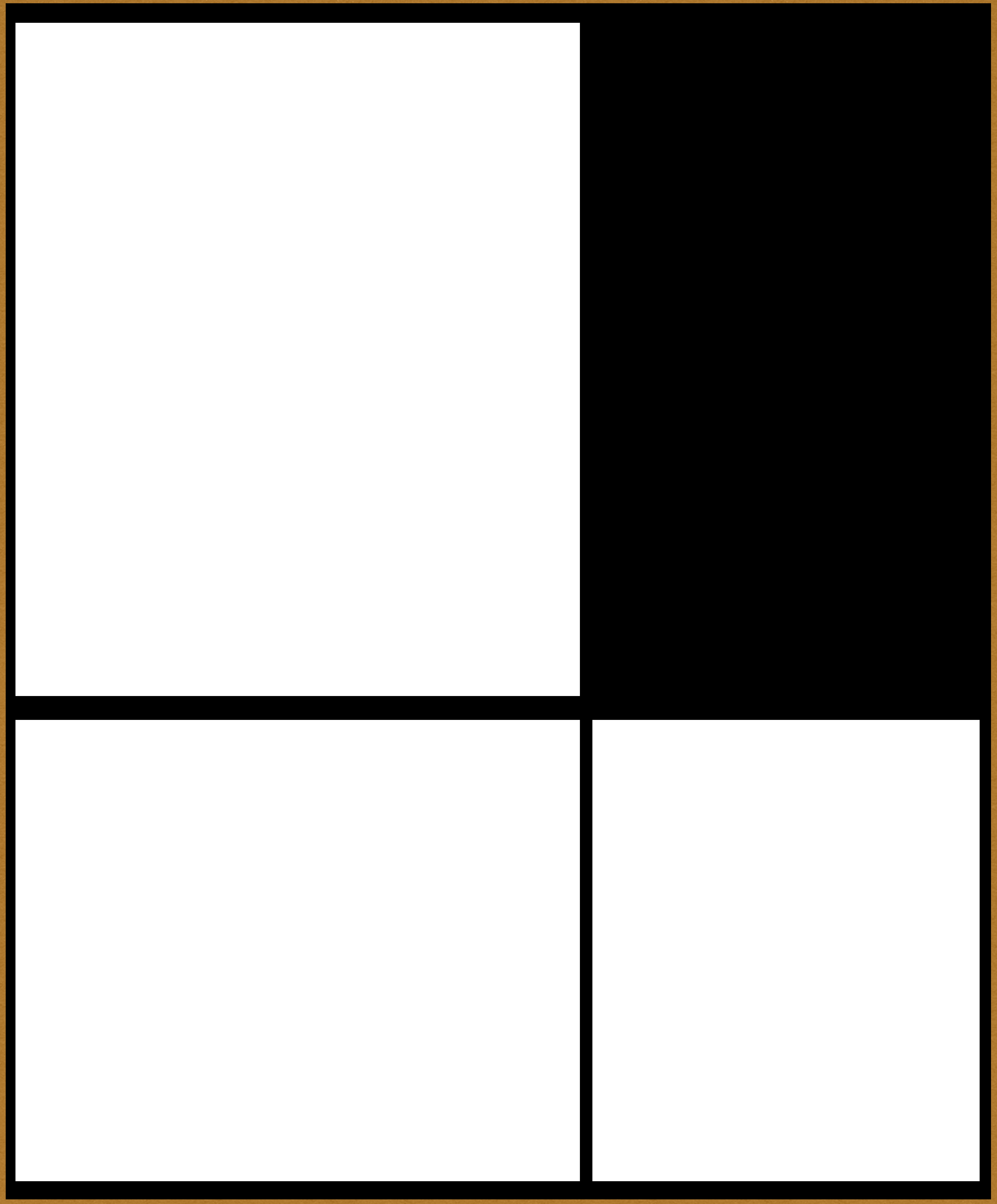
Application of fused web formalism

Rank can be obtained without explicit computation

Rank = # of Exponentiated Colour factors

We can obtain the # of exponentiated colour factors





Agarwal, Pal, Srivastav, AT ;
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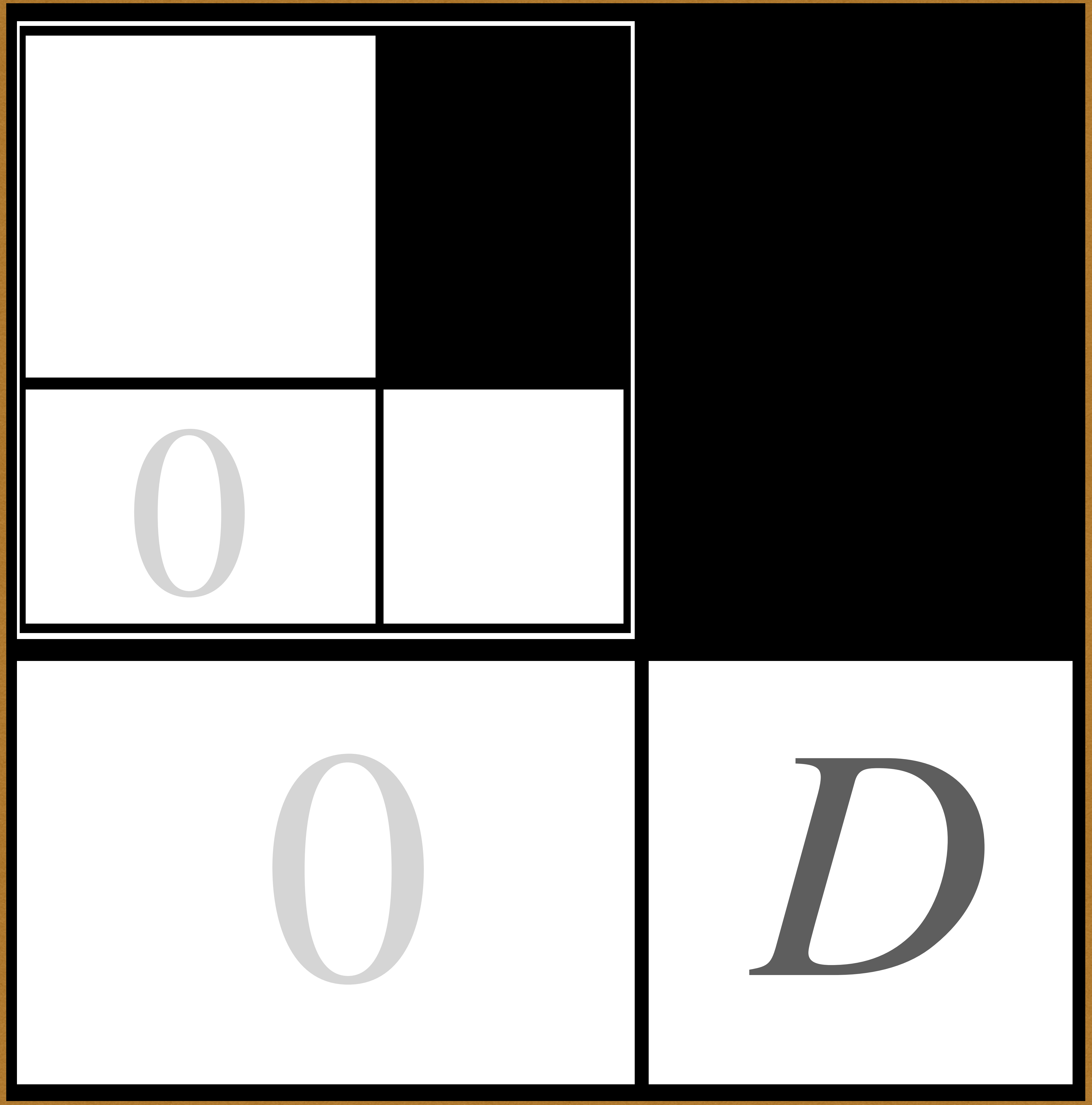

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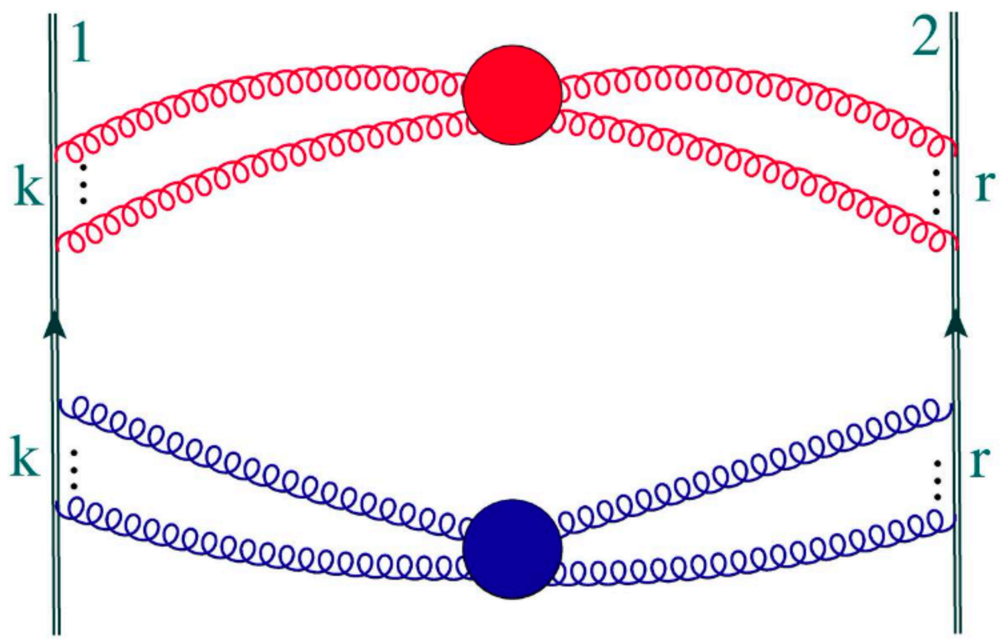
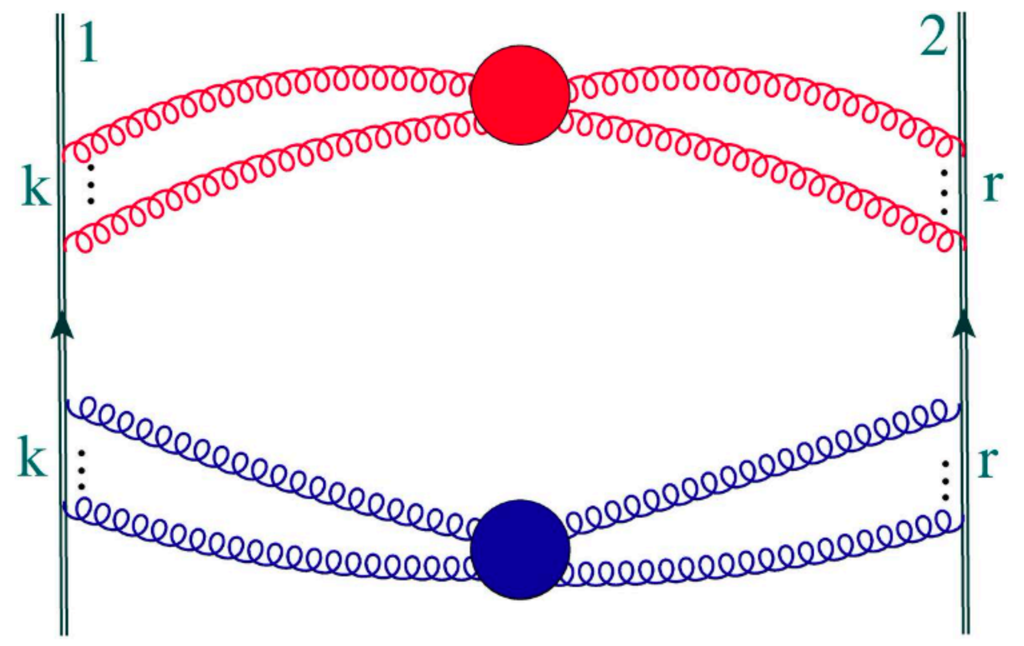
All order predictions for two special classes



$$R = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 1 \\ & & & & \end{pmatrix}$$

$$R = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 & -1/2 & -1/2 \\ 0 & 1 & 0 & \dots & 0 & -1/2 & -1/2 \\ 0 & 0 & 1 & \dots & 0 & -1/2 & -1/2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & -1/2 & -1/2 \\ 0 & 0 & 0 & \dots & 0 & 1/2 & -1/2 \\ 0 & 0 & 0 & \dots & 0 & -1/2 & 1/2 \end{pmatrix} .$$

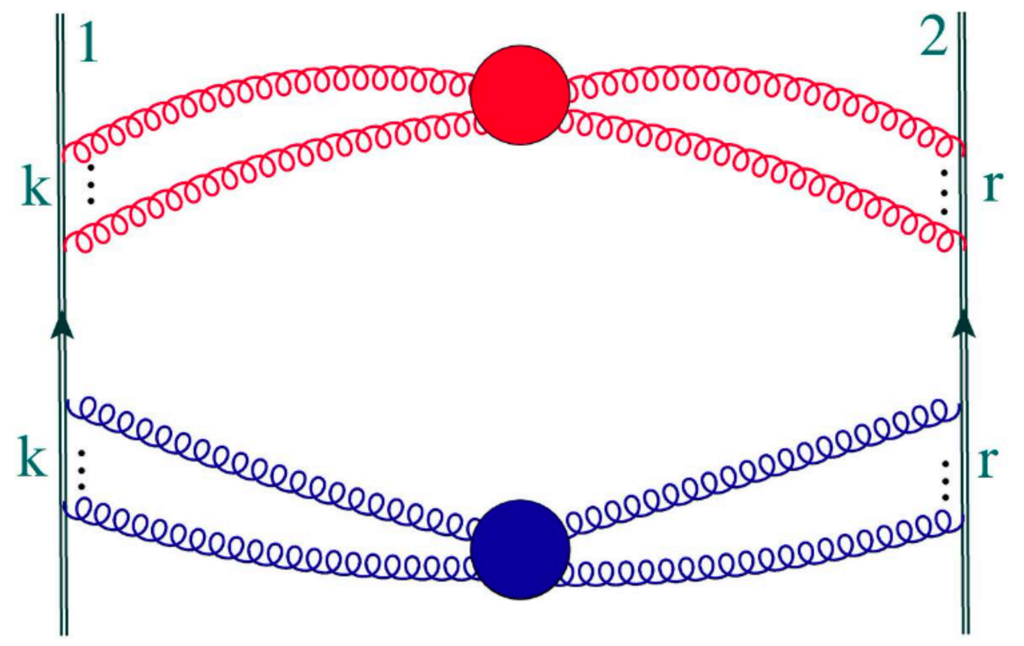
All order predictions for two special classes



$$R = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$R = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 & -1/2 & -1/2 \\ 0 & 1 & 0 & \dots & 0 & -1/2 & -1/2 \\ 0 & 0 & 1 & \dots & 0 & -1/2 & -1/2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & -1/2 & -1/2 \\ 0 & 0 & 0 & \dots & 0 & 1/2 & -1/2 \\ 0 & 0 & 0 & \dots & 0 & -1/2 & 1/2 \end{pmatrix}.$$

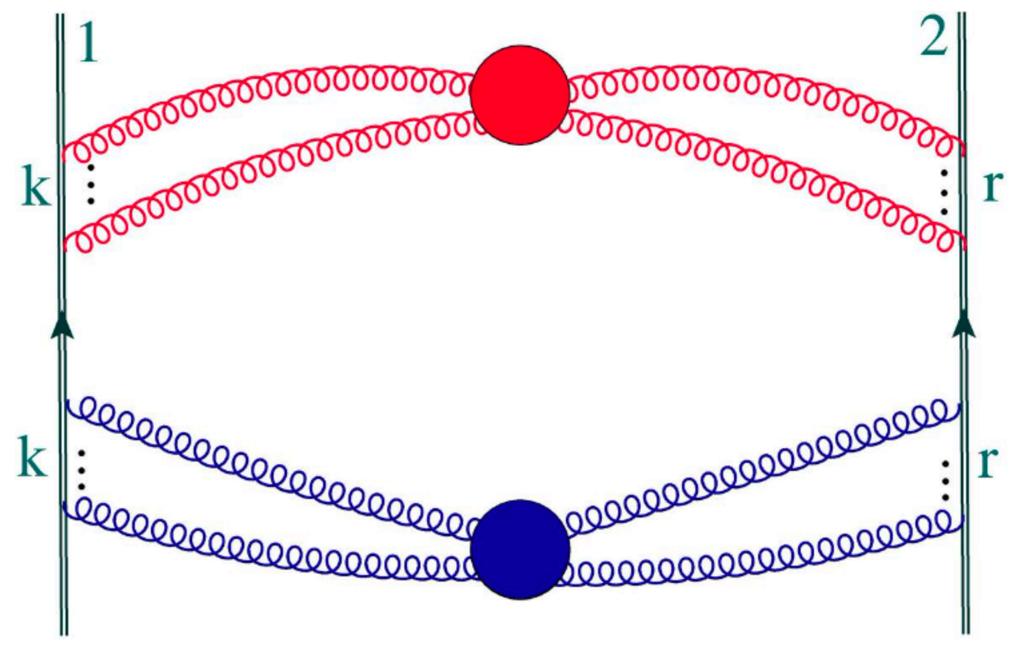
All order predictions for two special classes



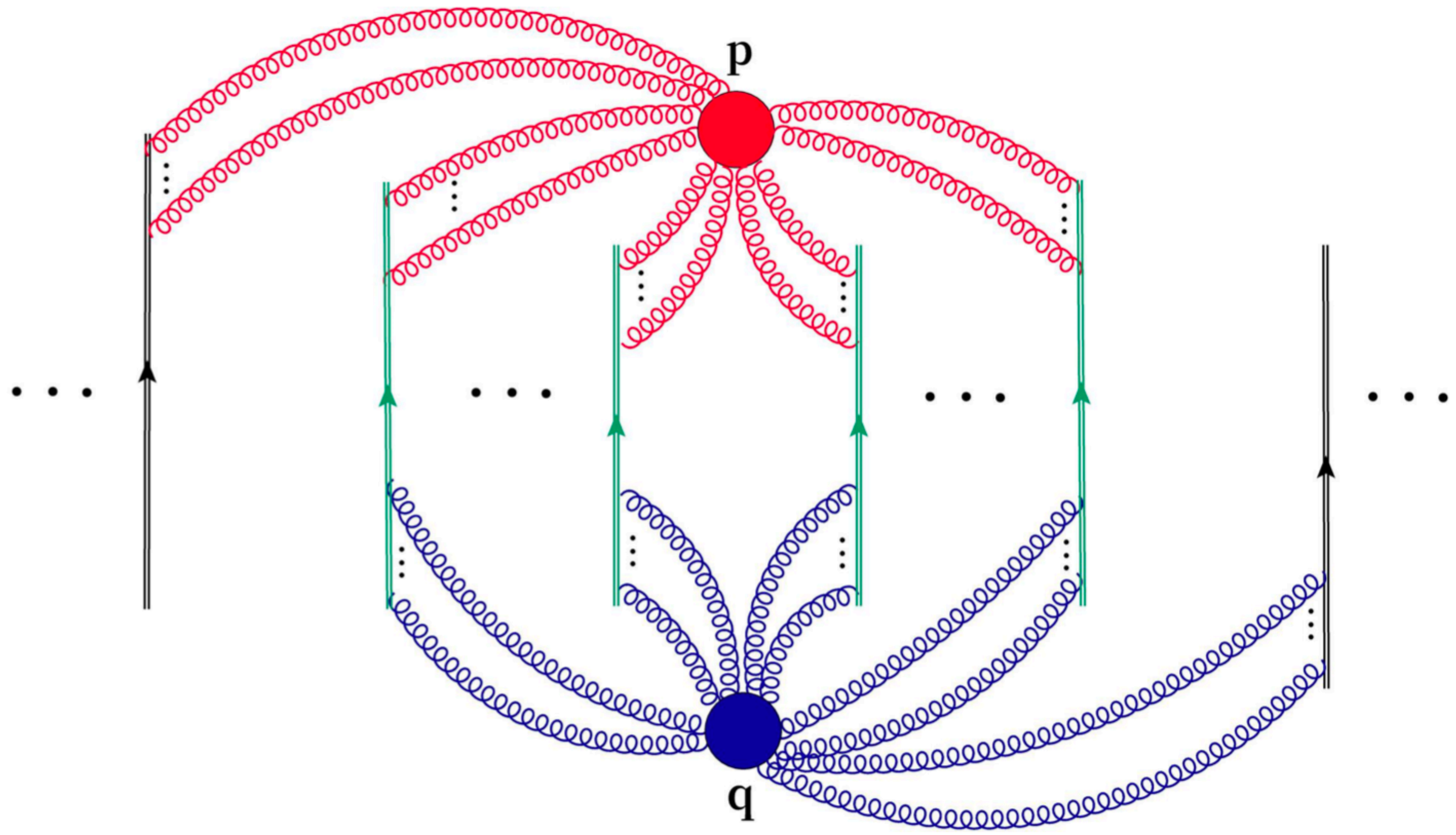
$$R = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$R = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 & -1/2 & -1/2 \\ 0 & 1 & 0 & \dots & 0 & -1/2 & -1/2 \\ 0 & 0 & 1 & \dots & 0 & -1/2 & -1/2 \\ \vdots & \vdots & \vdots & \vdots & \vdots\,\vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & -1/2 & -1/2 \\ 0 & 0 & 0 & \dots & 0 & 1/2 & -1/2 \\ 0 & 0 & 0 & \dots & 0 & -1/2 & 1/2 \end{pmatrix} .$$

All order predictions for two special classes



$$R = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$



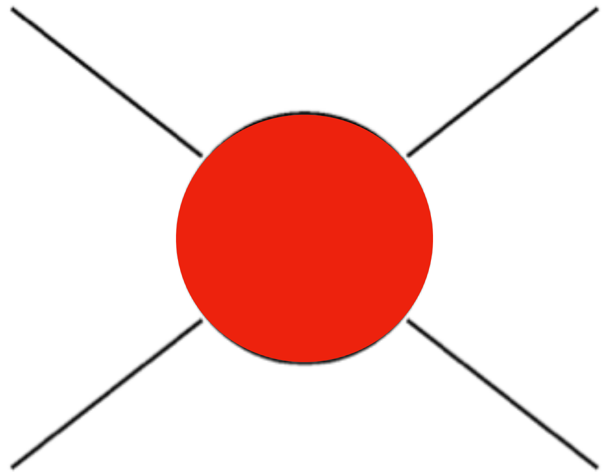
$$R = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 & -1/2 & -1/2 \\ 0 & 1 & 0 & \dots & 0 & -1/2 & -1/2 \\ 0 & 0 & 1 & \dots & 0 & -1/2 & -1/2 \\ \vdots & \vdots & \vdots & \vdots & \vdots\,\vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & -1/2 & -1/2 \\ 0 & 0 & 0 & \dots & 0 & 1/2 & -1/2 \\ 0 & 0 & 0 & \dots & 0 & -1/2 & 1/2 \end{pmatrix}.$$

Application of fused web formalism

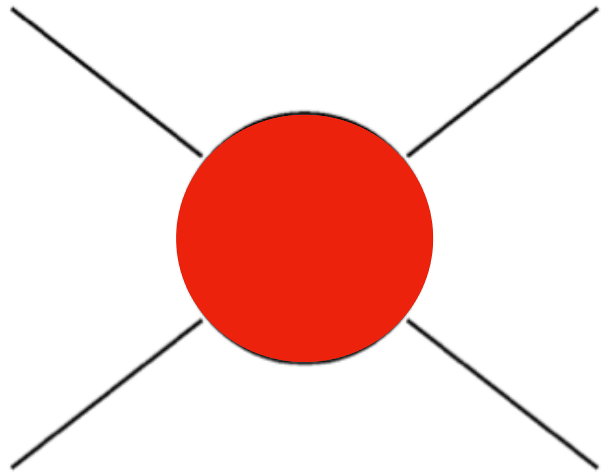
At 4 loops we can predict

- Diagonal blocks: 60% of the matrices
- Complete construction: ~50% of the matrices

Summary



- Using our Fused Web formalism we can obtain the diagonal blocks of R
- Diagonal Blocks are I or mixing matrices themselves
- # Exponentiated colour factors can be predicted using the diagonal blocks
- All order predictions can be made for special classes
- Important application of fused webs to boomerang webs



Thank You!

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The infrared structure of perturbative gauge theories ☆

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Thank You!

A R T I C L E I N F O

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A B ... T R A C T

Infra... di... perturbative expansion of gauge theory amplitudes and cross sections have been a focus of theoretical investigations for almost a century. New insights still continue to emerge, as higher perturbative orders are explored, and high-

Backup Slides

Most recent work by IITH QCD group

- We have improved the algorithm
- We have also calculated contributions for scattering of massive Wilson lines
- Under review

Mixing matrices

Cwebs — Set of diagrams built out of gluon correlators

Agarwal, Danish, Magnea, Pal, AT ; 2020

Replica Trick — N_r identical copies of gauge fields are introduced, Wilson lines are replicated

Gardi, Laenen, Stavenga, White, 2010

See also: Vladimirov, 2014-2017

Replicated correlator

$$\mathcal{S}_n^{\text{repl.}}(\gamma_i) = \left[\mathcal{S}_n(\gamma_i)\right]^{N_r} = \exp\left[N_r\, \mathcal{W}_n(\gamma_i)\right]$$

Order N_r term

$$= \mathbf{1} + N_r\, \mathcal{W}_n(\gamma_i) + \mathcal{O}(N_r^2)$$

Combinatorics to extract ECF

- Assign replica number i to each connected gluon correlator
- Replica ordering operator to order colour generators $\mathbf{T}_k^i$ on each line
- # of hierarchies $h(m)$ between m replica numbers
- …
- Algorithm gives ECF

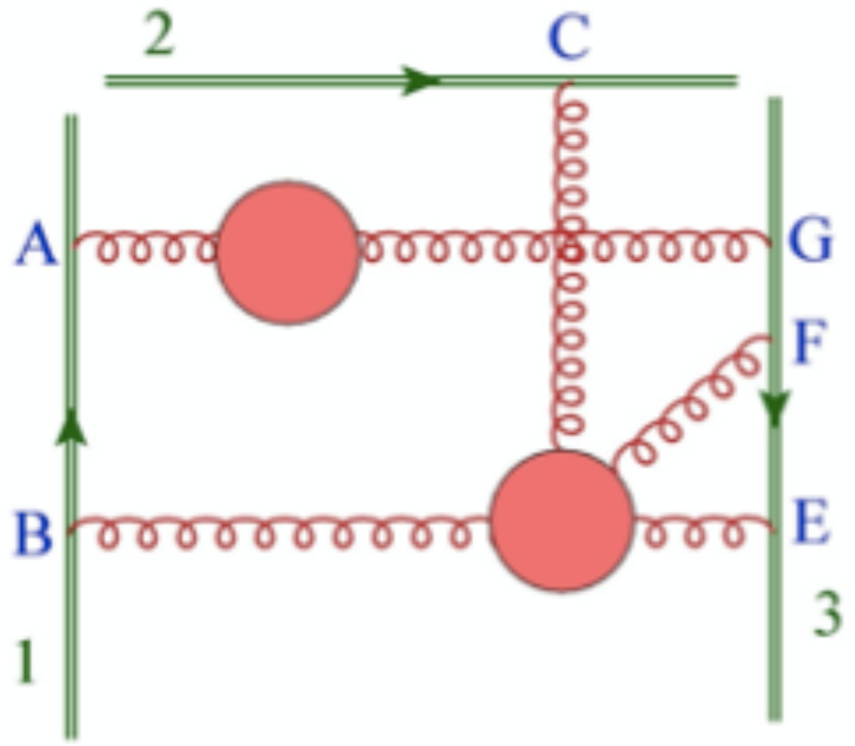
Inhouse Mathematica Code

The algorithm from generation of diagrams → computation ECF is implemented →Mixing matrices

New Results at 4 loops

(3 and 2-leg webs)

AT et al (to appear)



Diagrams	Sequences	S-factors
C_1	$\{\{BA\},\{GFE\}\}$	0
C_2	$\{\{BA\},\{FGE\}\}$	0
C_3	$\{\{BA\},\{FEG\}\}$	1
C_4	$\{\{AB\},\{GFE\}\}$	1
C_5	$\{\{AB\},\{FGE\}\}$	0
C_6	$\{\{AB\},\{FEG\}\}$	0

$$R = \begin{pmatrix} 1 & 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & -\frac{1}{2} & 1 & 0 \\ 0 & 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & 1 \end{pmatrix} \qquad D = (\mathbf{1}_5, 0)$$

Exponentiated Colour Factors

$$(YC)_1 = if^{afk}f^{bcg}f^{efg}\mathbf{T}_1^b\mathbf{T}_1^a\mathbf{T}_2^c\mathbf{T}_3^e\mathbf{T}_3^k + if^{aeh}f^{bcg}f^{efg}\mathbf{T}_1^b\mathbf{T}_1^a\mathbf{T}_2^c\mathbf{T}_3^h\mathbf{T}_3^f$$
$$-if^{abm}f^{bcg}f^{efg}\mathbf{T}_1^m\mathbf{T}_2^c\mathbf{T}_3^e\mathbf{T}_3^f\mathbf{T}_3^a$$

$$(YC)_2 = if^{afk}f^{bcg}f^{efg}\mathbf{T}_1^b\mathbf{T}_1^a\mathbf{T}_2^c\mathbf{T}_3^e\mathbf{T}_3^k - if^{abm}f^{bcg}f^{efg}\mathbf{T}_1^m\mathbf{T}_2^c\mathbf{T}_3^e\mathbf{T}_3^f\mathbf{T}_3^a$$

$$(YC)_3 = -if^{abm}f^{bcg}f^{efg}\mathbf{T}_1^m\mathbf{T}_2^c\mathbf{T}_3^e\mathbf{T}_3^f\mathbf{T}_3^a$$

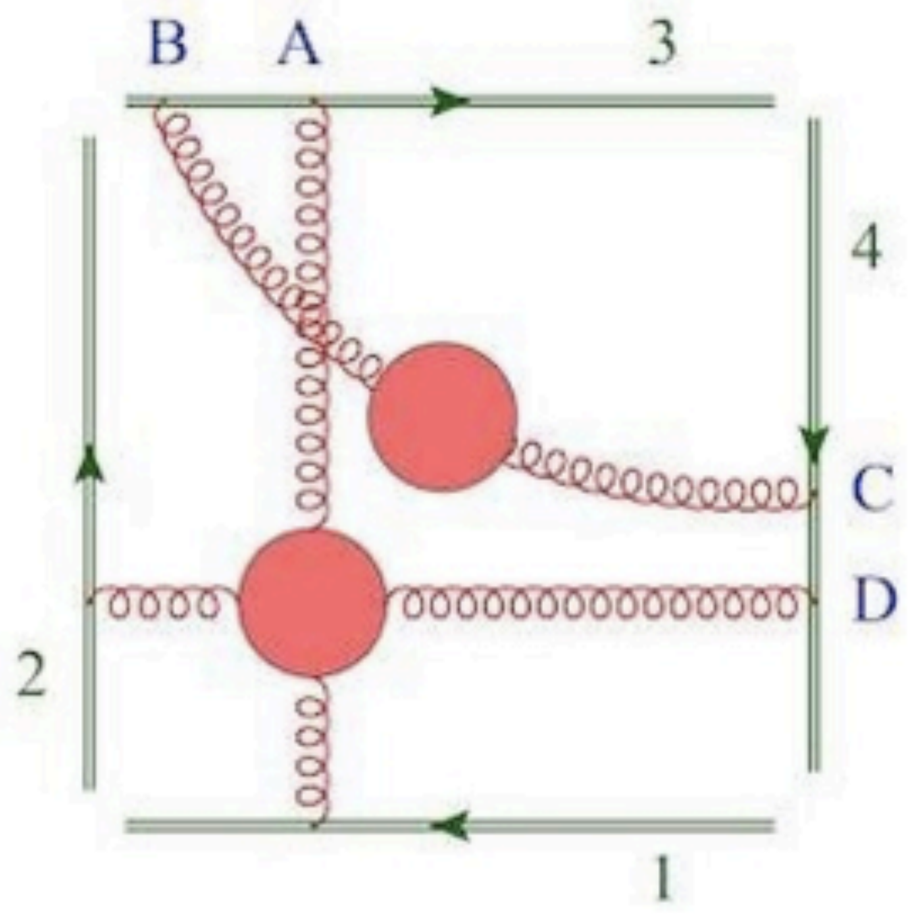
$$(YC)_4 = if^{afk}f^{bcg}f^{efg}\mathbf{T}_1^a\mathbf{T}_1^b\mathbf{T}_2^c\mathbf{T}_3^e\mathbf{T}_3^k + if^{aeh}f^{bcg}f^{efg}\mathbf{T}_1^a\mathbf{T}_1^b\mathbf{T}_2^c\mathbf{T}_3^h\mathbf{T}_3^f$$
$$(YC)_5 = if^{afk}f^{bcg}f^{efg}\mathbf{T}_1^a\mathbf{T}_1^b\mathbf{T}_2^c\mathbf{T}_3^e\mathbf{T}_3^k$$

Results at 4 loops

(One of the 4-leg webs)

$\mathbf{W}_{4,\mathrm{I}}^{(1,0,1)}(1,1,2,2)$

Agarwal, Danish, Magnea, Pal, AT ; 2020



Diagrams	Sequences	S-factors
C_1	$\{\{BA\},\{CD\}\}$	1
C_2	$\{\{BA\},\{DC\}\}$	0
C_3	$\{\{AB\},\{CD\}\}$	0
C_4	$\{\{AB\},\{DC\}\}$	1

$$R = \begin{pmatrix} \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad D = (\mathbf{1}_3, 0)$$

Exponentiated Color factors

$$(YC)_1 = i f^{abg} f^{cdg} f^{edh} \mathbf{T}_1^a \mathbf{T}_2^b \mathbf{T}_3^e \mathbf{T}_3^c \mathbf{T}_4^h - i f^{abg} f^{cdg} f^{cej} \mathbf{T}_1^a \mathbf{T}_2^b \mathbf{T}_3^j \mathbf{T}_4^d \mathbf{T}_4^e \,,$$

$$(YC)_2 = -i f^{abg} f^{cdg} f^{cej} \mathbf{T}_1^a \mathbf{T}_2^b \mathbf{T}_3^j \mathbf{T}_4^d \mathbf{T}_4^e \,,$$

$$(YC)_3 = i f^{abg} f^{cdg} f^{edh} \mathbf{T}_1^a \mathbf{T}_2^b \mathbf{T}_3^e \mathbf{T}_3^c \mathbf{T}_4^h - f^{abg} f^{cdg} f^{cej} f^{edh} \mathbf{T}_1^a \mathbf{T}_2^b \mathbf{T}_3^j \mathbf{T}_4^h \,.$$

Results at 4 loops

Agarwal, Danish, Magnea, Pal, AT ; 2020

Wilson line Correlators (**Cwebs**)	# of webs	Largest dimension of mixing matrix
5 legs	9	24
4 legs	21	24
3 legs	23	36
2 legs	8	36

Loop order (m)	Maximum number of hierarchies
1	**1**
2	3
3	13
4	75
5	541
6	**4683**

Fubini numbers
1,3,13,75,541,4683, …

Generating Function of Fubini numbers $h(m)$

$$\frac{1}{2-\exp(x)} - 1 \equiv \sum_{m=1}^{\infty} h(m)\,\frac{x^m}{m!}\,.$$

Results at 4 loops

Wilson line Correlators (**Cwebs**)	# of webs	Largest dimension of mixing matrix	Computational time (Upto 2021)
5 legs	9	24	7 days
4 legs	21	24	7 days
3 legs	23	36	9 days
2 legs	8	36	9 days

Agarwal, Danish, Magnea, Pal, AT ; 2020

in-house Mathematica code
Made in Hyderabad

Multi-parton Webs

Web (w) : A set of diagrams closed under permutations of the gluon attachments on the Wilson lines.

(Gardi, Smillie, White, *et al*)

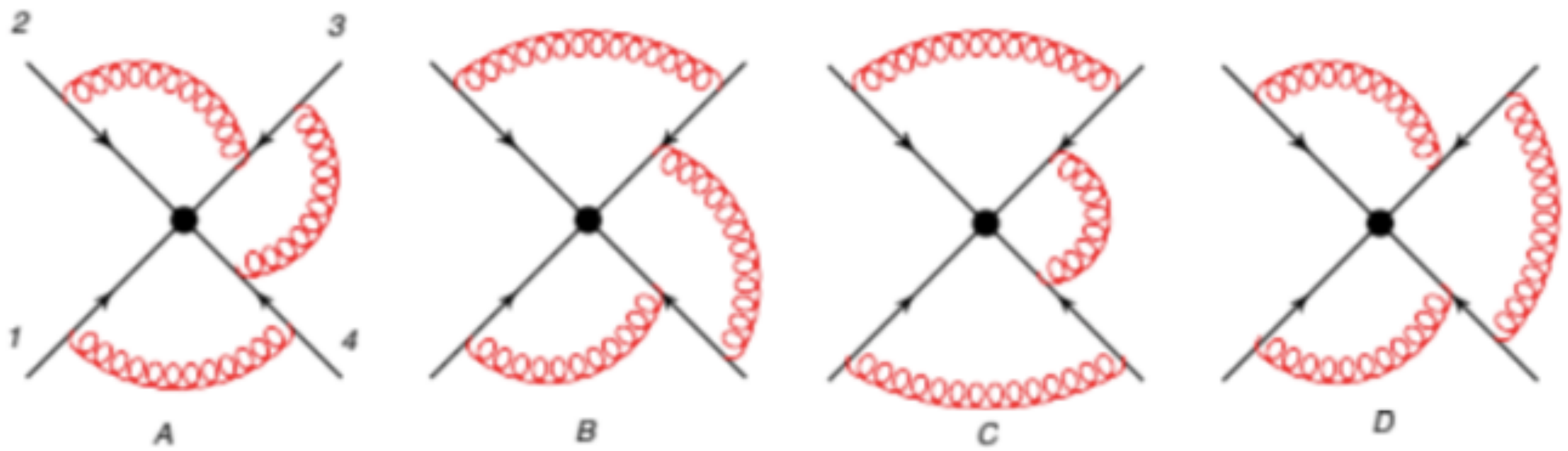
The exponent $W(\gamma_i)$ grouped into webs

$$S_n(\gamma_i) = \exp\left(\sum w\right)$$

$$= \exp\left(\sum_{D,D'\in w} K(D)\, R_w(D,D')\, C(D)\right)$$

$R_w(D,D')$
Web mixing matrix

A 3 loop web
4×4 **mixing matrix**



S=1 S=1 S=2 S=2