Application of thermal field theory to dark matter annihilation cross section

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1. Introduction

• **Motivation** : Relic abundance of dark matter in our universe is increasing accurately measured ($\Omega_c h^2 = 0.1200 \pm 0.0012$) by the successive generation of experiments. The Boltzmann equation determines the yields using the dark matter annihilation cross section as one of the input; the accurate computation of the latter including thermal contribution thus assumes importance.
• **Background** : We are interested in obtaining thermal correction to relic abundance of dark matter(DM). We present effect of thermal fluctuation on dark matter annihilation cross section at NLO, utilizing advanced techniques of thermal field theory(TFT), which is used as input in Boltzmann equation. We use generalized Grammer and Yennie (GY) technique, in order to deal with IR divergences encountered in annihilation cross section calculations .

2. Model and Feynman Rules

• **Model** : Extension of SM by an $SU(2)\times U(1)$ singlet Majorana fermion χ and scalar doublet $\phi = (\phi^+, \phi^0)^T$. f is SM fermion doublet [2].

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{f}(i\not{D} - m_f)f + \frac{1}{2}\bar{\chi}(i\not{\partial} - m_\chi)\chi$$
$$+ (D_\mu\phi)^\dagger(D_\mu\phi) - m_\phi^2\phi^\dagger\phi + (\lambda\bar{\chi}P_L f^- \phi^+ + h.c.)$$

• Feynman Rules for photon propagator in TFT

$$iD^{t_a,t_b}_{\mu\nu,k} = -g_{\mu\nu}\left\{\begin{bmatrix} i\Delta_k & 0 \\ 0 & i\Delta_k^* \end{bmatrix} + 2\pi\delta(k^2)N_B(|k^0|)\begin{bmatrix} 1 & e^{\frac{k^0}{2T}} \\ e^{\frac{k^0}{2T}} & 1 \end{bmatrix}\right\}$$

where $i\Delta_k = i/(k^2 + i\epsilon)$ and $N_B|k^0| = 1/(e^{|k^0|/T} - 1)$. Here $t_a, t_b (= 1,2)$ refers to fields thermal type.
• Feynman Rules for fermion propagator in TFT

$$i\mathcal{S}^{t_a,t_b}_{\mathrm{f(p,m)}} = \begin{bmatrix} S & 0 \\ 0 & S^* \end{bmatrix} - 2\pi S'\delta(p^2 - m^2)N_{\mathrm{F}}(|p^0|)\begin{bmatrix} 1 & \epsilon_{p_0} e^{\frac{|p^0|}{(2T)}} \\ -\epsilon_{p_0} e^{\frac{|p^0|}{(2T)}} & 1 \end{bmatrix},$$

where $S = i/(\not{p} - m + i\epsilon)$, $S' = (\not{p} + m)$ & $N_F(p^0) = 1/(e^{|p^0|/T} + 1)$

3. Techniques and tools

• The time path for real time formulation (RTF)

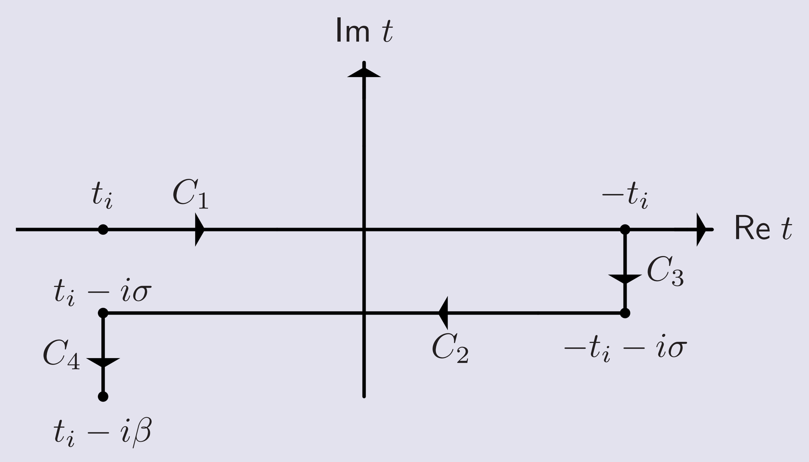

Fig1 : '$Im\ t = \beta'$, (the inverse temperature).

• **Grammer and Yennie(GY) technique**
• Grammer and Yennie(GY) technique for Rearrangement of $-ig_{\mu\nu}$ [Virtual Photon] [3–5]

$$-ig_{\mu\nu} \to -i\{g_{\mu\nu} - b_{k,p_i,p_f}k_\mu k_\nu + b_{k,p_i,p_f}k_\mu k_\nu\}$$
$$G_{\mu\nu} = g_{\mu\nu} - b_{k,p_i,p_f}k_\mu k_\nu \ \&\ K_{\mu\nu} = b_{k,p_i,p_f}k_\mu k_\nu$$

• Rearrangement of Photon polarization sum [Real Photon]

$$\sum_{\text{pol}}\epsilon^{*,\mu}(k)\,\epsilon^\nu(k) \to -g^{\mu\nu} \to -[\tilde{G}^{\mu\nu}_k + \tilde{K}^{\mu\nu}_k]$$

• Structure of $b[p_i, p_f.k]$

$$b_{p_i,p_f,k} = \frac{1}{2}\left[\frac{(2p_f - k).(2p_i - k)}{((p_f - k)^2 - m^2).((p_i - k)^2 - m^2)} + (k \leftrightarrow -k)\right]$$

• Useful formula for obtaining thermal correction

$$\int_0^\infty \omega \mathrm{d}\omega\, n_B(\omega) = \frac{\pi^2 T^2}{6}\ .\ and\ \int_0^\infty \omega \mathrm{d}\omega\, n_F(\omega) = \frac{\pi^2 T^2}{12}\ .$$

4. Details and Approximations

• In GY technique, IR divergence cancels between the *K-photon* virtual correction and real $\tilde{K}$*-photon* contribution order by order [3–5].

• We calculate "*G-photon*" contribution to NLO-virtual correction in order to obtain thermal correction to DM annihilation cross section [1].

• Calculation was first performed in Ref. [2] using alternate approach. Our approach is manifestly IR finite.

• In our calculations, we take scalar mass to be heavy compared to dark matter mass, $m_\phi \gg m_\chi$. We calculate DM annihilation cross section in following approximations on non-thermal scalar propagator [1]

• Non-dynamical scalar (Heavy scalar) approximation

$$i\Delta_{l+k} := \frac{i}{(l+k)^2 - m_\phi^2}) \to \frac{i}{(-m_\phi^2)}$$

• Dynamical scalar approximation

$$i\Delta_{l+k} := \frac{i}{((l+k)^2 - m_\phi^2)} \to \frac{i}{(l^2 - m_\phi^2)}\left[1 - \frac{(2l\cdot k + k^2)}{(l^2 - m_\phi^2)}\right].$$

5. Feynman diagrams

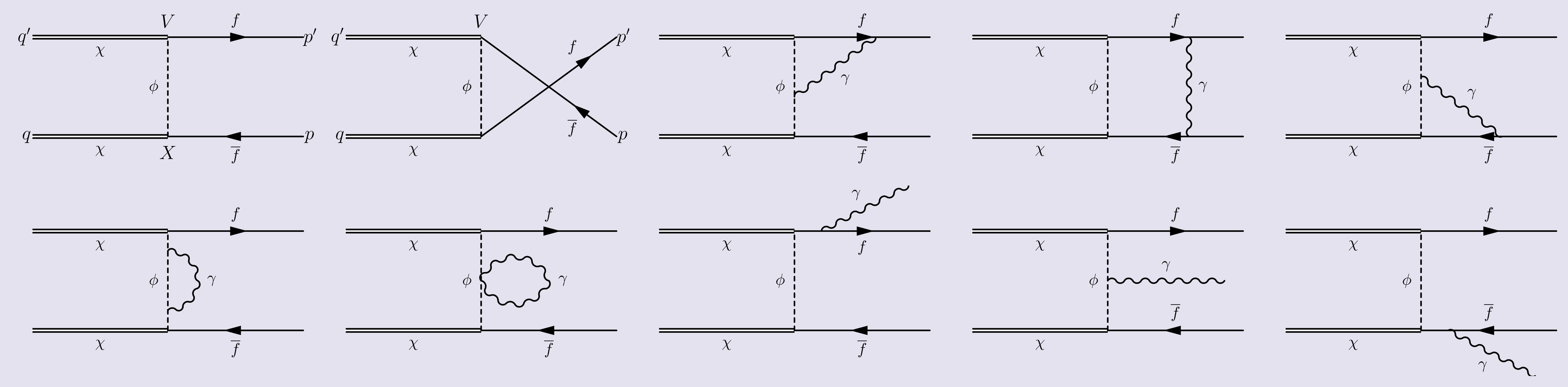

Fig.2 : The LO (t and u) and NLO t-channel virtual and real photon processes at NLO.

6. DM annihilation cross section for the process $\chi\chi \to f\bar{f}$ and $\chi\chi \to f\bar{f}\gamma$

• **Momenta in CM frame**: $q'^\mu = (H, 0, 0, P)$; $p'^\mu = (H, P'\sin\theta, 0, P'\cos\theta)$; $q^\mu = (H, 0, 0, -P)$; $p^\mu = (H, -P'\sin\theta, 0, -P'\cos\theta)$.
• **Tree level DM annihilation cross section :** σ_{LO}

$$\sigma_{LO}^{\text{heavy scalar}} = \frac{1}{12\pi s}\frac{P'}{P}\frac{\lambda^4}{m_\phi^4}\left[8H^2(H^2 - m_\chi^2) + m_f^2(5m_\chi^2 - 2H^2)\right] ;\ \sigma_{LO}(s) \overset{v\ small}{\longrightarrow} \frac{\lambda^4}{4\pi s}\frac{P'}{P}\left[\frac{m_\chi^2 m_f^2}{(m_\chi^2 + m_\phi^2 - m_f^2)^2} + \mathcal{O}(v^2)\right].$$

• **Cancellation of Soft IR divergences :** Soft IR divergences cancel between the real and virtual corrections at NLO.
• **Cancellation of collinear divergences :** Collinear divergences occurring from the virtual correction cancels with the collinear divergences due to companion real corrections at NLO.
• **DM annihilation cross section for virtual NLO corrections, in heavy scalar limit**
We obtain quadratic dependence, $\mathcal{O}(T^2)$, for the virtual contribution to the DM annihilation cross section. For example, Diagram 3 of Fig. 2 , with photon propagator thermal, gives a contribution:

$$\sigma^t_{NLO}(1,\gamma) = \frac{1}{32s(2\pi)^4}\frac{|\vec{p}'|}{|\vec{q}'|}\int \mathrm{d}\omega\,\{\omega\, n_B(\omega)\}\, I^t_{1,\gamma}\ . \implies \sigma^t_{NLO}(1,\gamma) = \frac{1}{32s(2\pi)^4}\frac{|\vec{p}'|}{|\vec{q}'|}\frac{\pi^2T^2}{6}\times I^t_{1,\gamma}\ ,$$
$$I^t_{1,\gamma} = \frac{64\pi e^2\lambda^4}{3{m_\phi}^6}\left[4\left(3H^4 + H^2P^2 - P^2m_f^2\right)\right].$$

• **DM annihilation cross section for real NLO corrections, due to emission and absorption of photons** [6]

$$\sigma^{t,12}_{NLO,real} = \frac{1}{32(2\pi)^4\sqrt{s}P}\int \mathrm{d}\omega\,\{\omega\, n_B(\omega)\}\frac{I^t_{\gamma,real}}{\omega} \implies \frac{1}{32(2\pi)^4\sqrt{s}P}\times\frac{\pi^2T^2}{6}\times I^t_{\gamma,real}$$
$$I^t_{\gamma,real} = \frac{32\pi e^2\lambda^4\omega(2H^2 - m_f^2)(3H^4 - H^2P^2 - P^2m_f^2)}{3H^2m_\phi^6}\ .$$

7. Virtual contribution to σ_{NLO} with dynamical scalars

• s-wave contribution, $\langle\sigma v_{rel}\rangle \sim a + bv_{rel}^2$, with $D = (m_\chi^2 - m_f^2 + m_\phi^2)$.

Diagram	γ/f	Int^a_{NLO} (T^2 contribution)	Int^a_{NLO} (T^4 contribution)
1	γ	$-8m_\chi^2m_f^2(m_f^2 - m_\phi^2)/D^4$	0
	f	$4m_\chi^2m_f^2(5m_\chi^2 - 5m_f^2 + m_\phi^2)/D^4$	0
	$\text{Total}_{\gamma+f}$	$2m_\chi^2m_f^2(5m_\chi^2 - 9m_f^2 + 5m_\phi^2)/D^4$	0
2	γ	$-8m_\chi^2m_f^2/D^3$	0
	f	$-6m_f^2(2m_\chi^2 - m_f^2)/D^3$	$-\frac{21\pi^2T^2}{10m_\chi^2D^3}m_f^2(2m_\chi^2 - m_f^2)$
	$\text{Total}_{\gamma+f}$	$-m_f^2(14m_\chi^2 - 3m_f^2)/D^3$	$-\frac{21\pi^2T^2}{10m_\chi^2D^3}m_f^2(2m_\chi^2 - m_f^2)$
3	γ	$-8m_\chi^2m_f^2(m_f^2 - m_\phi^2)/D^4$	0
	f	$4m_\chi^2m_f^2(3m_\chi^2 - 2m_f^2 + m_\phi^2)/D^4$	0
	$\text{Total}_{\gamma+f}$	$2m_\chi^2m_f^2(3m_\chi^2 - 6m_f^2 + 5m_\phi^2)/D^4$	0
4	γ	$32m_\chi^4m_f^2/D^4$	$-\frac{56\pi^2T^2}{15D^5}m_\chi^2m_f^2(m_\chi^2 - m_f^2)$
5	γ	$-16m_\chi^2m_f^2/D^3$	0
All	$\text{Total}_{\gamma+f}$	$\frac{1}{D^3}m_f^2(2m_\chi^2 + 3m_f^2)+$	$-\frac{21\pi^2T^2}{10m_\chi^2D^3}m_f^2(2m_\chi^2 - m_f^2)$
		$\frac{2}{D^4}m_f^2m_\chi^2(10m_\phi^2 + 24m_\chi^2 - 15m_f^2)$	$-\frac{56\pi^2T^2}{15D^5}m_\chi^2m_f^2(m_\chi^2 - m_f^2)$

8. Conclusion

• We presented thermal correction to annihilation cross section of DM due to pure thermal fluctuations at NLO for virtual corrections utilizing thermal field theory.
• We utilized generalized Grammer and Yennie technique in order to deal with IR divergence encountered and get thermal correction to IR finite G-photon contribution.
• Our calculations are performed in two approximations on scalar propagator, first considering scalar to be heavy and taking scalar mass term in the propagator and further in Dynamical scalar approximation.
• We took thermal part of the propagator for photon, fermion or anti-fermion for obtaining thermal correction due to thermal fluctuations. Only one of the propagator among them can be taken as thermal at a time.
• We find virtual NLO thermal correction terms have a T^2 dependence, but are suppressed by additional powers of (heavy scalar) propagator, $D^n \sim m_\phi^{2n}$ and square of the fermion mass, m_f^2, both in LO and NLO.
• Thermal correction due to real photon emission and absorption, i.e. $\tilde{G}-photon$ contribution is ongoing part of the work, which is necessary for relic abundance calculations of DM utilizing Boltzmann equation. A sample calculation indicates no helicity suppression.
• While the thermal corrections are small at freeze-out ($m_\chi/T \sim 20$), it is possible to have significant corrections from the real photon diagrams.
• Freeze-in scenarios may be even more significantly affected by thermal corrections.

9. References

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