# SURFACE MAGNETIC FIELD OF NEUTRON STARS


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## Basic dynamics of a Radio pulsar

The magnetic fields of neutron stars play a pivotal role in all of the observed activities of these stars.

- The magnetospheric structure of a NS is believed to be made of closed and open field lines, where the closed field lines form a torus around the star and co- rotate with it.

- This region is charge saturated, with the charges mainly originating from the pair creation of electrons and positrons due to the strong surface magnetic field.

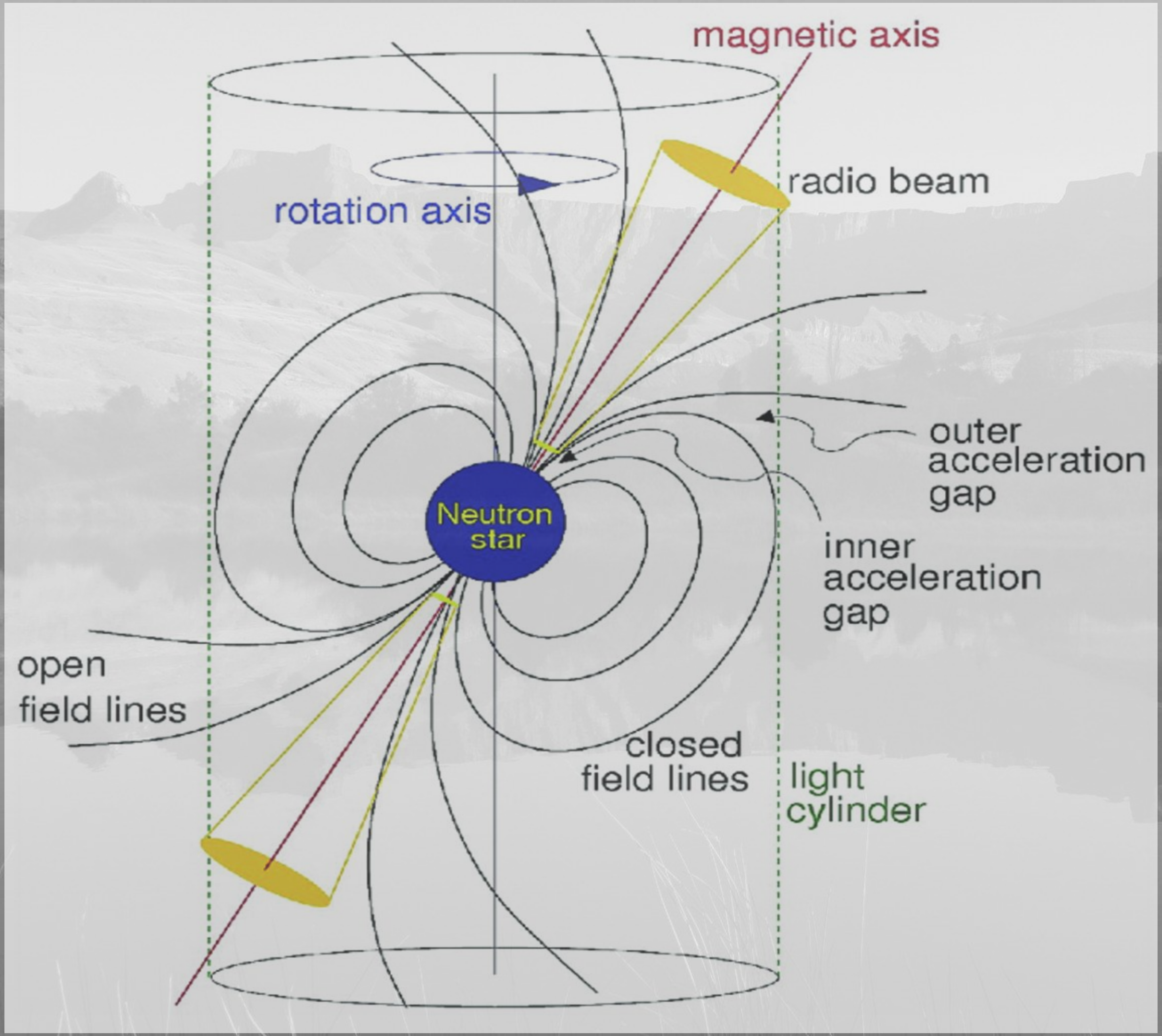
- The closed field lines 'open' up at the light cylinder, and the observed emission emerges from this conical zone.

- Though mostly the emission from the light cylinder is in the form of a wind of plasma, a small fraction of this plasma is converted to a coherent beam of radio waves arising due to the growth of plasma instabilities in the inner magnetosphere, from regions below 10% of the light cylinder radius

- This beam is ejected relentlessly, which we finally observe as the pulse profile, also suggests that the pair creation process generating the plasma has to be continuous.

- The magnetic field of the magnetosphere is the strongest at the NS surface, where the pair creation (and hence the generation of the plasma) is maximum.

- Since the closed field lines are charge-saturated, the surface magnetic field at the polar caps plays a seed role in the subsequent processes for the creation of highly polarized radio beams (and the winds)



magnetic axis
radio beam
rotation axis
outer
acceleration
gap
Neutron
star
inner
acceleration
gap
open
field lines
closed
field lines
light
cylinder

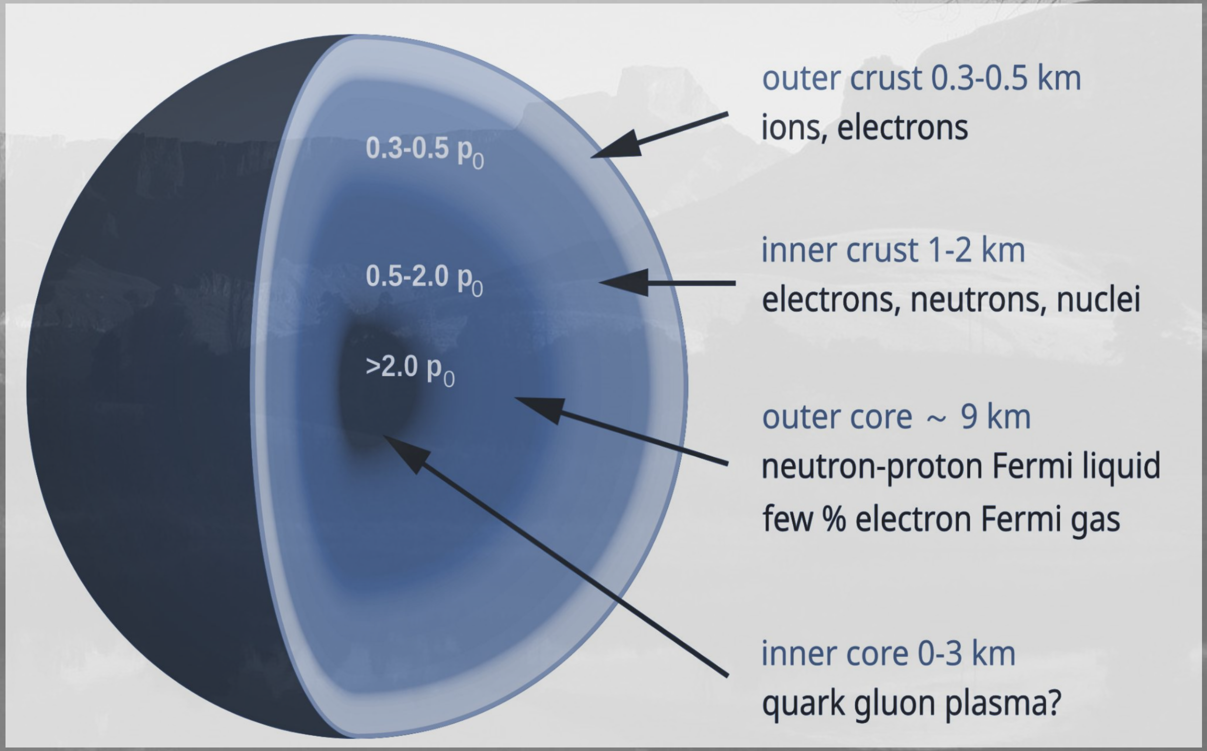
# Neutron star matter distribution (general understanding)

**Inner core** - ρ ~ 10^15 g/cm^3 - Nuclear matter? Quark matter?

**Outer core** - ρ ~ A few times 10^14 g/cm^3 - Nuclear matter - with about 90% Neutrons and 10% Protons

**Inner crust** - ρ ~ 10^14 g/cm^3 to 10^11 g/cm^3 (neutron drip)

**Outer crust** - ρ ~ 10^11 g/cm^3 to 10^4 g/cm^3




So one can see that the matter density inside the core varies by about 1 order of magnitude, whereas that in the crust is 10 orders of magnitude.

So the density variation in the outer crust is more drastic where it changes by 7 orders of magnitude in a span of a few hundred metres.

# Known problems of hosting a strong magnetic field inside a NS

There is no conclusive theory to explain the actual origin of the NS magnetic field.

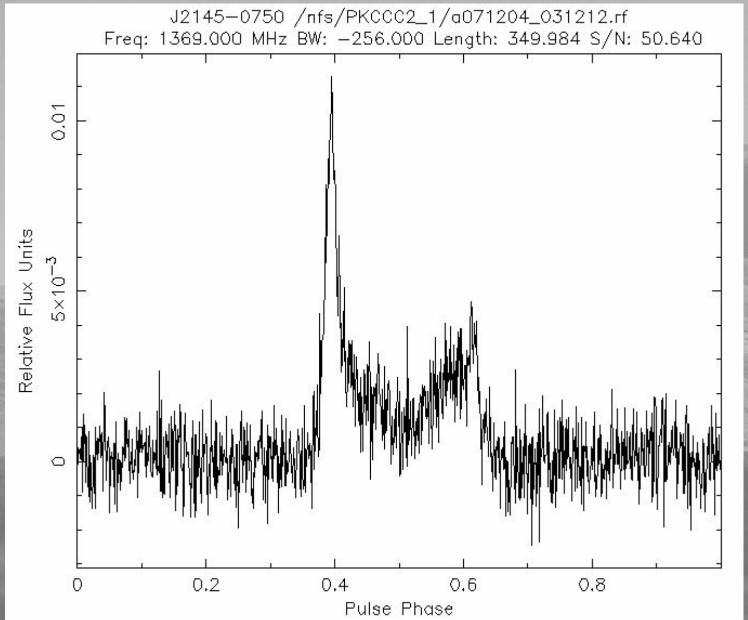
It is generally accepted that the strong magnetic field of a NS is due to the flux conservation of the original star from which it was born.

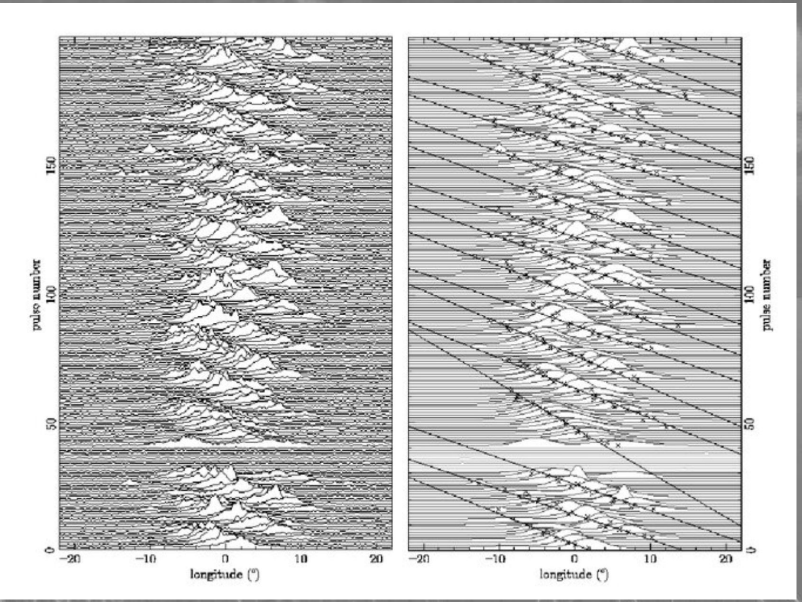
Said so, there isn't any a priori knowledge about the distribution of the magnetic field inside the NS.

The primary problem in hosting a super strong magnetic field in the NS core is due to the fact that the nuclear matter in the core is highly superconducting, resulting in repulsion of flux to a lower density regime.

Another challenge may come from general relativity (GR), where the vectorial nature of the dipolar magnetic field does not allow interior solutions for a spherically symmetric configuration for realistic equations of state.

# Radio pulsar observation and few known unresolved issues




The new generation telescopes and advanced analysis techniques have brought into limelight some observational features that need deeper understanding and more robust explanation about their emission mechanism.

Some of these are: (a) Mode changing (b) Nulling (c) Subpulse drifting and the issue of (d) microstructures

Within a single pulse profile we observe microstructures the source of which is still unknown. (In the integrated pulse profile they are smeared out as ‘noise’)

Sub pulses are observed with almost every pulsar and are believed to be related to the fundamental property of the pulsars emission. However we also often see a drift in the sub pulses in many of the pulsars.  These sub pulse drifts can also be related to the mode changing and nulling.

Here, in our initial approach, we tried to see the magnetic field decay and its variability on the surface of a NS.

This variability should affect the particle production rate which will affect the curvature radiation and subsequently the formation of the coherent radio beam that we finally observe.

# Magnetic field evolution through the outer crust

Considering the magnetic field structure of a pulsar to be a dipole, the induction equation gives:

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left[\frac{c^2}{4\pi\sigma}\nabla \times \mathbf{B} + \frac{c}{4\pi n_e e}(\nabla \times \mathbf{B}) \times \mathbf{B}\right]$$

The second term on the r.h.s. is related to the Hall drift, which is effective on the toroidal field component.

The first term on the r.h.s. is the Ohmic diffusion and is relevant for our study of magnetic field decay through the outer crust in particular through the polar caps.

So the induction equation simplifies to :

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left(\frac{c^2}{4\pi\sigma}\nabla \times \mathbf{B}\right)$$

For a purely dipolar field with axial symmetry, we will have the vector potential for the magnetic field as :

$$\mathbf{A} = [0, 0, A_\phi(r, \theta, t)].$$

Using a Stokes stream function, one can obtain a functional form of the potential as:

$$A_\phi = \frac{f(r,t)}{r}\sin\theta.$$

This gives us a differential equation, which we solve to obtain the magnetic field decay through the surface of the star

$$\frac{c^2}{4\pi R^2 \sigma(x)}\left[\frac{\partial^2 f(x,t)}{\partial x^2} - \frac{2}{x^2} f(x,t)\right] = \frac{\partial f(x,t)}{\partial t}$$

*R* -> radius; *σ(x)* -> electrical conductivity along the radial direction; *x=r/R*

For a curved spacetime, and using a background Schwarzschild metric, the equations change to the following :

The axially symmetric vector potential is: $$A_\phi = -g(r,t)\sin^2\theta.$$

where *g(r,t)* is analogous to *f(r,t)* as for the flat spacetime obtained before

The diffusion equation to solve for the surface magnetic field decay becomes:

$$\frac{c^2}{4\pi R^2\sigma(x)}\left(1-\frac{y}{x}\right)^{\frac{1}{2}}\left[\left(1-\frac{y}{x}\right)\frac{\partial^2 g\left(x,t\right)}{\partial x^2}+\frac{y}{x^2}\frac{\partial g\left(x,t\right)}{\partial x}-\frac{2}{x^2}g\left(x,t\right)\right]=\frac{\partial g\left(x,t\right)}{\partial t}$$

These diffusion equations are solved numerically, with proper choice of the model for electrical conductivity and the boundary conditions.

# The electrical conductivity in NS outer crust

The matter density of the NS crust varies over 7 orders of magnitude in the outer crust which is only about a few hundred metres.

Thus, for such a drastic variation one needs to have a consistent model matching the density with the depth.

Datta, Thampan & Bhattacharya ( 1995 ) computed this variation for various equations of state of nuclear matter.

A simple but elegant model was given by Urpin & Yakovlev ( 1979 ), which closely resembles the results of Datta, Thampan & Bhattacharya ( 1995 ).

We have adopted this model in order to find our solutions.

The analytical formula describing this model are as follows:

$$\chi = \sqrt{z(z+2)} = \left( \frac{\rho}{\mu_e 10^6 \,\mathrm{g\,cm^{-3}}} \right)^{\frac{1}{3}}$$

$$z = \frac{d}{H_R}$$

$$H_R = \frac{m_e R^2}{m m_p \mu_e} \left( 1 - \frac{2m}{R} \right)^{1/2}$$

$$\mu_e = A/Z$$

The above reduces to:

$$\rho(r) = [(R-r)(R-r+2H_R)]^{3/2} \frac{1}{H_R^3} \mu_e 10^6 \,\mathrm{g\,cm^{-3}}$$

The electrical conductivity σ has two components.

The first $\sigma_{ph}$ is due to the phonon scattering or oscillatory scattering of the electrons that happens more in the denser region, where electrons are more free.

The second $\sigma_{imp}$ is due to the impurity scattering.

Following prescription from Yakovlev & Urpin,1980:

$$\sigma_{\mathrm{ph}} = \frac{1.57 \times 10^{23} \chi^4}{T_6 (2 + \chi^2)} \frac{\sqrt{0.017 + \delta^2}}{13\delta} \ \mathrm{s}^{-1}$$

Here $T_6 = T/10^6$ K and $\delta = 0.45(T/T_D)$

The Debye temperature $T_D$ is given by:

$$T_{\mathrm{D}} = 0.45 \frac{\hbar}{k_{\mathrm{B}}} \left( \frac{4\pi Z^2 e^2 n_i}{A m_{\mathrm{p}}} \right)^{\frac{1}{2}} = 2.4 \times 10^6 \left( \frac{2}{\mu_{\mathrm{e}}} \right)^{\frac{1}{2}} \chi^{\frac{3}{2}} \ \mathrm{K}.$$

The conductivity for impurity scattering $\sigma_{imp}$ , which dominates at lower temperatures is given by:

$$\sigma_{\mathrm{imp}} = \frac{8.53 \times 10^{21} \chi^3}{\Lambda_{\mathrm{imp}}(1+\chi^2)} \frac{Z}{Q} \,\mathrm{s}^{-1}$$

Here, $\Lambda_{imp}$ is the Coulomb logarithm, which is $\Lambda_{imp} \cong 2$ for $\rho \geq 10^5$ g cm $^{-3}$ and

*Q* is the impurity parameter:

$$Q = \frac{1}{n} \sum_i n_i (Z - Z_i)^2$$

where *n* and *Z* are the number density and electric charge, respectively, of background ions in the crust lattice without impurity and $n_i$ and $Z_i$ are the density and charge of the *i*th impurity species.

Due to lack of an exact understanding of the extent of the impurity content in the crust and the types of impurity species present there, an approximate range of Q varying from 0.1 to 0.001 has been assumed in most of the earlier studies found in the literature.

The net electrical conductivity within the crust is then given by:

$$\sigma = \left( \frac{1}{\sigma_{\rm ph}} + \frac{1}{\sigma_{\rm imp}} \right)^{-1}$$

Making substitutions for $\sigma_{\mathbf{ph}}$ and $\sigma_{\mathbf{imp}}$ , we can write the electrical conductivity in terms of the density ρ as:

$$\sigma(\rho) = \Bigg[ 8.28 \times 10^{-29} T \left( \frac{\mu_e 10^6}{\rho} \right)^{4/3} \left[ 2 + \left( \frac{\rho}{\mu_e 10^6} \right)^{2/3} \right] \times \left[ 9.67 \times 10^{11} \frac{1}{T^2 \mu_e^2} \frac{\rho}{10^6} + 1 \right]^{-1/2}$$
$$+ 2.34 \times 10^{-22} \frac{Q}{Z} \frac{\mu_e 10^6}{\rho} \left[ 1 + \left( \frac{\rho}{\mu_e 10^6} \right)^{2/3} \right] \Bigg]^{-1}$$

# Numerical Results

The diffusion equations are generally solved by the Crank-Nicolson method.

For our work, we made use of a finite-differencing method known as the ***method of lines***.

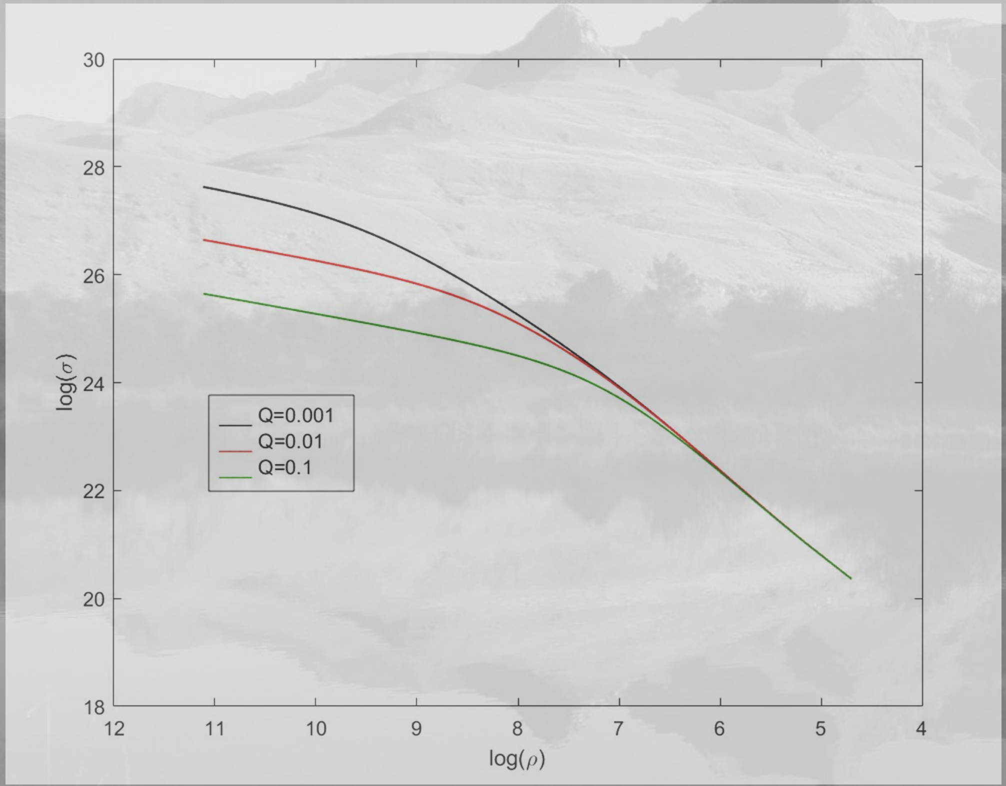
It involves discretising all dimensions except one and then integrating the resulting semi-discrete problem as a system of ODEs or differential-algebraic equations.

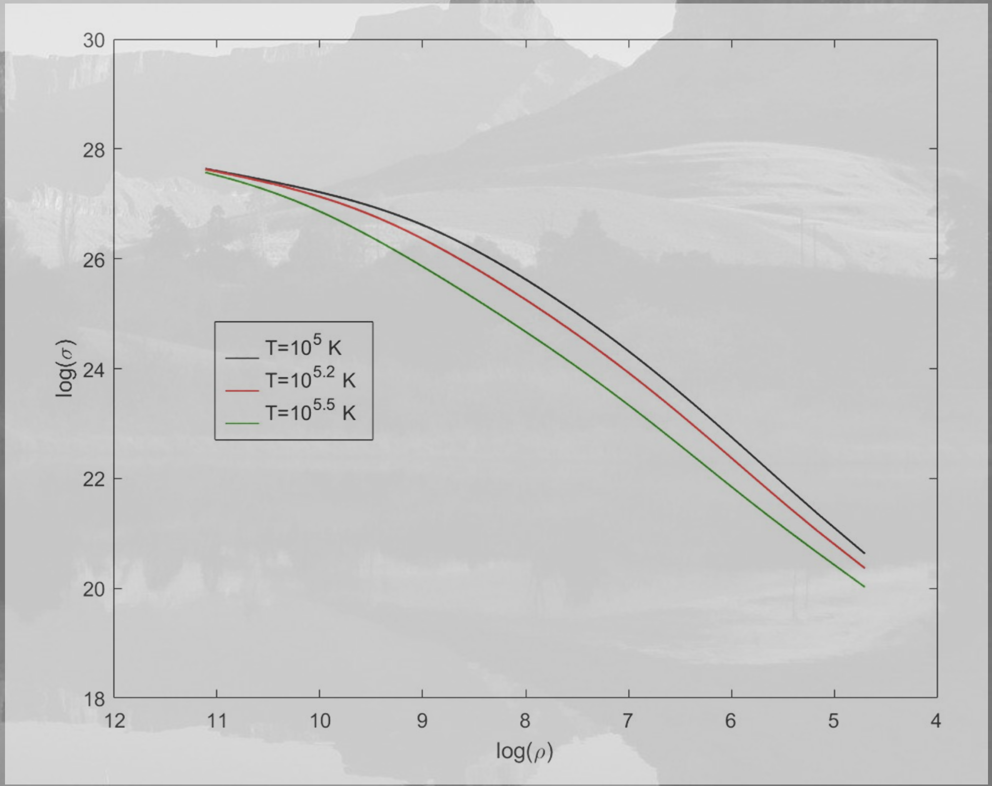
For the density profile, we considered two EOSs for nuclear matter - The Waleck model (abbreviated here as WAL) and the Bethe-Johnson Model V (abbreviated here as BJV) for comparison.

Unless specifically mentioned, we have used the impurity parameter Q=0.001 and the temperature $T=10^{\mathbf{5.2}}$ K

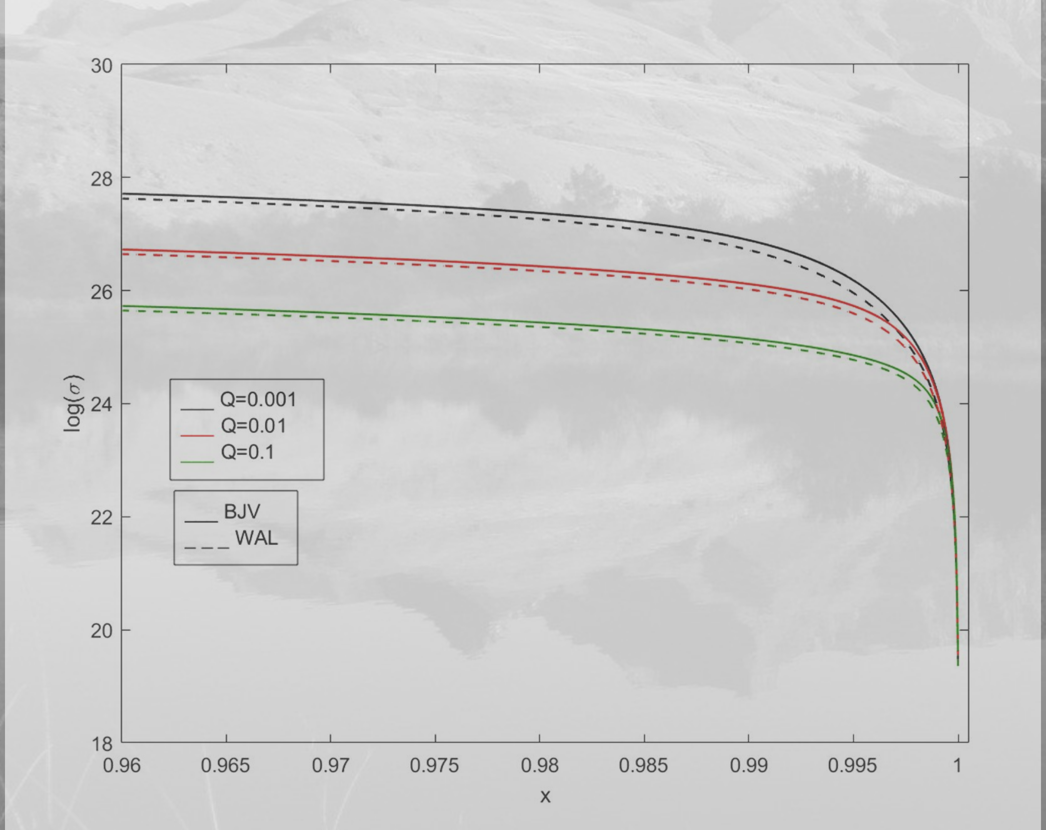
It can be found from literature that a NS within about 100 years from its birth, cools rapidly and settles at around $T=10^{\mathbf{5.5}}-10^{5.2}$K, and remains nearly unchanged for the rest of its lifetime (~$10^8$ years).

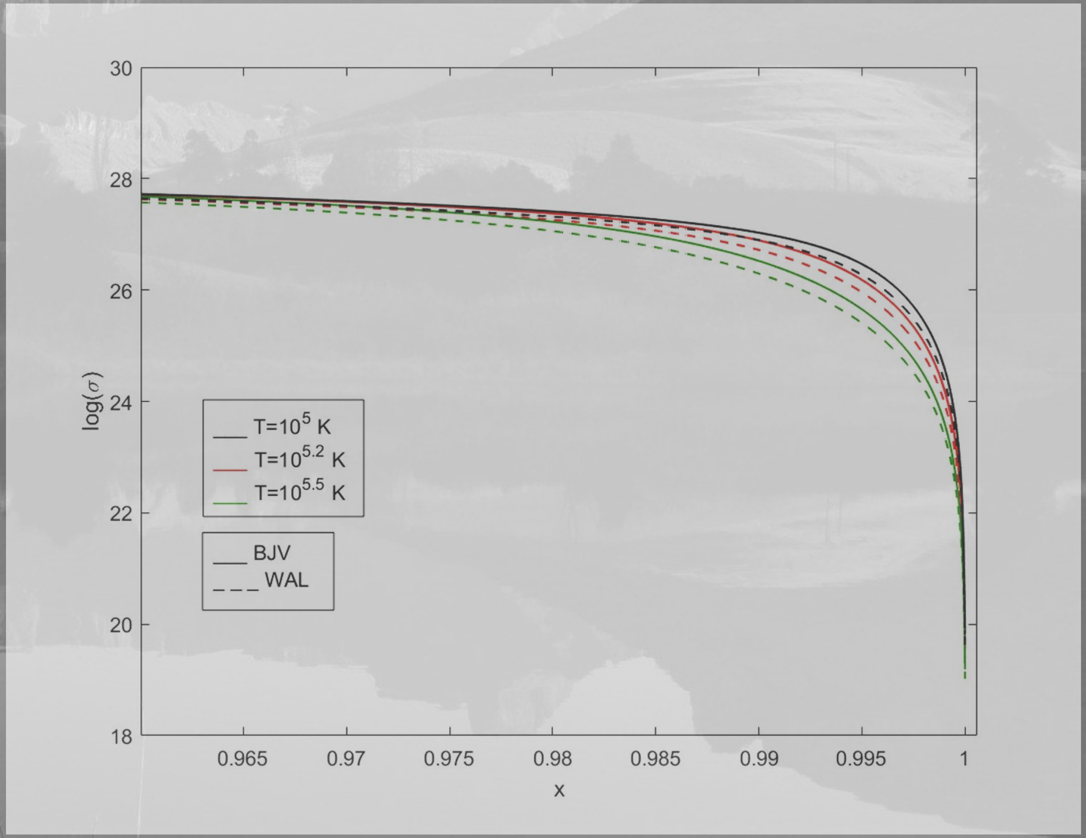
# Variation of the electrical conductivity σ with the matter density of the outer crust

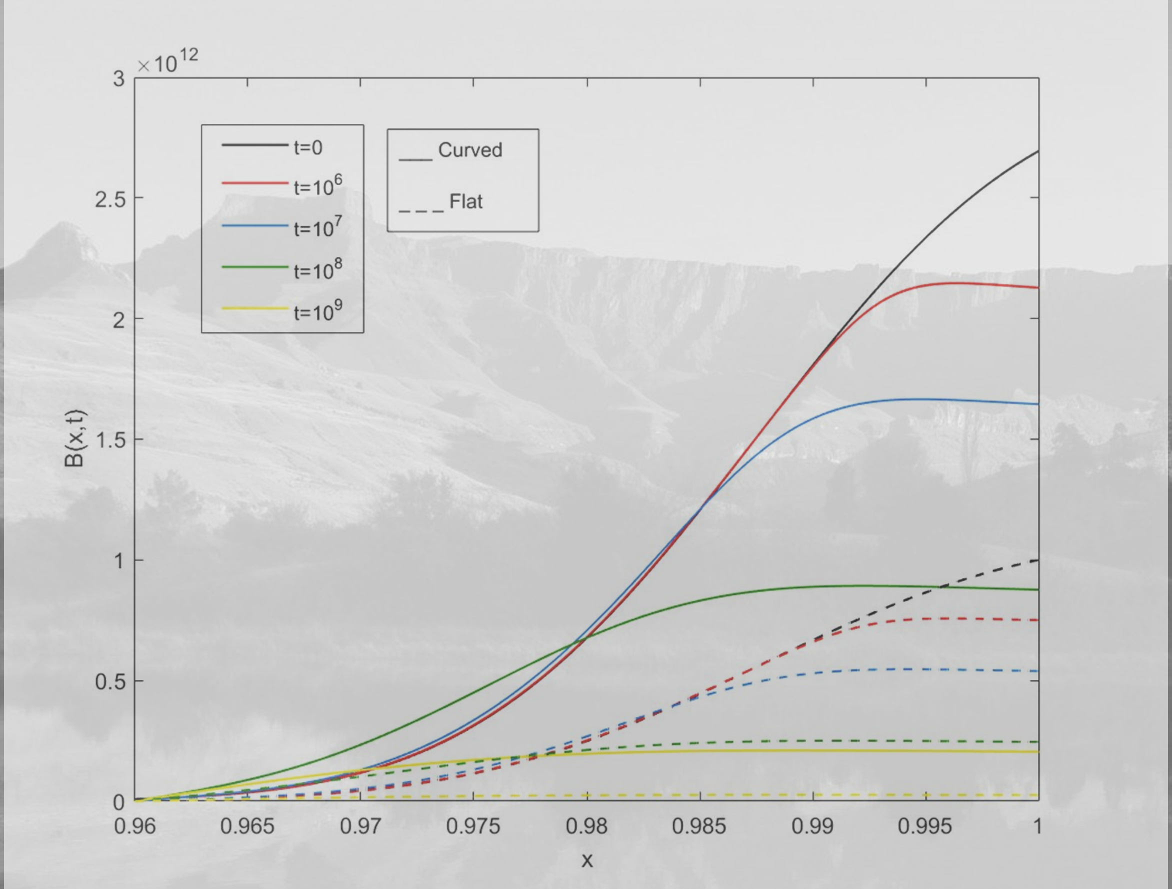


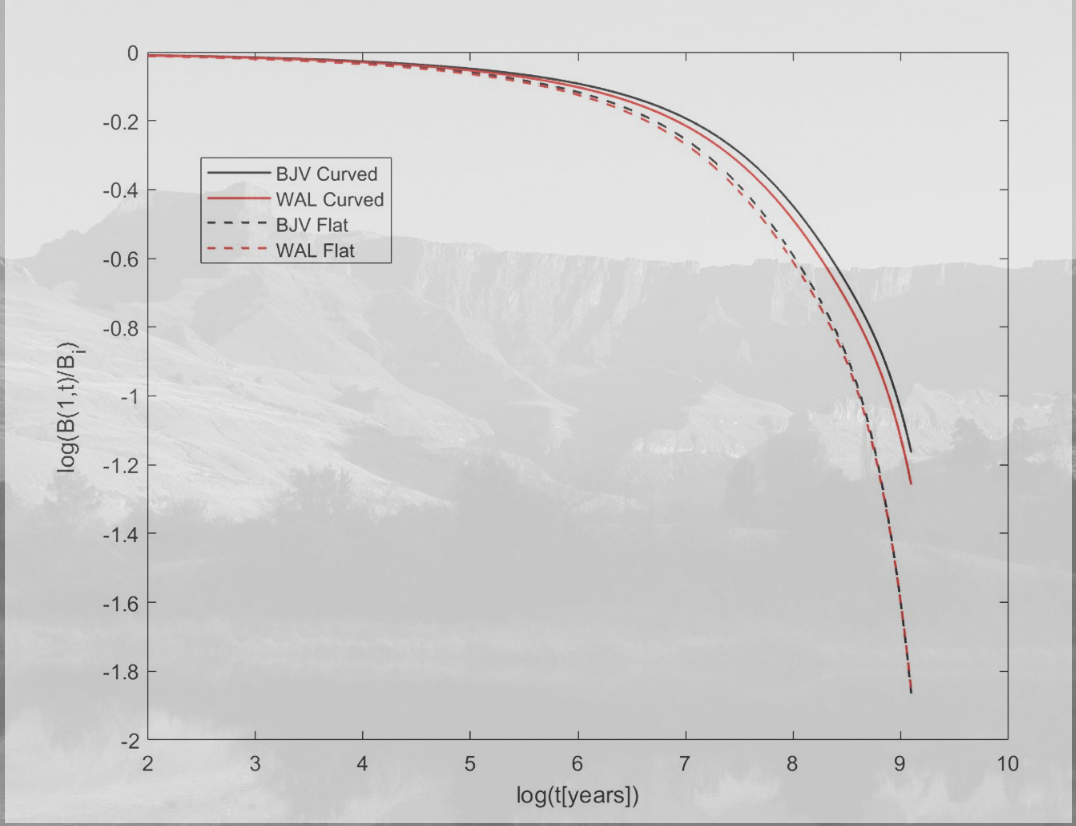
# Variation of the electrical conductivity σ along the outer crust (for two different EOSs)






Magnetic field evolution through the outer crust for both the flat and curved spacetimes.
The different coloured lines with corresponding t values represent points in time of the NS evolution in years.




Surface magnetic field decay for both WAL and BJV for the flat and curved spacetimes.

The electrical conductivity of matter in the outer crust of a NS plays a decisive role on the behaviour of the surface magnetic field if we consider the magnetic field has evolved from this outer layer.

This conductivity is highly sensitive to the matter density which increases with depth and along with it increases the incompressibility of matter- making it structurally more rigid as we go deeper.

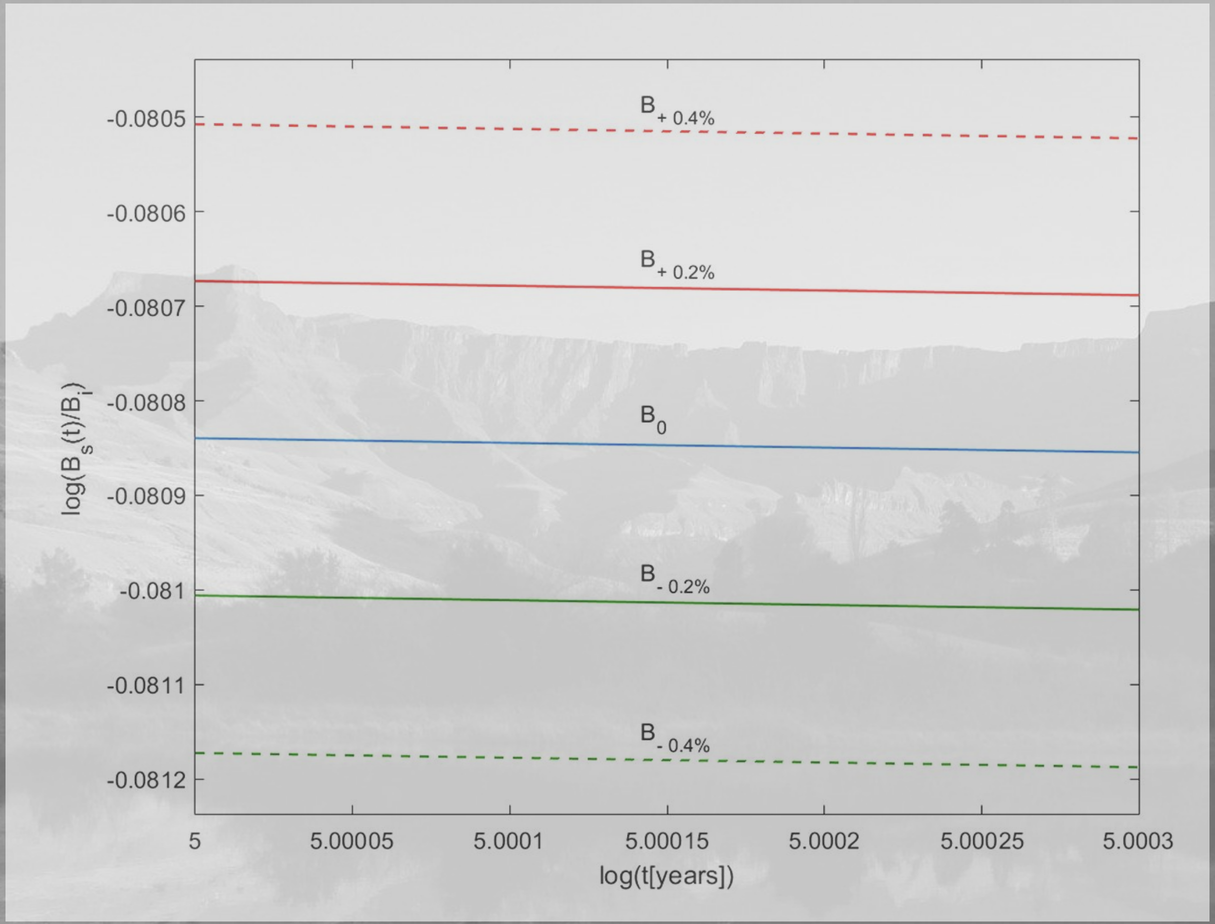
The core of the NS has much higher incompressibility than the crust.

The nuclear matter core of a NS is like a giant hyper nuclei, which is susceptible to various vibration and oscillation modes.

Among them the breathing modes or the radial modes of oscillation will be the most effective contributor to the crustal fluctuations in the outermost layer.

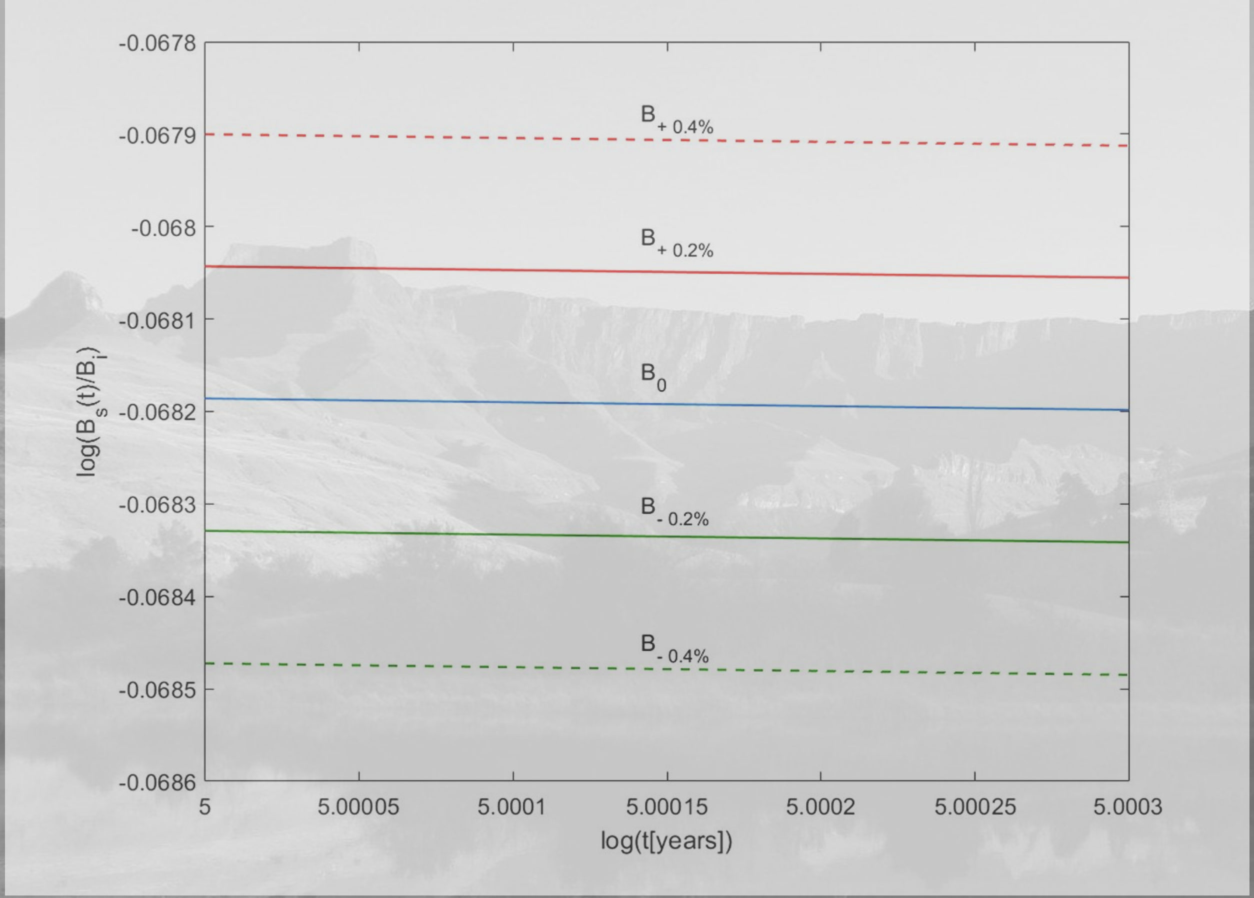
These radial fluctuations coming from the oscillations will affect the electrical conductivity which will directly influence the crustal magnetic field evolution and hence the surface magnetic field will also show rapid fluctuation.

The frequency of such fluctuation will be directly proportional to the oscillation frequency, which had been estimated to be of the order of Kilohertz- a value that also fits well with the observed sub-pulse variability.

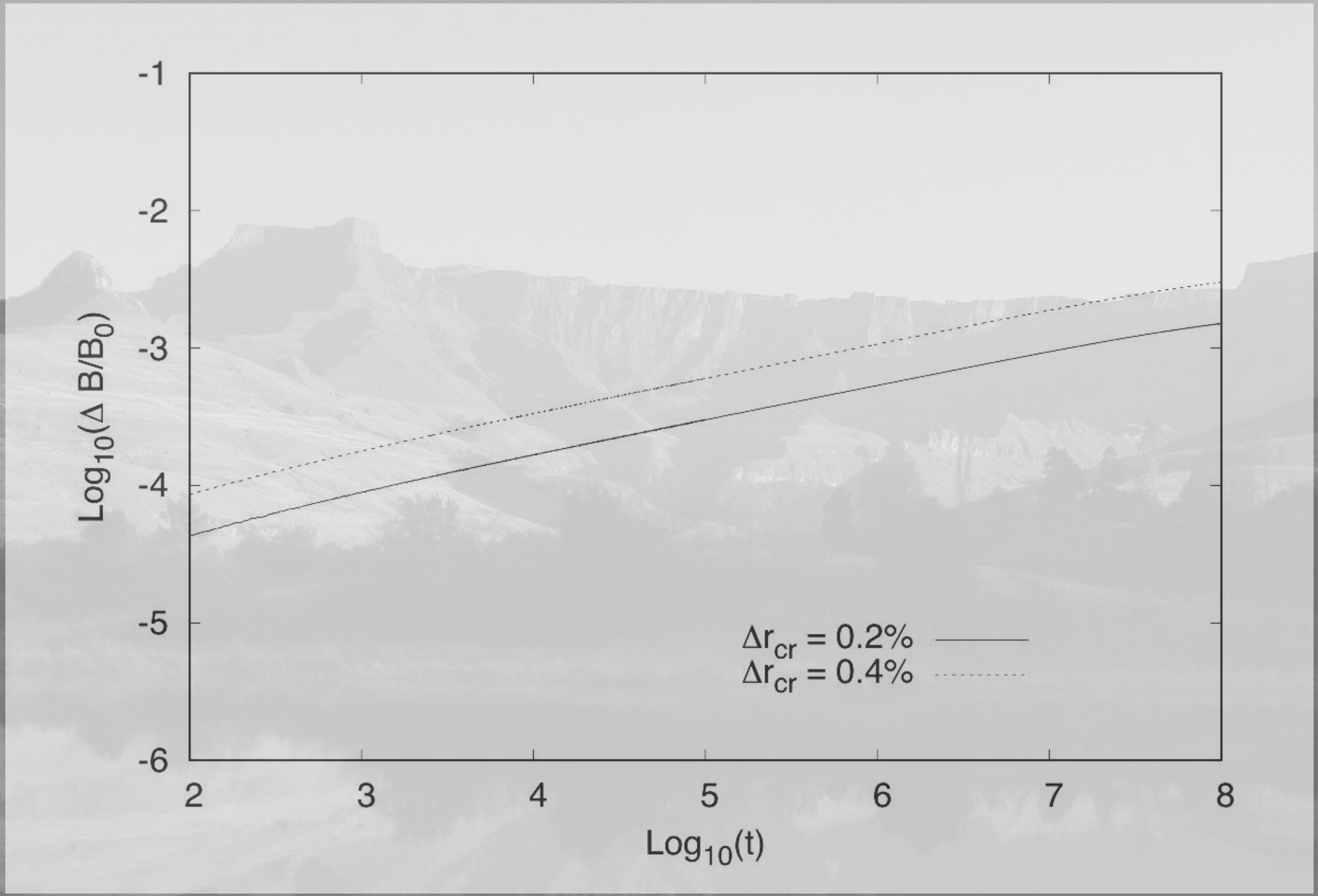



Surface magnetic field variations due to radial fluctuations of 0.2% and 0.4% of the outer crust over a period of ~ 70 years for WAL, for the flat spacetime.

$B_0$ is the unperturbed field and $B_+$ and $B_-$ are the maximum and minimum fields respectively due to the oscillations.




Surface magnetic field variations due to radial fluctuations of 0.2% and 0.4% of the outer crust over a period of ~ 70 years for WAL for the curved spacetime.




Variation in the surface magnetic field ΔB due to the radial fluctuation of 0.2% and 0.4% of the outer crust, including the general relativistic effects.

It is seen that there is an increase in the magnetic field fluctuation in the later stage of the NS's lifetime.

The intensity of the varying surface field is directly related to size of the fluctuation of the radial length of the crust as can be seen from the 0.2% and 0.4% variations in crust width.

Additionally to this, the GR effects add to the compactness of the star which leads to smaller fluctuations in the magnetic field strength.

Notable result of this study is the fact that as the star ages and the magnetic field strength decreases, the magnitude of the change in the magnetic field strength $\Delta B$ due to the radial oscillations increases.

This result matches well with observational data, as reported in the work of Song et al. [2023]

Song et al. [2023] conducted a study on 1198 pulsars using the Thousand-Pulsar-Array programme on MeerKAT.

They found that 418 pulsars ( ~35%) of these pulsars exhibit drifting sub- pulses and that these sub-pulses are more pronounced towards the deathline, as is consistent with other previous studies (Rankin 1986 ; Weltevrede, Edwards & Stappers 2006 ).

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Thank you for your attention !

- A very brief statement of isolated radio pulsars emission – magnetosphere, formation of charged plasma and subsequent emission of coherent radio beam
- Problems of hosting a strong dipolar magnetic field inside a neutron star
- Understanding of the origin of the magnetic fields (or do we?)
- Behaviour of the crustal matter
- Transport property of dense matter – up to the density of neutron drip
- Decay of Magnetic field through the NS crust – the Ohmic diffusion
- Role of Hall drift
- Effect of radial oscillation on the surface magnetic field