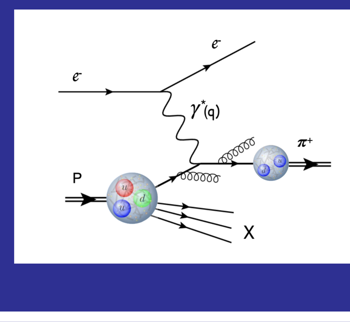
NNLO QCD Corrections to Semi-Inclusive Deep-Inelastic Scattering

Saurav Goyal†, Roman N. Lee, Sven-Olaf Moch, Vaibhav Pathak, Narayan Rana, V. Ravindran
The Institute of Mathematical Sciences Chennai, India

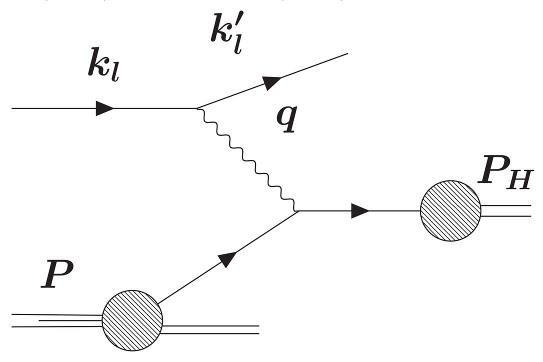


Introduction

Deep-inelastic scattering (DIS) provides valuable information on the internal structure of hadrons at high energies in terms of their partonic constituents namely quarks, anti-quarks and gluons, and also of the underlying strong interaction dynamics through QCD through structure functions (SF). In DIS we sum up all the final states except the scattered lepton. In semi-inclusive DIS (SIDIS) experiments, one observes the state of a specific hadron in the final state, in addition to that of scattered lepton. Such an observable will be sensitive to dynamics that governs the fragmentation of parton into a hadron.

Differential Hardonic Cross section

Differential Hadronic cross section for $e^-(k_l) + H(P) \to e^-(k_l') + H'(P_H) + X'$ is written as,



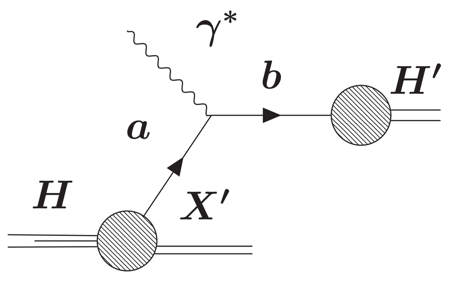
$$\frac{d^3\sigma_{e-H}}{dxdydz} = \frac{2\pi y\alpha_e^2}{Q^4}\mathcal{L}_{\mu\nu}(k_l, k'_l, q)\mathcal{W}^{\mu\nu}(q, P, P_H)$$

$\mathcal{L}_{\mu\nu}$ is the leptonic tensor and $\mathcal{W}^{\mu\nu}$ is hadronic tensor. Using the property that $W^{\mu\nu}$ is 2^{nd} rank tensor, current conservation and symmetries, it can be parameterized in terms of SFs:

$$W^{\mu\nu} = F_1\Big[-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\Big] + F_2\Big[\frac{1}{P.q}(P^\mu - \frac{P.q}{q^2}q^\mu)(P^\nu - \frac{P.q}{q^2}q^\nu)\Big]$$
$$- F_3\Big[\frac{i}{P.q}\varepsilon^{\mu\nu\sigma\lambda}q_\sigma P_\lambda\Big] - g_1\Big[\frac{i}{P.q}\varepsilon^{\mu\nu\sigma\lambda}q_\sigma S_\lambda\Big] - g_2\Big[\frac{i}{P.q}\varepsilon^{\mu\nu\sigma\lambda}q_\sigma(S_\lambda - \frac{S\cdot q}{P\cdot q}P_\lambda)\Big]$$

These dimensionless SFs are Lorentz invariant which are not calculable in perturbation theory.

Parton Model



We'll use Parton Model to write SFs as:

$$F_I = x^{I-1}\sum_{a,b}\int_x^1 \frac{dx_1}{x_1} f_a(x_1, \mu_F^2)\int_z^1 \frac{dz_1}{z_1} D_b(z_1, \mu_F^2)\mathcal{F}_{I,ab}\Big(\frac{x}{x_1}, \frac{z}{z_1}, Q^2, \mu_F^2\Big).$$

Here, μ_F^2 is the factorization scale.

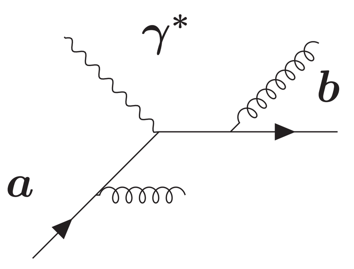
- $f_a dx_1$: The probability of finding a parton of type 'a' which carries a momentum fraction x_1 of the parent hadron H.
- $D_b dz_1$: The probability that a parton of type 'b' will fragment into hadron H' which carries a momentum fraction z_1 of the parton.
- $\mathcal{F}_{I,ab}$ are the finite CFs that can be computed perturbatively, it is related to partonic cross section.

Motivation

- CFs were known only upto NLO accuracy[1].
- Adding more corrections will decrease scale uncertainty, making perturbation theory more reliable.
- Extracting fragmentation function D_b.

Partonic Cross section

Computation of CFs starts from the parton level cross section denoted by $\hat{\sigma}_{I,ab}$, where we defined,



$$\hat{\sigma}_{I,ab} = \frac{\mathcal{P}_I^{\mu\nu}}{4\pi}\int \text{dPS}_{X'+b}\,\overline{\Sigma}\big|M_{ab}\big|^2_{\mu\nu}\,\delta\Big(\frac{z}{z_1} - \frac{p_a\cdot p_b}{p_a\cdot q}\Big)$$

Here, $\mathcal{P}_I^{\mu\nu}$ are the projectors to project out corresponding CFs and $|M_{ab}|^2$ is the squared amplitude for the process $a(p_a) + \gamma^*(q) \to$ '$b'(p_b) + X'$. Beyond leading order PCS gets contribution from loop diagrams as well as real emission diagrams.

- In the high momentum region the loop integral gives divergences, which are removed by renormalization procedure.
- Presence of massless particles gives rise to Infrared divergences (soft and collinear).
- Infrared divergences cancel among virtual and real emission processes, except for the collinear divergences related to the a and b partons in the initial state and the final fragmentation state respectively.

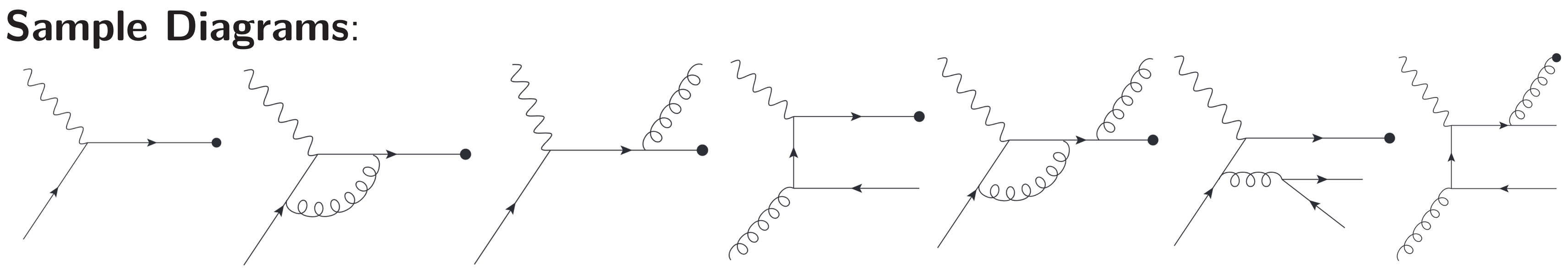
Mass factorisation: Left over divergences can be factored out into Altarelli-Parisi (AP) kernels at μ_F scale,

$$\frac{\hat{\sigma}_{I,ab}(\epsilon)}{x'^{I-1}} = \Gamma_{c\leftarrow a}(\mu_F^2, \epsilon)\otimes\mathcal{F}_{I,cd}(\mu_F^2, \epsilon)\tilde{\otimes}\tilde{\Gamma}_{b\leftarrow d}(\mu_F^2, \epsilon)$$

Flow Chart

- Generation of set of Feynman diagrams using QGRAF.
- Output of QGRAF to FORM form to get amplitude for individual diagrams.
- Used FORM extensively to do symbolic calculation like Lorentz contractions, Dirac algebra, handling Gell-Mann matrices.

Sample Diagrams:



Loop and Phase-Space Integrals

Using, the fact that the integral of a total derivative vanishes within DimReg. and the property of scaleless integral, one gets linear Integration-by-parts (IBP) identities to write loop integrals in terms of the basis of integrals called Master Integrals (MIs).

$$\int d^D l\frac{\partial}{\partial l^\mu}\Big[\frac{\{l^\mu, p^\mu\}}{D_1^{\nu_1}D_2^{\nu_2}...D_n^{\nu_n}}\Big] = 0$$

For Phase-space integrals, we used Reverse Unitarity method for converting the integrals into loop integrals and performing reduction to get set of MIs.

$$\delta(p^2 - m^2) \to \frac{i}{p^2 - m^2 + i\delta} - \underset{\text{can be almost forgot}}{\text{c.c.}}$$

We used LiteRed to generate IBP identities and got total '21' MIs in phase space calculation. To solve them we used Differential equation method.

Solving MIs

For phase-space integrals, we can set up differential system by taking derivative w.r.to external variables (x', z') and also one can make a basis transformation, $\vec{f} \to \vec{J} = \hat{T}^{-1}\vec{f}$ such that the differential eq. is in ϵ-form,

$$\begin{Bmatrix}\frac{\partial\vec{f}}{\partial x'} = \hat{A}_1(x', z', \epsilon)\vec{f} \\ \frac{\partial\vec{f}}{\partial z'} = \hat{A}_2(x', z', \epsilon)\vec{f}\end{Bmatrix} \to \begin{Bmatrix}\frac{\partial\vec{J}}{\partial x'} = \epsilon\,\hat{\mathcal{A}}_1(x', z')\vec{J} \\ \frac{\partial\vec{J}}{\partial z'} = \epsilon\,\hat{\mathcal{A}}_2(x', z')\vec{J}\end{Bmatrix}$$

where, $\hat{\mathcal{A}}_1 = \hat{T}^{-1}(\hat{A}_1\hat{T} - \frac{d\hat{T}}{dx'})$, Similar of $\hat{\mathcal{A}}_2$. **Integrability condition**:

$$\frac{\partial\hat{A}_1}{\partial z'} - \frac{\partial\hat{A}_2}{\partial x'} + [\hat{A}_1, \hat{A}_2] = 0$$

The solution of the system is as follows:

$$\vec{J}(x', z', \epsilon) = \mathbb{P}\exp\Big[\epsilon\int_{x_0', z_0'}^{x', z'}(\hat{\mathcal{A}}_1 dx_1' + \hat{\mathcal{A}}_2 dz_1')\Big]\vec{J}(x_0', z_0', \epsilon)$$

Boundary integrals are calculated at $x' \to 1$ and $z' \to 1$ using conventional method (choosing an appropriate frame).

Plots

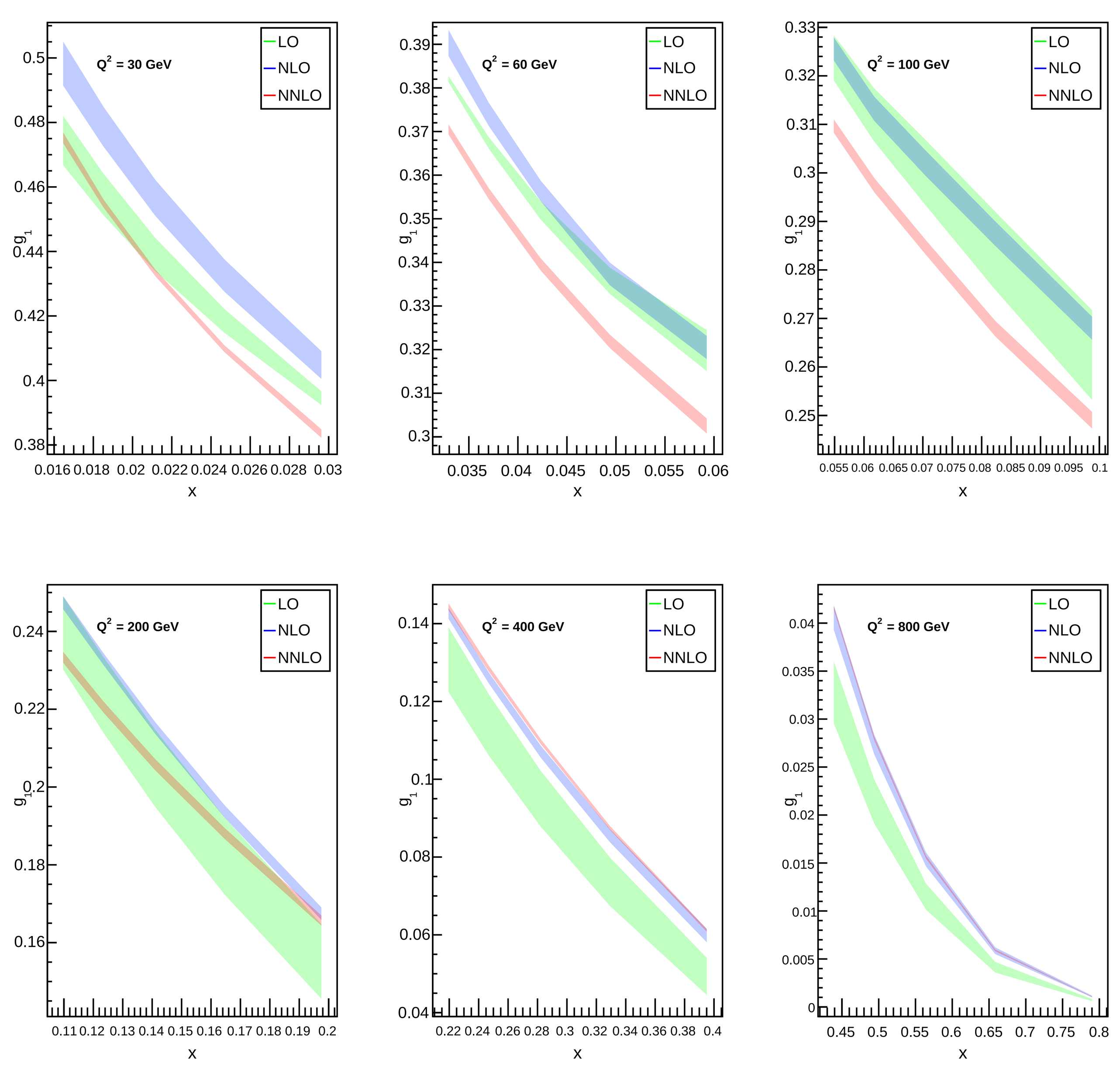


Figure 1: Dependence of $g_1^{\pi^+}$ on μ_R^2 and μ_F^2 in 7-point variation with $\sqrt{s} = 45$ GeV.

- Inclusion of NNLO corrections reduces scale dependence compared to previous orders.
- QCD Corrections for polarized case are negative.

1) Guido Altarelli et al. "Processes Involving Fragmentation Functions Beyond the Leading Order in QCD". In: Nucl. Phys. B 160 (1979), pp. 301–329.
2) Saurav Goyal et al. "Next-to-Next-to-Leading Order QCD Corrections to Semi-Inclusive Deep-Inelastic Scattering". In: Phys. Rev. Lett. 132.25 (2024), p. 251902. arXiv: 2312.17711.
3) Saurav Goyal et al. "NNLO QCD corrections to polarized semi-inclusive DIS". arXiv: 2404.09959.

†sauravg@imsc.res.in