Exploring the Impact of Extra Dimensions on the Equation of State of Baryonic Matter and the Structure of Neutron Stars

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Exploring the Impact of Extra Dimensional spacetime on Neutron Star Structure and Equation of State

arXiv: 2403.07174

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Extra Spatial Dimensions?

- There are many theoretical considerations that support the existence of extra spatial dimensions (Lugones & Arbañil 2017, Chakravarti et al. 2018, Arbañil et al. 2019)

- Initially, these extra dimensions naturally emerged in string theory, which mandates ten or more dimensions.

- Later, the presence of extra dimensions was revisited to address the 'gauge hierarchy problem' (Arkani et al 1998, Antoniadis1988,Perez-Lorenzana2005). This problem arises due to the significant and seemingly unrelated hierarchy between the electro-weak symmetry breaking scale (around 10^3GeV) and the Planck scale (about 10^{18}GeV).

Extra Spatial Dimensions?

- There are a few observational arguments that favor the presence of extra spatial dimensions (GW170817) [Pardo et al. 2018, Abbott et al 2019]

- GW luminosity distance of 40^{+8}_{-14} Mpc (Abbott et al. 2017)

  Vs

  EM luminosity distance of $40.7^{+2.4}_{-2.4}$ Mpc (Cantiello et al. 2018)

➡ *Possibility of unequal luminosity distances due to extra spatial dimension !!*

cannot be dismissed

(Deffayet and Menou 2007, Liu et al. 2023)

ries of gravity with a characteristic length scale R_c of the order of the Hubble radius $R_H \sim 4\,\mathrm{Gpc}$, such as the well known Dvali-Gabadadze-Porrati (DGP) models of dark energy [88, 89], small transition steepnesses ($n \sim \mathcal{O}(0.1)$) are excluded by the data. Our analysis cannot conclusively rule out DGP models that provide a sufficiently steep transition ($n > 1$) between GR and the onset of gravitational leakage. Future LIGO-Virgo observations of binary neutron star mergers, especially at higher redshifts, have the potential to place stronger constraints on higher-dimensional gravity.

B. P. Abbott et al. Phys. Rev. Lett. 123, 011102 (2019)

M-R curves for Neutron Stars in 4D GR

- The Tolman-Oppenheimer-Volkoff (TOV) equations for a spherically symmetric and static neutron star (NS)

$$\frac{dm}{dr} = 4\pi\rho r^2 \qquad \text{- (A)}$$

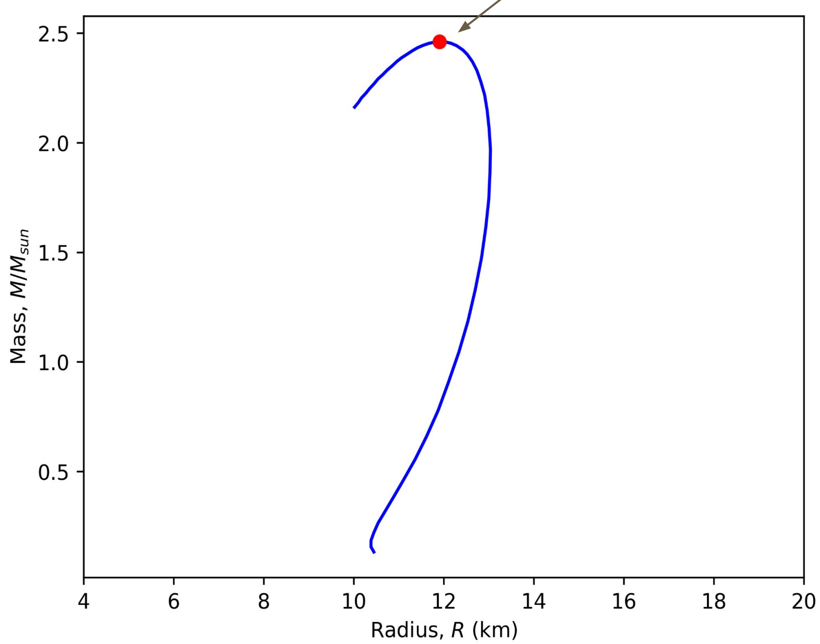
<= Mass balance Equation

$$\frac{dp}{dr} = -\frac{(\rho + p)(m + 4\pi r^3 p)}{r(r - 2m)}$$

<= Force Balance Equation

- (B)

Maximum Mass



- Equation of State: To explain the strong nuclear force within NSs we have considered realistic DD2 EoS model, which describes the nuclear matter, consisting of baryonic components like n, p, alongside leptons such as e and μ. Here we consider baryons interact through the exchange of σ, ω and ρ mesons.

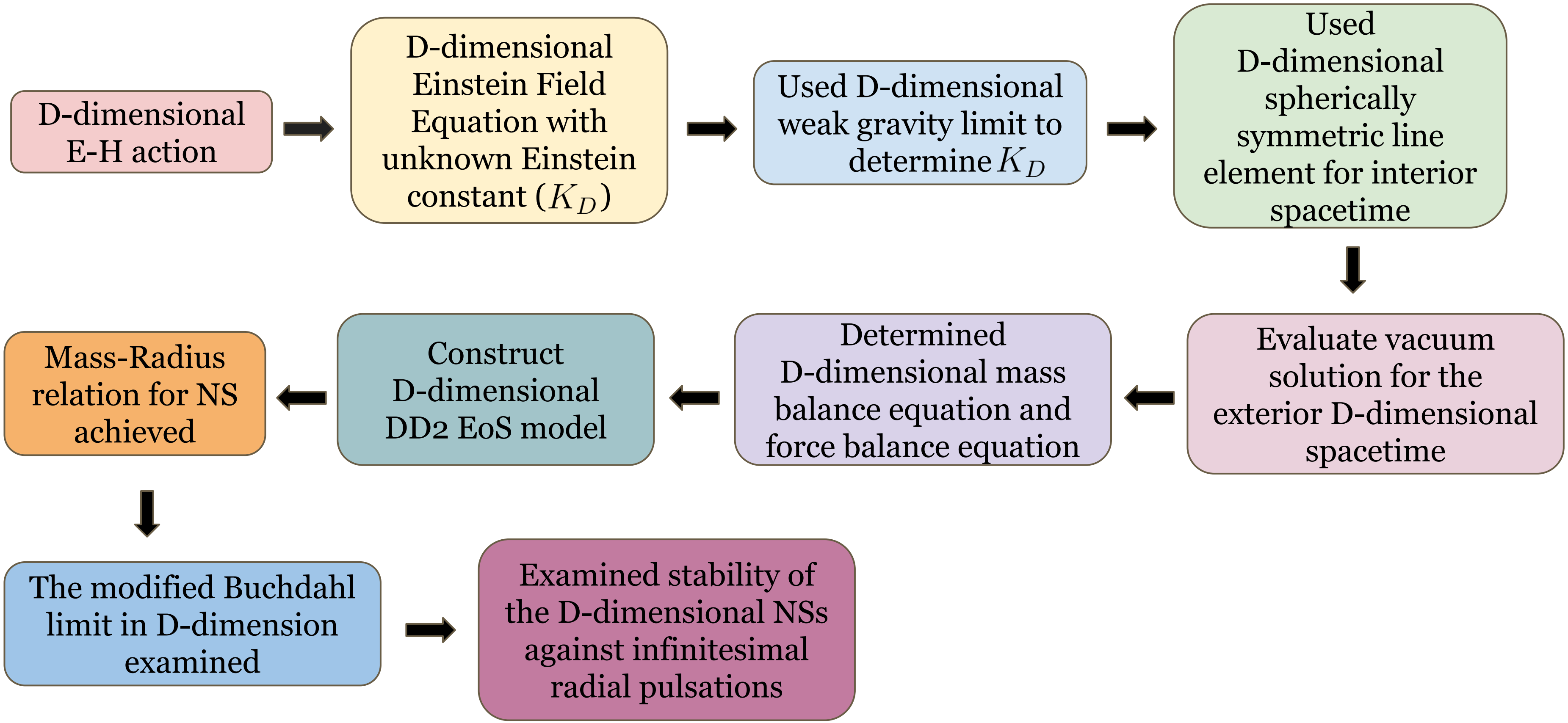
What did we do in https://arxiv.org/abs/2403.07174

- ❏ Investigated the influence of higher-dimensional spacetime on both the macroscopic (e.g., mass, radius) and microscopic (e.g., density, pressure) properties of neutron stars (NSs) within a higher-dimensional framework.

- ❏ Extended a realistic nuclear equation of state (EoS) to higher dimensions to accurately describe the strong interactions within neutron matter.

- ❏ Finally, verified the stability of NSs in higher-dimensional spacetime using the Buchdahl condition and stability analysis against infinitesimal radial pulsations.

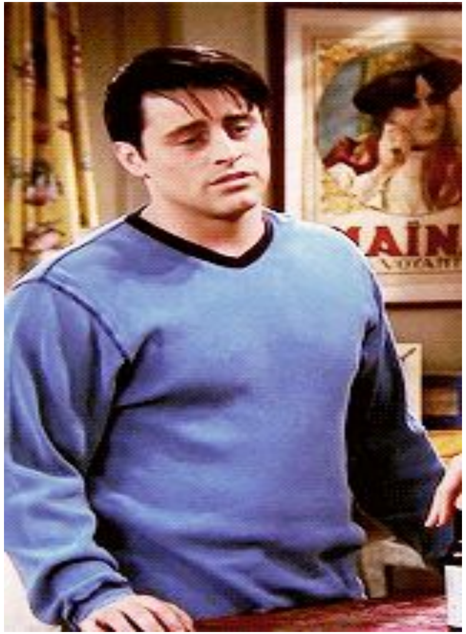
Plan of present work



Challenges

What is the value of Gravitational constant in D-dimension?

What is the value of planck length in D-dimension?



$$\xi_D = (l_c)^{D-4} = G_D/G_4 \Rightarrow l_c$$ is the length of extra compact dimension

(B Zwiebach (2009), A First Course in String Theory)

D-dimensional Einstein Field Equation

- Generalized Einstein-Hilbert (EH) Action in D-dimensions:

$$S = \frac{1}{K_D}\int d^D x \mathcal{R}\sqrt{-g} + \int d^D x \mathcal{L}_m \sqrt{-g}$$

- (1)

Einstein's Constant: $K_D = 2\,\frac{D-2}{D-3}\frac{G_D}{c^4} S_{D-2}$ - (2)

Area of Unit Sphere: $S_{D-2} = \dfrac{2\,\pi^{\frac{D-1}{2}}}{\Gamma\left(\frac{D-1}{2}\right)}$ - (3)

- Stress-Energy Tensor for Isotropic Fluid: $T_{\mu\nu} = \left(\rho_D + \dfrac{p_D}{c^2}\right) u_\mu u_\nu - p_D g_{\mu\nu}$ - (4)

- Generalized Einstein Field Equation: $\mathcal{R}_{\mu\nu} - \dfrac{1}{2}\mathcal{R} g_{\mu\nu} = \dfrac{D-2}{D-3} S_{D-2}\dfrac{G_D}{c^4} T_{\mu\nu}$ - (5)

Basic Stellar Equations in D-dimension

- The exterior Schwarzschild spacetime metric in D-dimensional spacetime

$$ds^2 = \left(1 - \frac{2MG_D}{c^2(D-3)r^{D-3}}\right) dt^2 - \left(1 - \frac{2MG_D}{c^2(D-3)r^{D-3}}\right)^{-1} dr^2 - r^2 \sum_{i=1}^{D-2} \left(\prod_{j=1}^{i-1} \sin^2\theta_j d\theta_i^2\right) \quad \text{- (6)}$$

- Stellar Structure Equations:

- Mass Balance Equation:

$$\frac{dm}{dr} = S_{D-2}\rho_D r^{D-2} \quad \text{- (7)}$$

- Force Balance Equation:

$$\frac{dp_D}{dr} = -\left(\rho_D c^2 + p_D\right) \frac{G_D\left[S_{D-2}p_D r^{D-1} + c^2 m(D-3)\right]}{c^2 r\left[c^2(D-3)r^{D-3} - 2mG_D\right]} \quad \text{- (8)}$$

- Simplifying Assumptions:
  - Mass, density, and pressure in D-dimensions:

$$\tilde{m} = mG_D/(D-3), \tilde{\rho} = \rho_D G_D, \tilde{p} = p_D G_D \quad \text{- (9)}$$

  - Set $G_4 = 1$ and $c = 1$ for simplicity.

DD2 EoS model in D-Dimension

- Relativistic Mean-Field (RMF) Approximation in D-Dimensions is employed to solve Meson Field Equations:

$$m_\sigma^2 \sigma = \sum_{B=n,p} g_{\sigma B} \rho_{DB}^s, \quad \text{- (10)}$$

$$m_\omega^2 \omega_0 = \sum_{B=n,p} g_{\omega B} \rho_{DB}, \quad \text{- (11)}$$

$$m_\rho^2 \rho_{03} = \sum_{B=n,p} g_{\rho B}^2 \tau_{3B} \rho_{DB}. \quad \text{-(12)}$$

- Baryon Number Density (ρ_{DB}) and Scalar Number Density ($\rho^{\mathbf{s}}_{\mathbf{DB}}$):

$$\rho_{DB}^s = \frac{2J_B+1}{(2\pi)^{D-1}} S_{D-2} \int_0^{k_B} \frac{k^{D-2} m_B^\star}{\sqrt{k^2 + {m_B^\star}^2}} dk, \quad \text{- (13)}$$

$$\rho_{DB} = \frac{2J_B+1}{(2\pi)^{D-1}} S_{D-2} \int_0^{k_B} k^{D-2} dk. \quad \text{- (14)}$$

- Effective Baryon Mass : $m_B^\star = m_B - g_{\sigma B} \sigma$ - (15)

DD2 EoS model in D-Dimension

- Dirac Equation for Baryon Fields: $\left[\left(i\gamma^{\mu}\partial^{\mu} - \gamma^{0}\Sigma_0^{\tau}\right) - m_B^{\star}\right]\psi_B = 0$ - (16)

  Here:

$$\Sigma_0^{\tau} = g_{\omega B}\omega_0 + g_{\rho B}\tau_{3B}\rho_{03} + \Sigma_0^{R}$$
- (17)

Where,

$$\Sigma_0^{R} = \sum_B \left[-g_{\sigma B}{}'\sigma\rho_{BD}^{s} + g_{\omega B}{}'\omega_0\rho_{DB} + g_{\rho B}{}'\tau_{3B}\rho_{03}\rho_{DB}\right]$$
(Rearrangement term) - (18)

- The energy density (ρ_D) for NS matter

$$\rho_D = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_\omega^2{\omega_0}^2 + \frac{1}{2}m_\rho^2{\rho_{03}}^2 + S_{D-2}\sum_{B=n,p}\frac{2J_B+1}{(2\pi)^{D-1}}\int_0^{k_B} k^{D-2}\sqrt{k^2 + {m_B^{\star}}^2}dk$$
$$+ S_{D-2}\sum_{l=e,\bar{\mu}}\frac{2J_l+1}{(2\pi)^{D-1}}\int_0^{k_l} k^{D-2}\sqrt{k^2 + m_l^2}dk,$$
- (19)

- the pressure (p_D)

$$p_D = -\frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_\omega^2{\omega_0}^2 + \frac{1}{2}m_\rho^2{\rho_{03}}^2 + \Sigma_0^R\sum_{B=n,p}\rho_{DB}$$
$$+ \frac{S_{D-2}}{D-1}\sum_{B=n,p}\frac{2J_B+1}{(2\pi)^{D-1}}\int_0^{k_B}\frac{2(D-2)}{\sqrt{k^2 + {m_B^{\star}}^2}}dk + \frac{S_{D-2}}{D-1}\sum_{l=e,\bar{\mu}}\frac{2J_l+1}{(2\pi)^{D-1}}\int_0^{k_l}\frac{2(D-2)}{\sqrt{k^2 + m_l^2}}dk$$
- (20)

DD2 EoS model in D-Dimension

The nucleon-meson density-dependent couplings (Typel et al 1999, 2005, 2010) :

$$g_{\alpha B}(n) = g_{\alpha B}(n_0) f_\alpha(x) \quad - (21)$$

where $x = n/n_o$

n = total baryon density

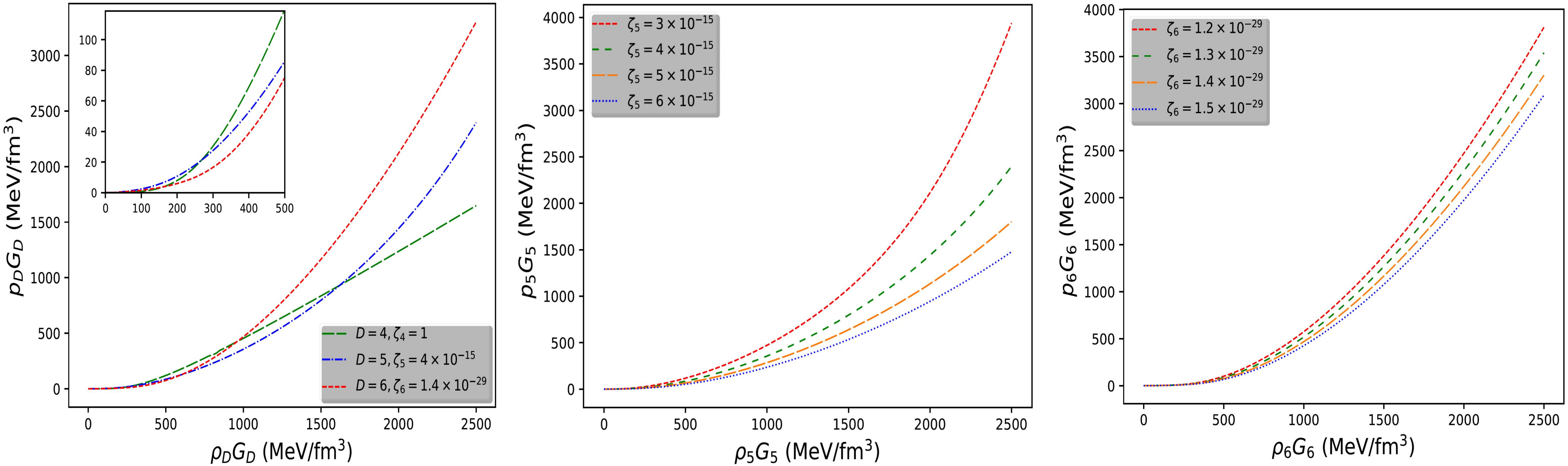
n_o = saturation density

For σ and ω meson: $$f_\alpha(x) = a_\alpha \frac{1 + b_\alpha(x + d_\alpha)^2}{1 + c_\alpha(x + d_\alpha)^2} \quad - (22)$$

For ρ meson: $$f_\alpha(x) = \exp[-a_\alpha(x - 1)] \quad - (23)$$

The values of the parameters $g_{\alpha B}(n_o)$, a_α, b_α, c_α, and d_α for $\alpha = \sigma$, ω and ρ are obtained through fitting the nuclear properties.

DD2 EoS model in D-Dimension



Mass-radius curve for Neutron Stars in D-dimensions

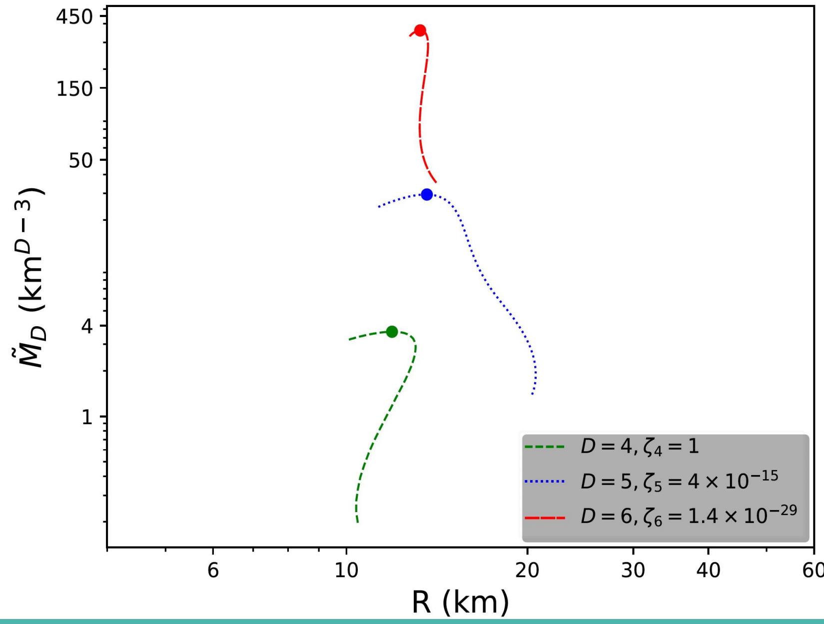
EH action => Einstein field Equation => Schwarzschild exterior metric=> mass balance equation => From EFE to TOV equation => modified EOS => MR curve ⟵ Need to explore all in D-dimensional spacetime

Variation of the total mass ($\log(MG_D/(D-3))$) in km$^{(D-3)}$ with the total radius R in km for the different D-dimensional cases, such as D=4, 5 and 6

- $G_4 \neq G_5 \neq G_6 \ldots \neq G_D$,
  we don't know the values for G_D

- We can employ dimensional analysis to introduce a scaling parameter ζ_D given by

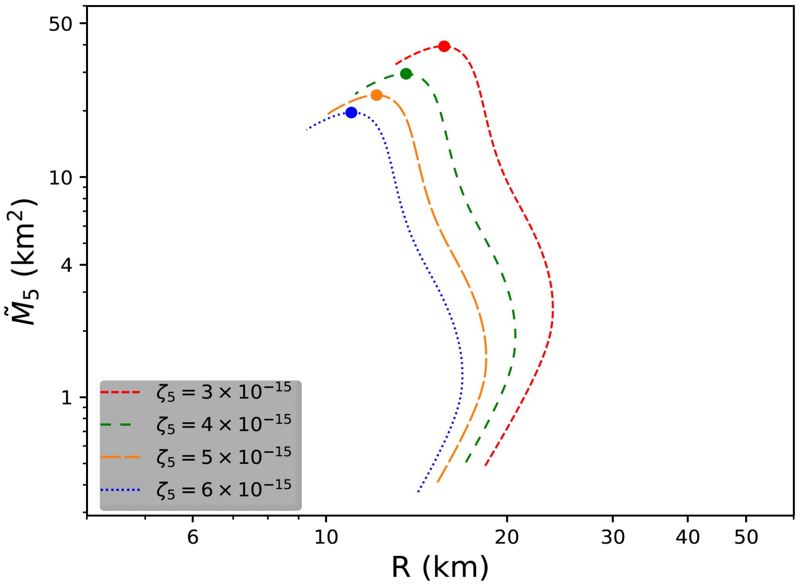
$$G_D/G_4 = \zeta_D\,\mathrm{km}^{D-4} \quad \text{- (C)}$$



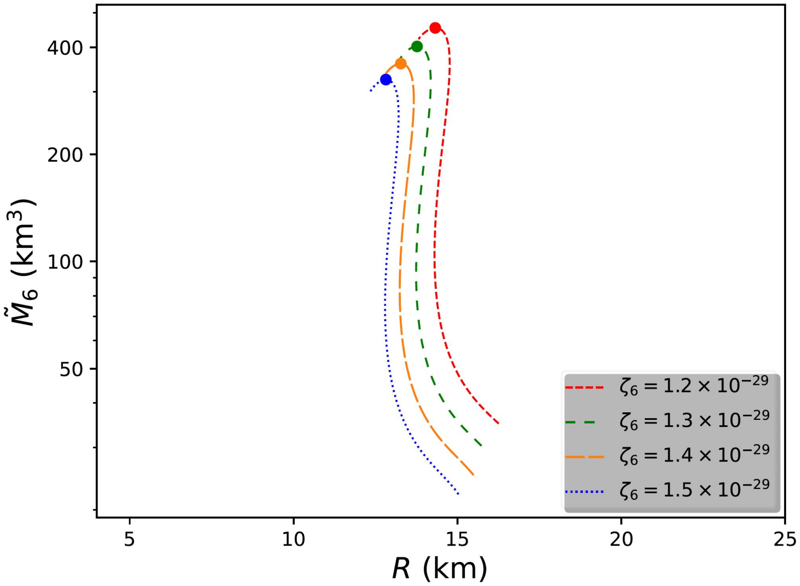
Mass-radius curve for neutron stars in D-dimension

Variation of the total mass ($\log(MG_D/(D-3))$) in km$^{(D-3)}$ with the total radius R in km for the D= 5 and 6 dimensional cases:

D = 5



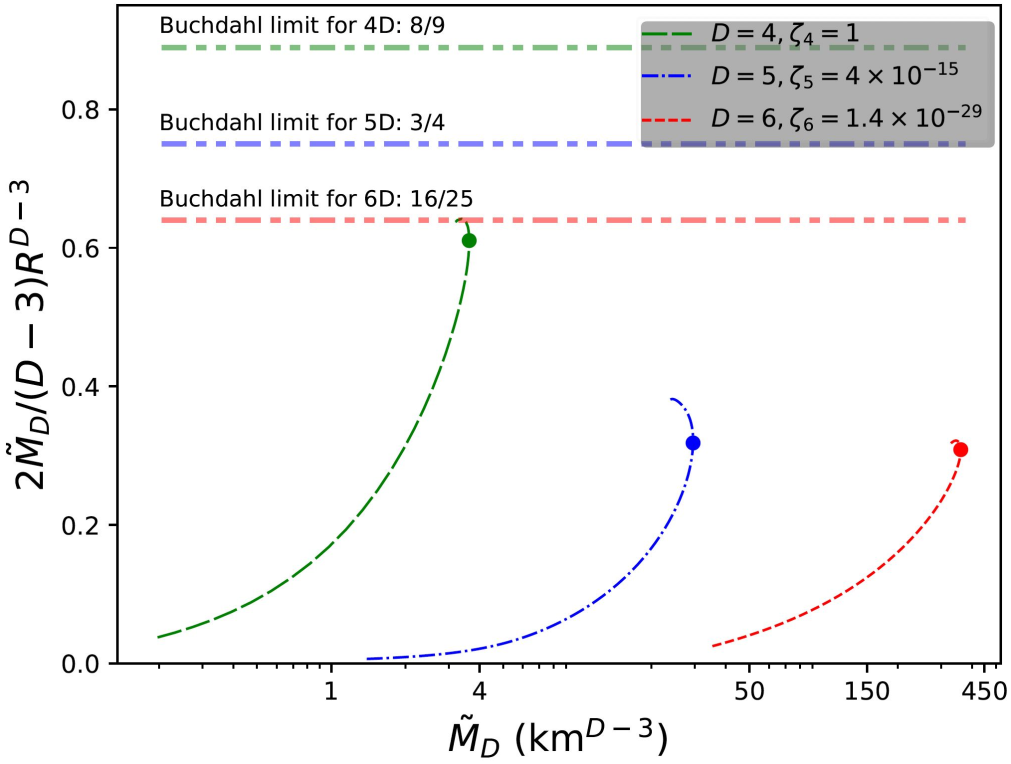
D = 6



Buchdahl Limit and Surface Redshift for NSs in D-dimension

Buchdahl limit for D-dimension spacetime: $$\frac{MG_D}{c^2(D-3)R^{D-3}} < \frac{2(D-2)}{(D-1)^2} \qquad \text{- (D)}$$

Buchdahl limit ensures that a neutron star is stable against gravitational collapse and provides an upper limit on how compact a neutron star can be without collapsing into a black hole

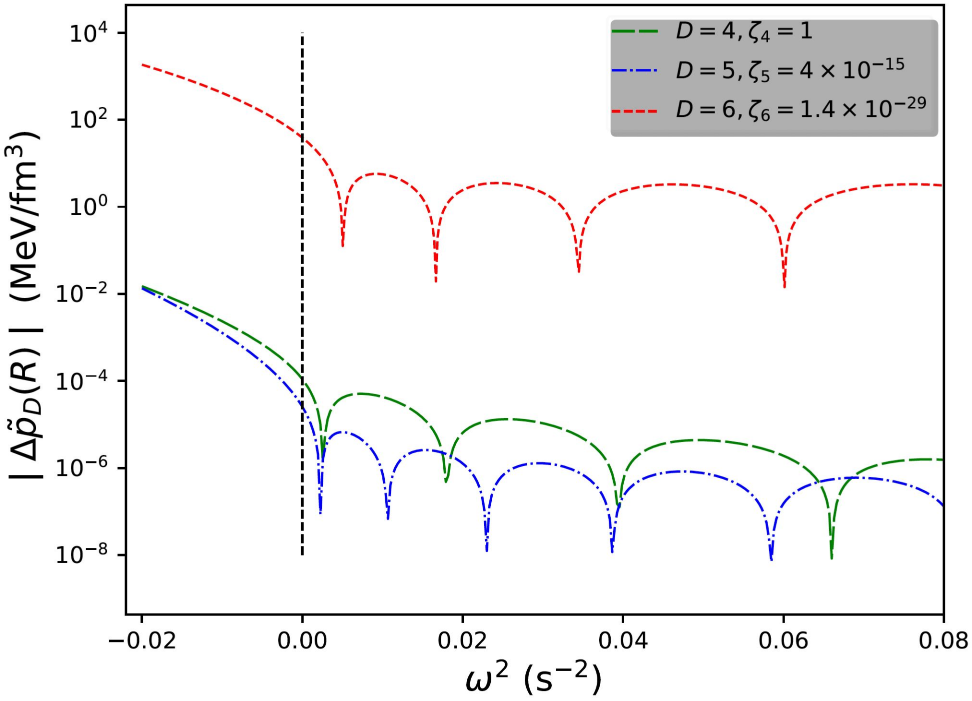


Testing stability against infinitesimal Radial Pulsation

Lagrangian perturbation of the radial pressure ($\Delta\tilde{P}(R) = \Delta P(R) G_D$) for a range of test values ω^2 within different D-dimensional spacetimes

ω = eigenfrequency

Lagrangian perturbation of the radial pressure $\Rightarrow$



$\omega^2 > 0$ ensures neutron star is stable against radial perturbations for D =4, 5 and 6.

Numerical values of some physical parameters

Value of D	Value of Maximum Mass $MG_D/(D-3)$ in km$^{(D-3)}$	Value of Corresponding Radius (km)	Central Density $\widetilde{\rho}_{D,c}$ (MeV fm^{-3})	Central Pressure $\widetilde{p}_c$ (MeV fm^{-3})	$\frac{2MG_D}{(D-3)R^{D-3}}$	Surface Redshift	Compactified Maximum Mass in 4-dimension ($M_\odot$)
4	3.63	11.91	1064.85	501.78	0.61	0.60	2.46
5	29.49	13.58	1459.85	754.40	0.32	0.21	2.50
6	360.25	13.27	2291.54	2782.94	0.31	0.20	3.31

Value of ζ_5	Value of Maximum Mass $MG_D/(D-3)$ in km$^{(D-3)}$	Value of Corresponding Radius (km)	Central Density $\widetilde{\rho}_{D,c}$ (MeVfm^{-3})	Central Pressure $\widetilde{p}_c$ (MeVfm^{-3})	$\frac{2MG_D}{(D-3)R^{D-3}}$	Surface Redshift	Compactified Maximum Mass in 4-dimension ($M_\odot$)
3×10^{-15}	39.31	15.72	1077.23	547.73	0.32	0.21	2.88
4×10^{-15}	29.49	13.58	1459.85	754.40	0.32	0.21	2.50
5×10^{-15}	23.59	12.13	1841.52	960.35	0.32	0.21	2.24
6×10^{-15}	19.66	11.01	2296.87	1245.24	0.32	0.22	2.05

A brief Summary

- ❑ Higher dimensions significantly affect the structural properties of NSs, including density and pressure profiles, as well as enclosed masses, due to the modified DD2 model EoS.

- ❑ The mass-radius analysis shows a progressive increase in total mass for a constant central density as the dimensionality increases, maintaining the stability criterion

  $$dM/d\rho_c > 0$$

- ❑ Higher-dimensional NSs exhibit stability against radial oscillations, adhering to the modified Buchdahl limit for higher dimensions.

- ❑ *We are exploring the potential applications how to match with any existing observational signatures*

Thank you !

Video credit: Interstellar movie