IMSc, Sep 2024

Neutrino Flavour and Anderson Localisation

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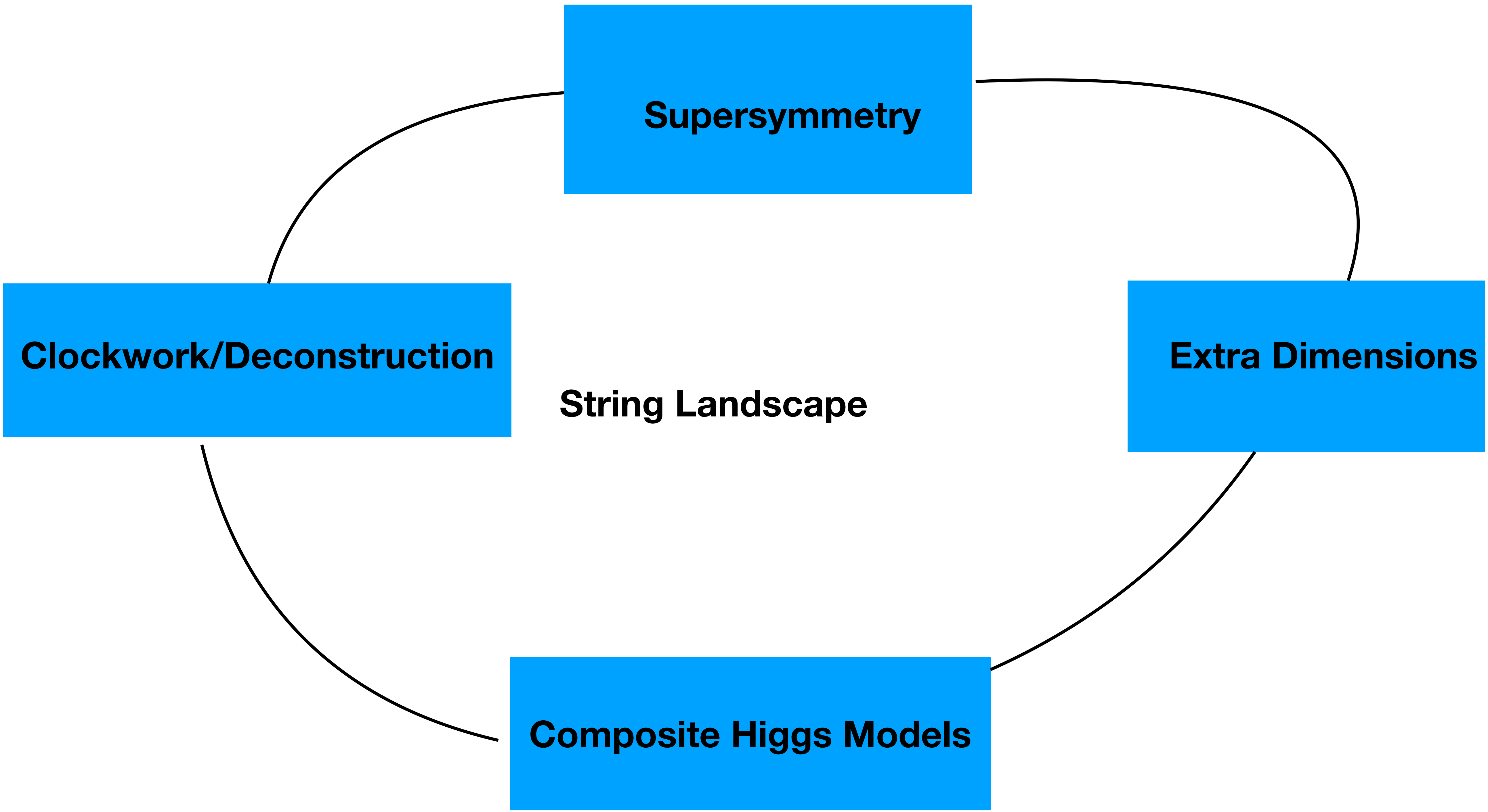
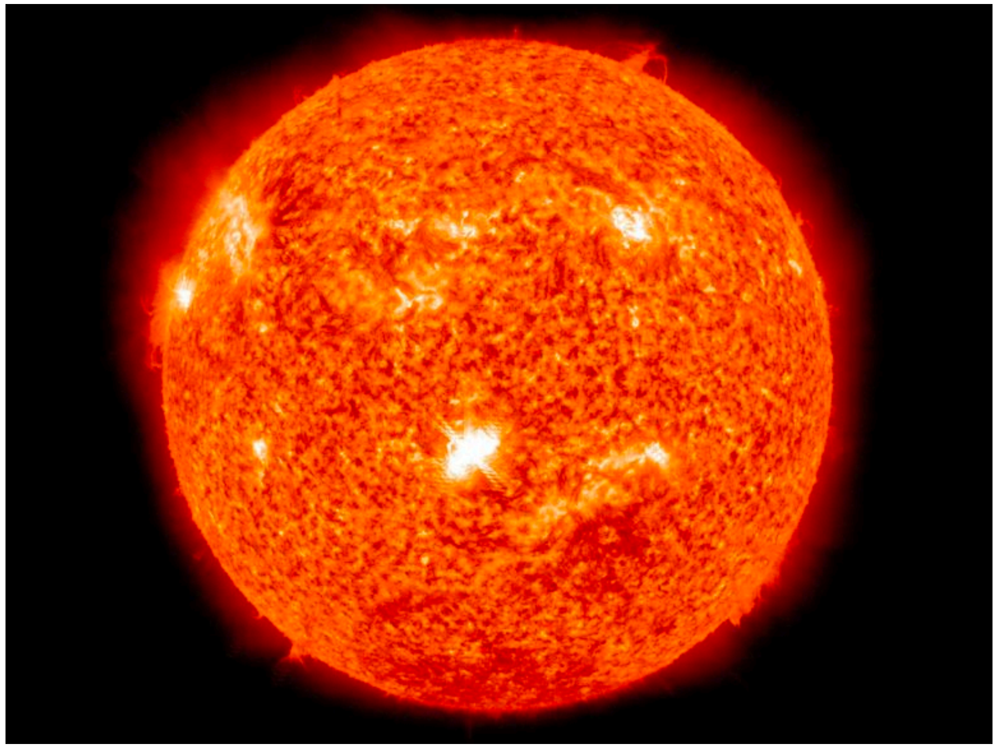
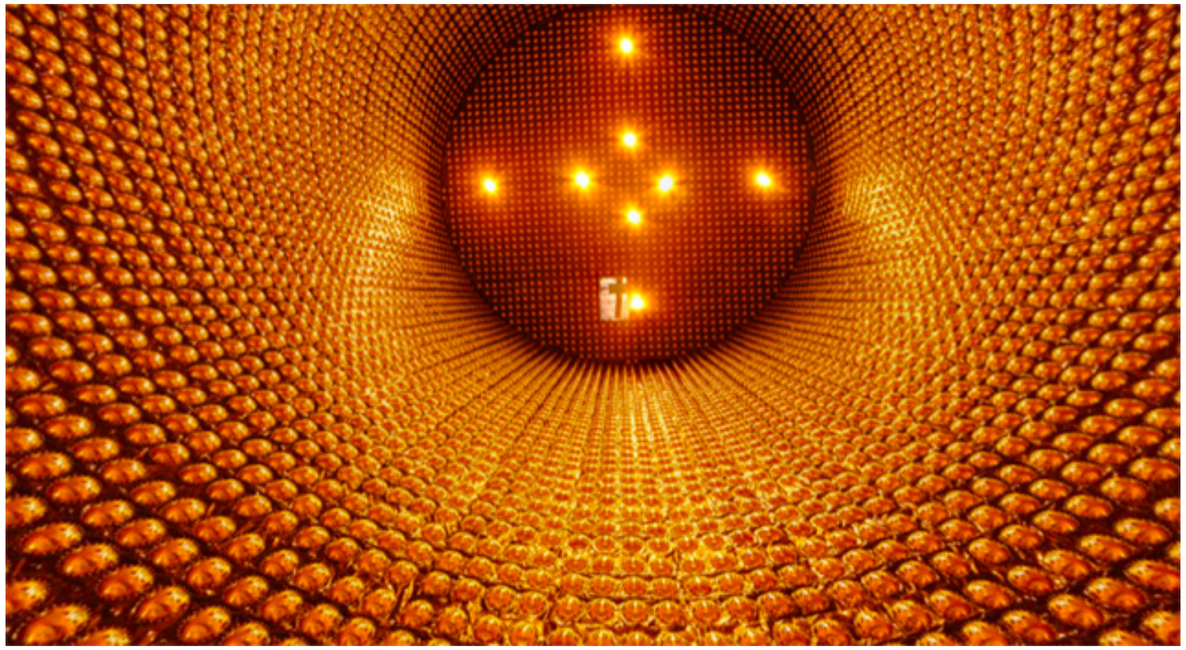
Landscape of Beyond Standard Models



BSM ======= Barely Seen Physics





$$m_{\nu 2} \sim \sqrt{\Delta m^2_{\odot}} \sim 0.008 \text{ eV}$$

$$m_{\nu_3} \sim \sqrt{\Delta m^2_{\textbf{ATM}}} \sim 0.05 \textbf{ eV}$$

Neutrinos Oscillate and this can be explained by tiny Sub eV masses

(assuming normal hierarchy)

Sources of randomness in couplings in Effective Field Theories :

(I) stringy landscapes — Balasubramaniam et.al

(II) dark sectors — Dienes, kumar et.al

Anderson localisation in particle physics — Craig Sutherland 2017

Using randomness in couplings to generate exponential hierarchies.
Applications to neutrino masses

"Anderson localisation" in 4D

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$$S=\sum_{j=1}^{N}\int d^4x\{\bar{\psi}\left(i\gamma^\mu D_\mu\right)\psi+\left(\overline{L_j}\Phi_{j,j+1}R_{j+1}+\overline{L_{j+1}}\Phi_{j+1,j}R_j\right)$$

$$+\overline{L_j}MR_j+h.c.\}$$

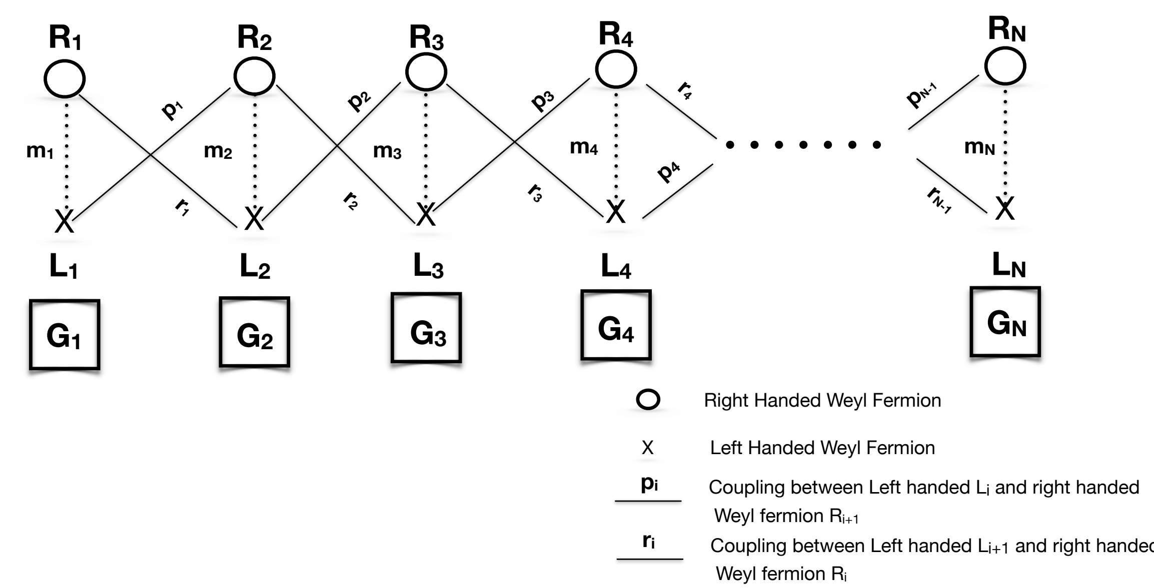
$$\mathcal{L}_{NP}=\mathcal{L}_{kin}-\sum_{i,j=1}^{n}\overline{L_i}\mathcal{H}_{i,j}R_j+h.c.$$

$$\mathcal{H}_{i,j}=\epsilon_i\delta_{i,j}-t_i(\delta_{i+1,j}+K\delta_{i,j+1})$$

$$M_{mass}=\begin{bmatrix}0 & M_A\\ M_A & 0\end{bmatrix}$$

$$M_A=\begin{bmatrix}\epsilon_1 & -t & 0 & \ldots & 0\\ -t & \epsilon_2 & -t & \ldots & 0\\ 0 & -t & \epsilon_3 & \ldots & 0\\ \ldots & \ldots & \ldots & \ldots & \ldots\\ 0 & \ldots & \ldots & -t & \epsilon_N\end{bmatrix}$$

Deconstruction Model



Sutherland and Craig, 2017

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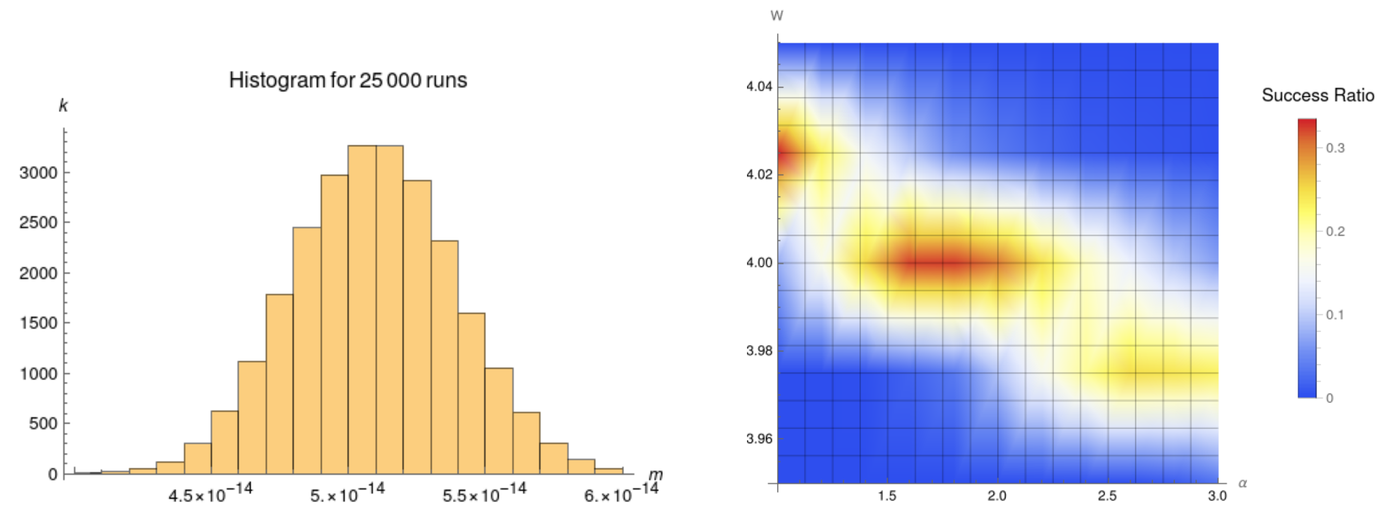


Plot 3 - Histogram for mass distribution of hierarchical mass produced by lattice with 2% randomness in ϵ_i for 25000 runs [Left]. Heat density plot for success ratio for values of W (TeV) and α (% randomness in ϵ_i) [Right].

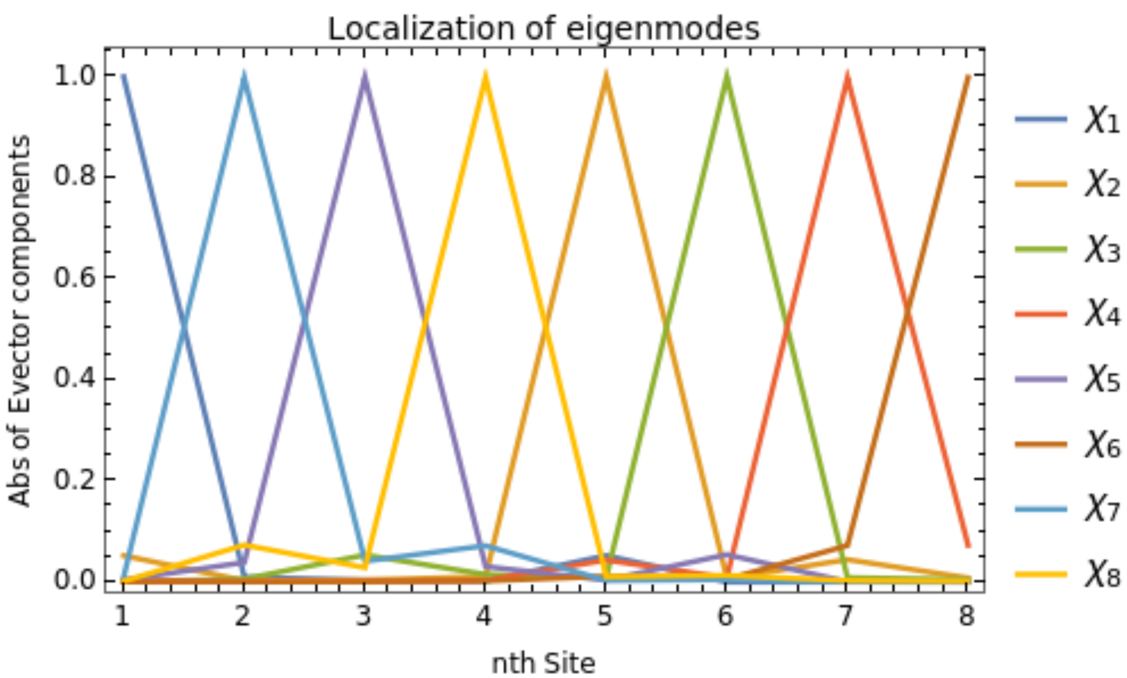
$$\epsilon_i \in [0, W]$$

For t=1, W = 3, N = 8

Strong localisation limit

$$W \gg t$$

$$L\left(m_i^2, t, W\right) \sim \left(\ln \frac{W}{2t} - 1\right)^{-1}$$



(1) Generalised Clockwork

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$$L_{CW} = L_{kin} - \sum_{i=1}^{n} \bar{\psi}_{L_i} H_{ij} \psi_{R_j} + H.C$$

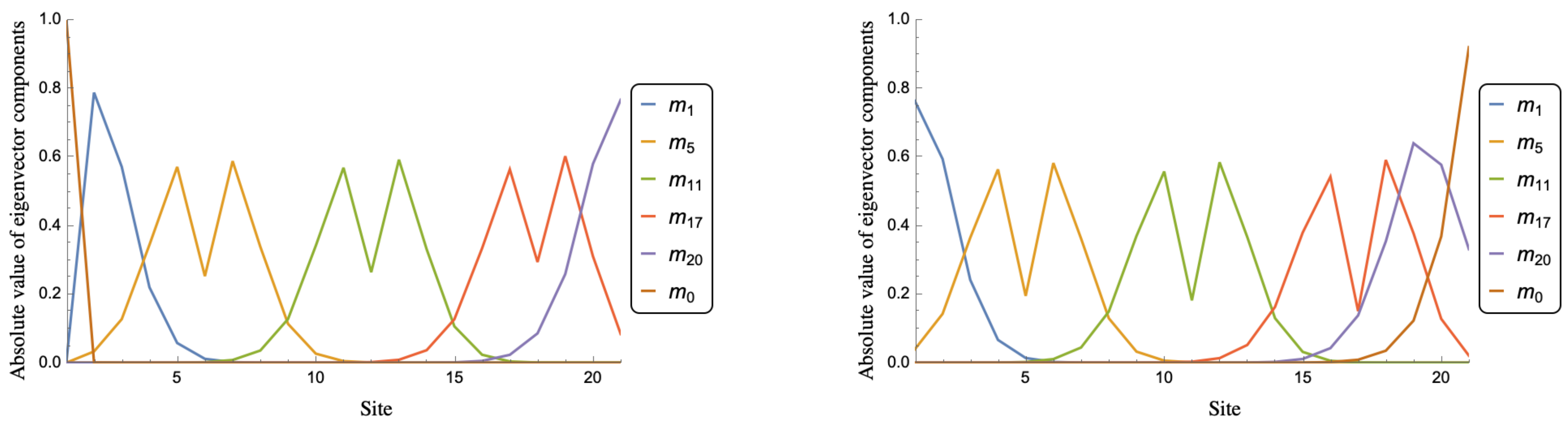
$$H_{ij} = m_i \, \delta_{ij} + q_i m_i \, \delta_{i+1,j}$$

Tiny Dirac neutrino masses !

Zero Mode !

Hong, Kurup, Perelstein

Localisation possible for regions
of parameters (no large hierarchies)



Plot 1(B) - Left plot shows the absolute value of left-handed mass eigenvectors in terms of CW fields and the right plot for right-handed mass eigenbasis with y = 0.1.

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(2) Two Sided Clockwork

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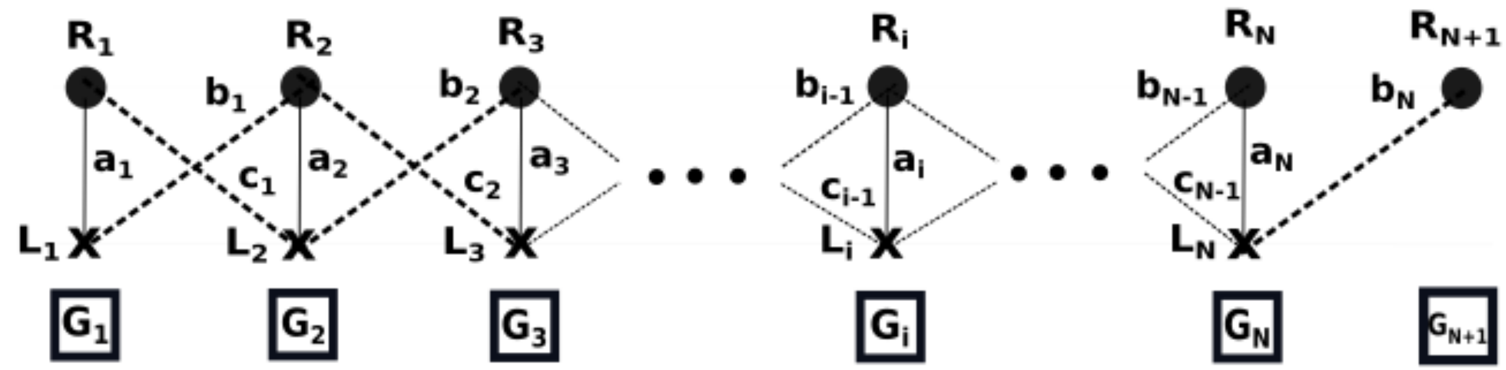
$$L_{CW} = L_{kin} - \sum_{i=1}^{n} \bar{\psi}_{L_i} H_{ij} \psi_{R_j} + H.C \qquad H_{ij} = m(\delta_{ij} + q_i \delta_{i+1,j} + q' \delta_{i,j+1})$$

Tiny Dirac neutrino masses !



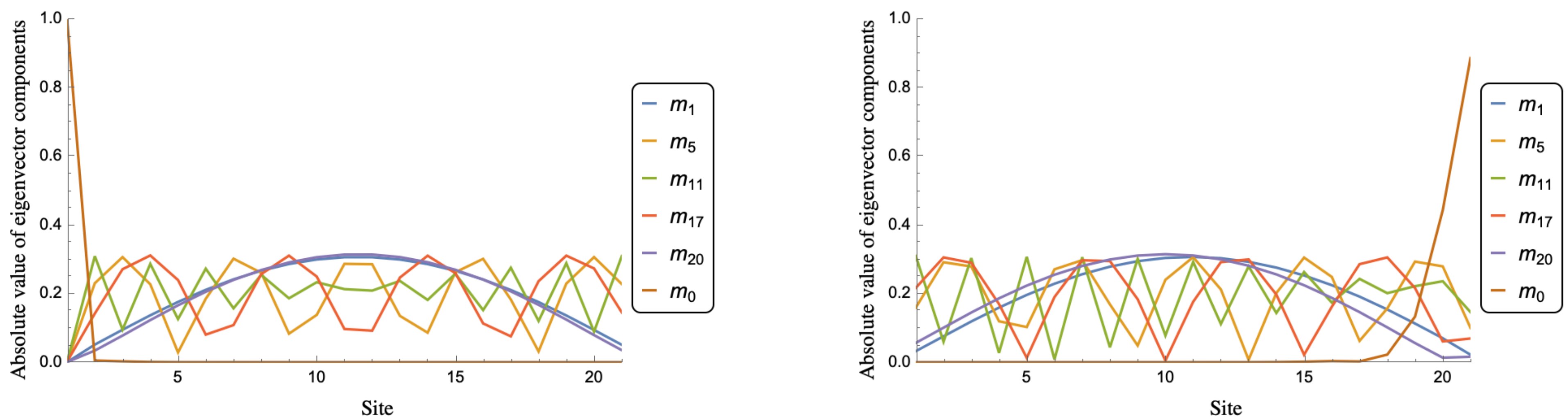
Deconstruction Model
Linear Moose

Zero Mode !

X - Left Handed CW Fermion

● - Right Handed CW Fermion

Localisation possible for regions of parameters (no large hierarchies)



Plot 2(B) - Left plot shows the absolute value of left-handed mass eigenvectors in terms of CW fields and the right plot for right-handed mass eigenbasis with y = 0.1.

Extremely efficient localisation with randomness/disorder

Gero Gresdroff

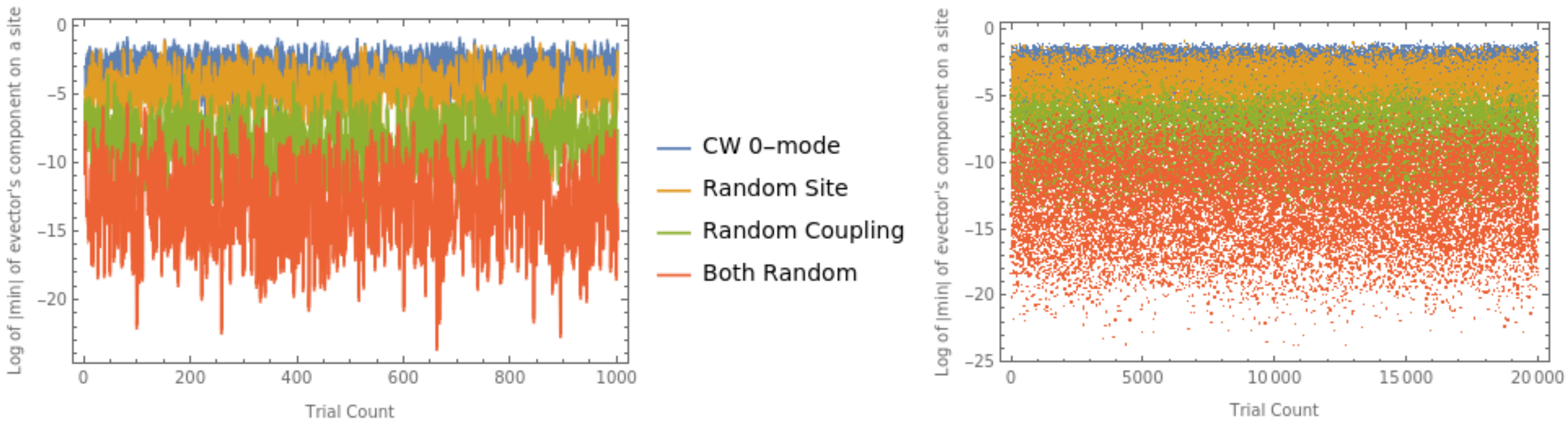
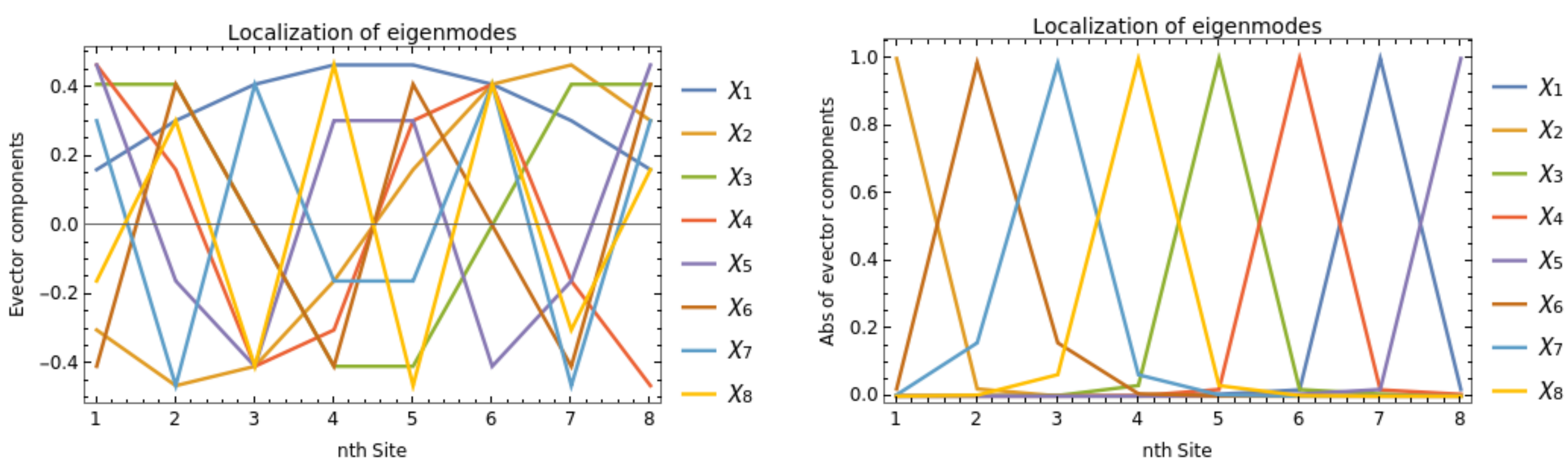


Fig.6 - Figure shows the Log of minimum component 0-mode of CW and lightest mode of disorder models achieved with n = 10 sites.



All modes localised

Fig.2 - Mass modes of Local lattice with uniform sites ϵ_i = W & $t_i = t$ (left) and random sites $t_i = t$ & $\epsilon_i \in$ [2W, -2W] (right) for W = 4 and t = 1/4 with N = 8 sites..

Singh and vempati, 2023

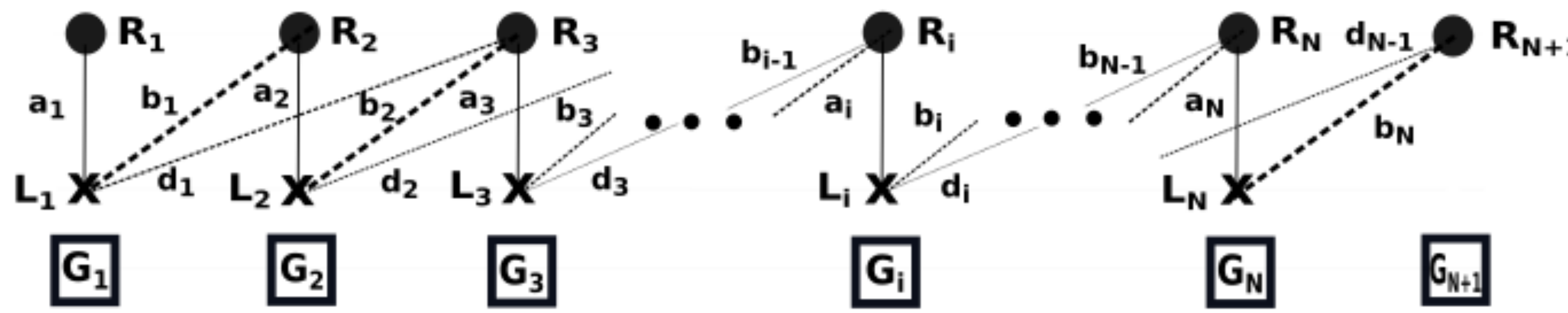
(3) Non-local Interactions

$$\mathcal{H}_{i,j} = a_i\delta_{i,j} + b_i\delta_{i+1,j} + d_i\delta_{i+2,j}$$

Tiny Dirac neutrino masses !

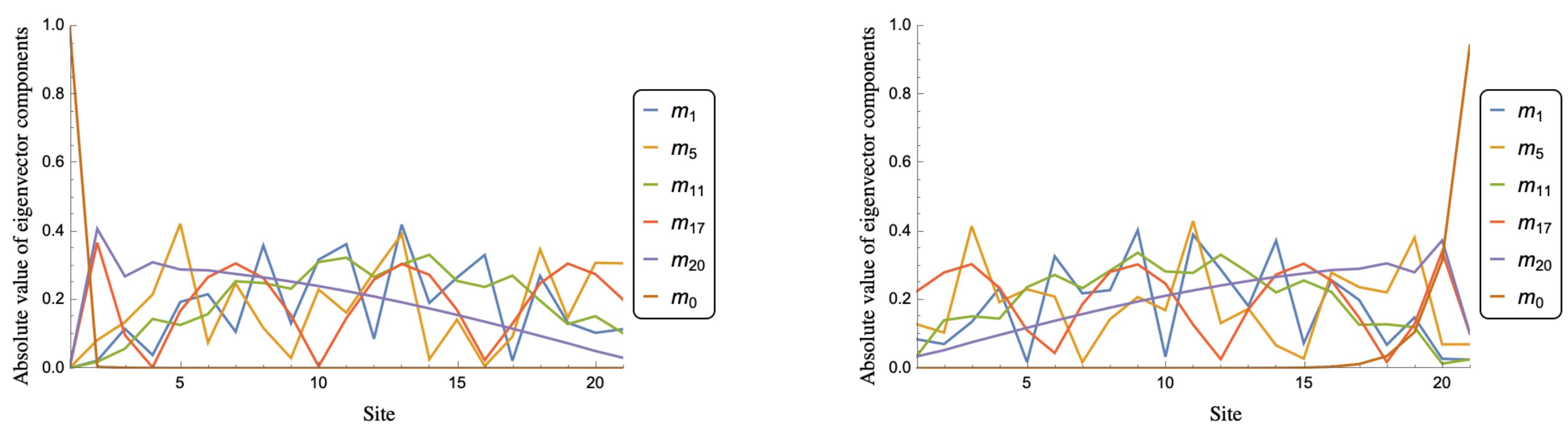
Zero Mode !

Localisation possible for regions of parameters (no large hierarchies)



X - Left Handed CW Fermion

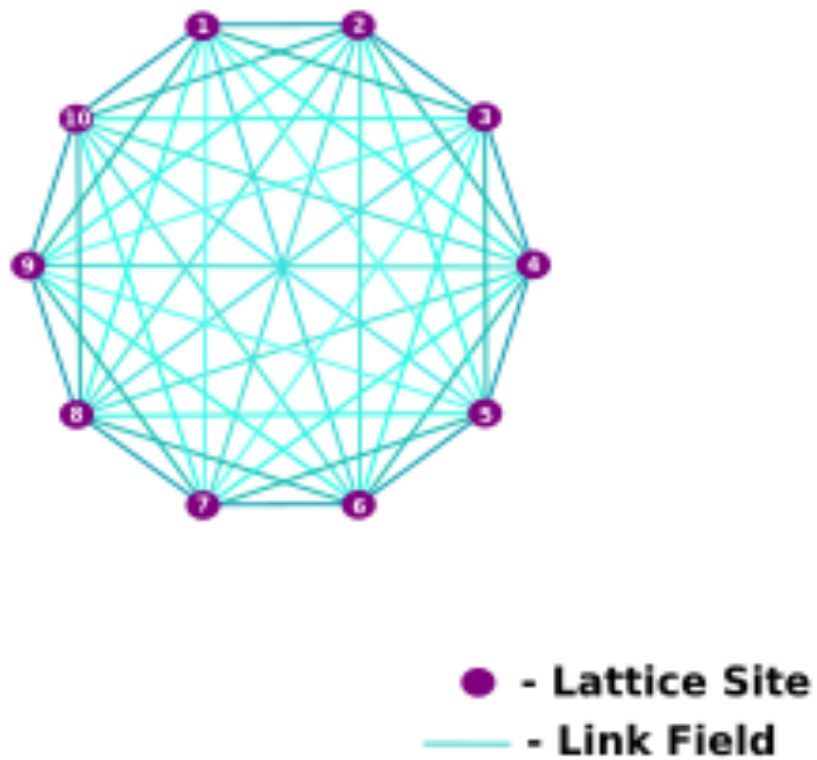
● - Right Handed CW Fermion



Plot 3(B) - Left plot shows the absolute value of left-handed mass eigenvectors in terms of CW fields and the right plot for right-handed mass eigenbasis with y = 0.1.

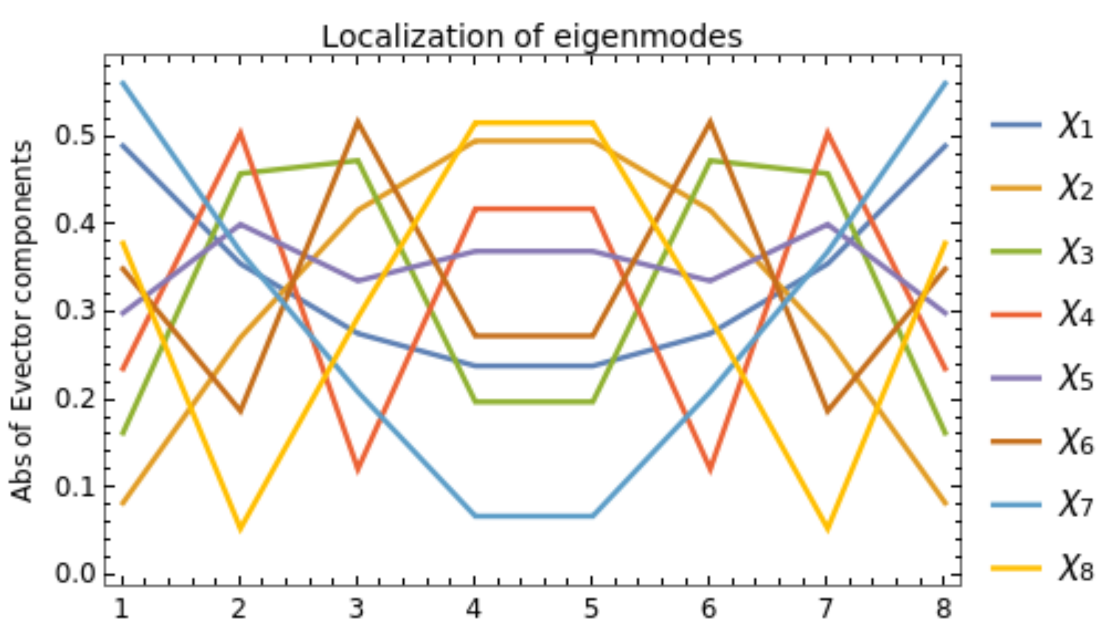
Singh and vempati, 2024

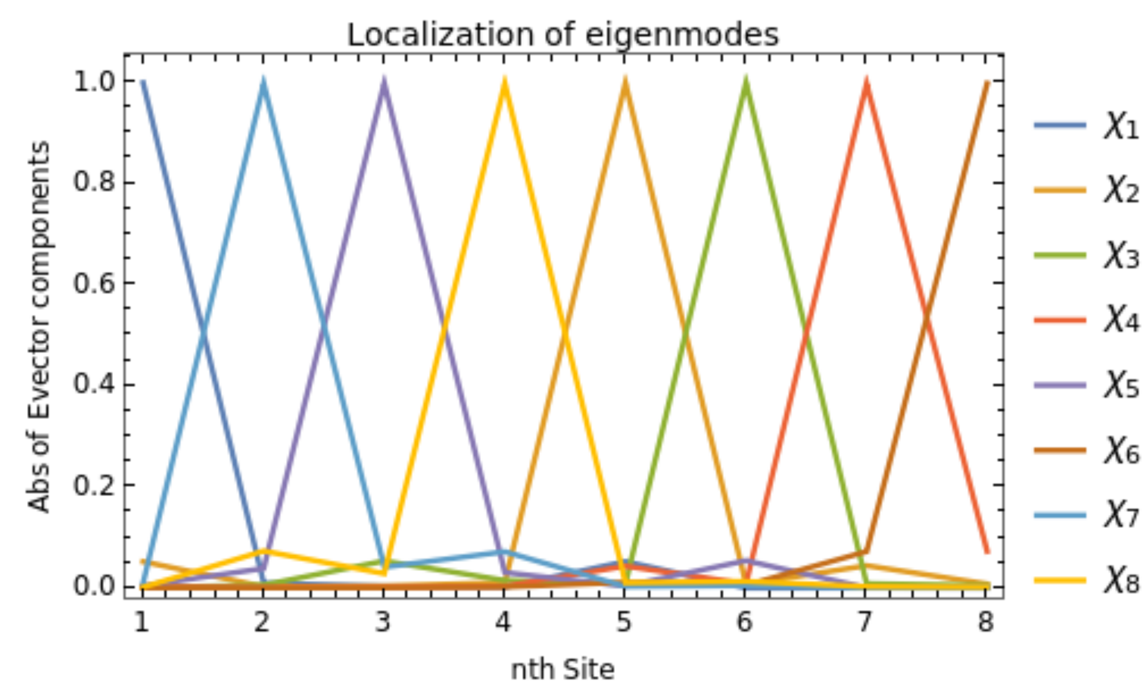
(3a) Completely Non-local



$$\mathcal{L}_{long-range} = L_{Kin} - \sum_{i,j=1}^{N} \overline{L_i}\epsilon_{i,j}R_j - \sum_{i,j=1}^{N} \overline{L_i}\frac{g}{b^{|i-j|}}\left(1-\delta_{i,j}\right)R_j + h.c.$$

$$M_{long-range} = \begin{bmatrix} \epsilon_1 & \frac{g}{b} & \frac{g}{b^2} & \cdots & \frac{g}{b^{N-1}} \\ \frac{g}{b} & \epsilon_2 & \frac{g}{b} & \cdots & \frac{g}{b^{N-2}} \\ \frac{g}{b^2} & \frac{g}{b} & \epsilon_3 & \cdots & \frac{g}{b^{N-3}} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{g}{b^{N-1}} & \cdots & \cdots & \frac{g}{b} & \epsilon_N \end{bmatrix}$$





$\epsilon_i = 2\mathrm{W}$, $\mathrm{g} = 1$, $\mathrm{N} = 8$ b=0.7, W = 4

Ji Ji fan et.al

Singh and Vempati, 2410.XXXX

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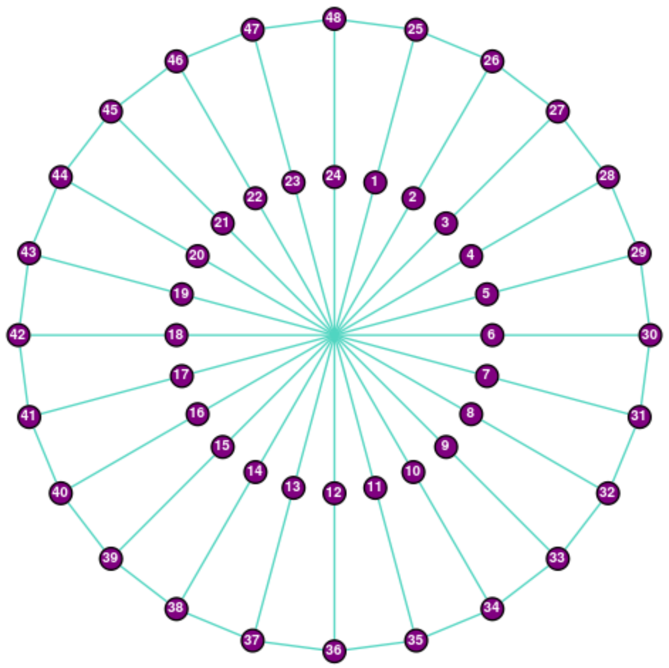
Pietersen Graph



$$L_{Petersen} = L_{Kin} - \sum_{i,j=1}^{N} \overline{L_i}\epsilon_{i,j}R_j - \sum_{i,j=1}^{N/4} \overline{L_i}\frac{g}{b^{|i-j|}}\left(\delta_{i,j+N/4} + \delta_{i+N/4,j}\right)R_j$$
$$- \sum_{i,j=1}^{N/2} \overline{L_i}\frac{g}{b^{|i-j|}}\left(\delta_{i,j+N/2} + \delta_{i+N/2,j}\right)R_j - \sum_{i,j=N/2+1}^{N} \overline{L_i}\frac{g}{b^{|i-j|}}\left(\delta_{i,j+1}\right)R_j$$
$$- \sum_{i,j=N/2+1}^{N} \overline{L_i}\frac{g}{b^{|i-j|}}\left(\delta_{i+1,j}\right)R_j + h.c.$$

Partially non local

$$M_{Petersen} = \begin{bmatrix} \epsilon_1 & 0 & \frac{g}{b^2} & 0 & \frac{g}{b^4} & 0 & 0 & 0 \\ 0 & \epsilon_2 & 0 & \frac{g}{b^2} & 0 & \frac{g}{b^4} & 0 & 0 \\ \frac{g}{b^2} & 0 & \epsilon_3 & 0 & 0 & 0 & \frac{g}{b^4} & 0 \\ 0 & \frac{g}{b^2} & 0 & \epsilon_4 & 0 & 0 & 0 & \frac{g}{b^4} \\ \frac{g}{b^4} & 0 & 0 & 0 & \epsilon_5 & \frac{g}{b} & 0 & \frac{g}{b^3} \\ 0 & \frac{g}{b^4} & 0 & 0 & \frac{g}{b} & \epsilon_6 & \frac{g}{b} & 0 \\ 0 & 0 & \frac{g}{b^4} & 0 & 0 & \frac{g}{b} & \epsilon_7 & \frac{g}{b} \\ 0 & 0 & 0 & \frac{g}{b^4} & \frac{g}{b^3} & 0 & \frac{g}{b} & \epsilon_8 \end{bmatrix}$$

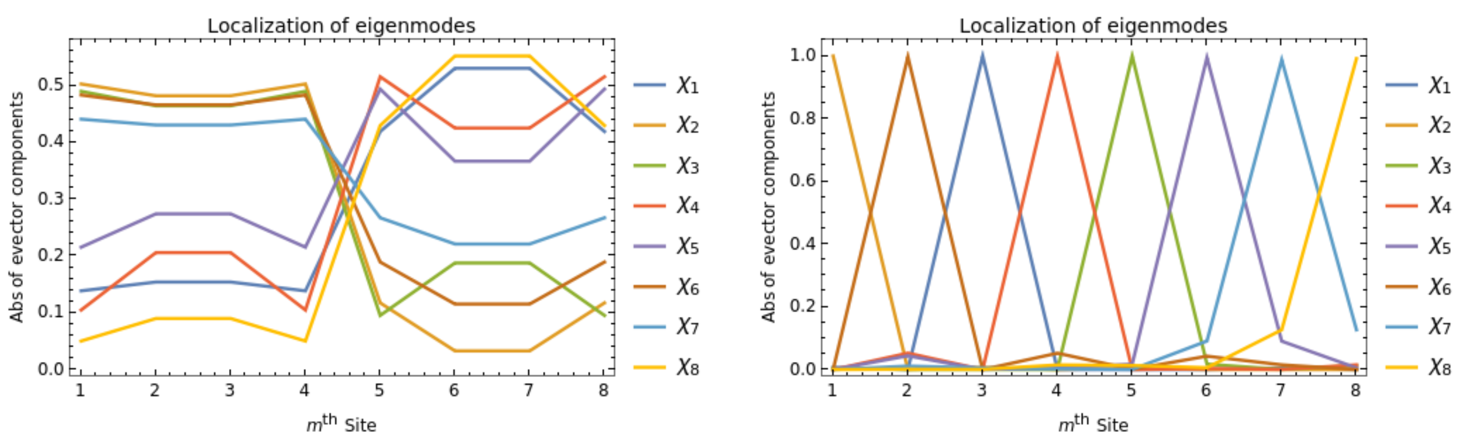


Fig.6 - Mass modes of Petersen graph with uniform sites (left) and random sites(right) for N = 8, W = 5, g = 1/4 and b = 1.4.

Singh and Vempati, 2401.XXXX

Strong Localisation Limit :

$$\epsilon \gg t$$

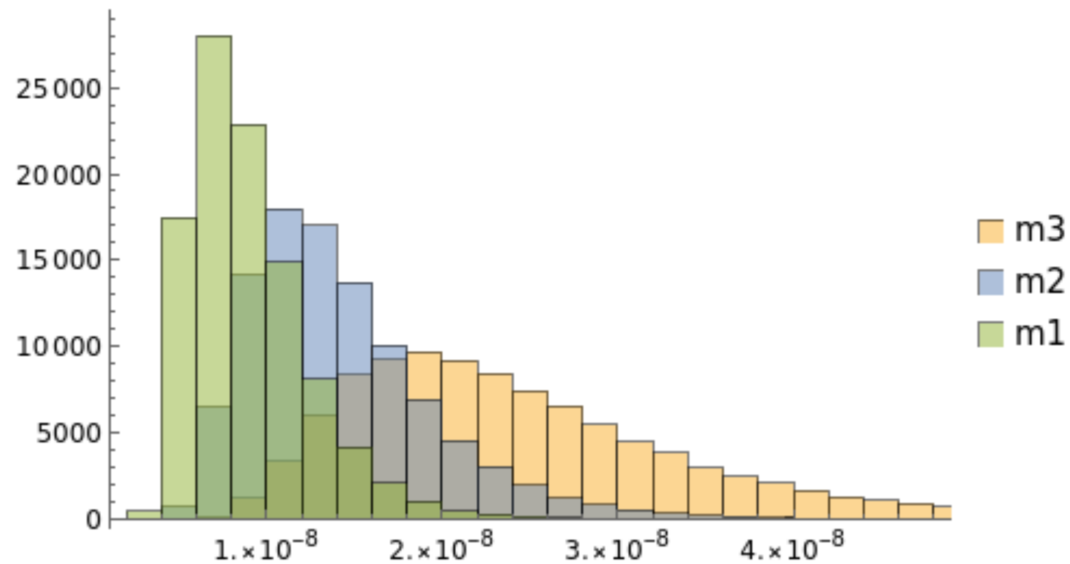
$$\mathcal{H}_{i,j} = \epsilon_i \delta_{i,j} - t_i(\delta_{i+1,j} + K\delta_{i,j+1})$$

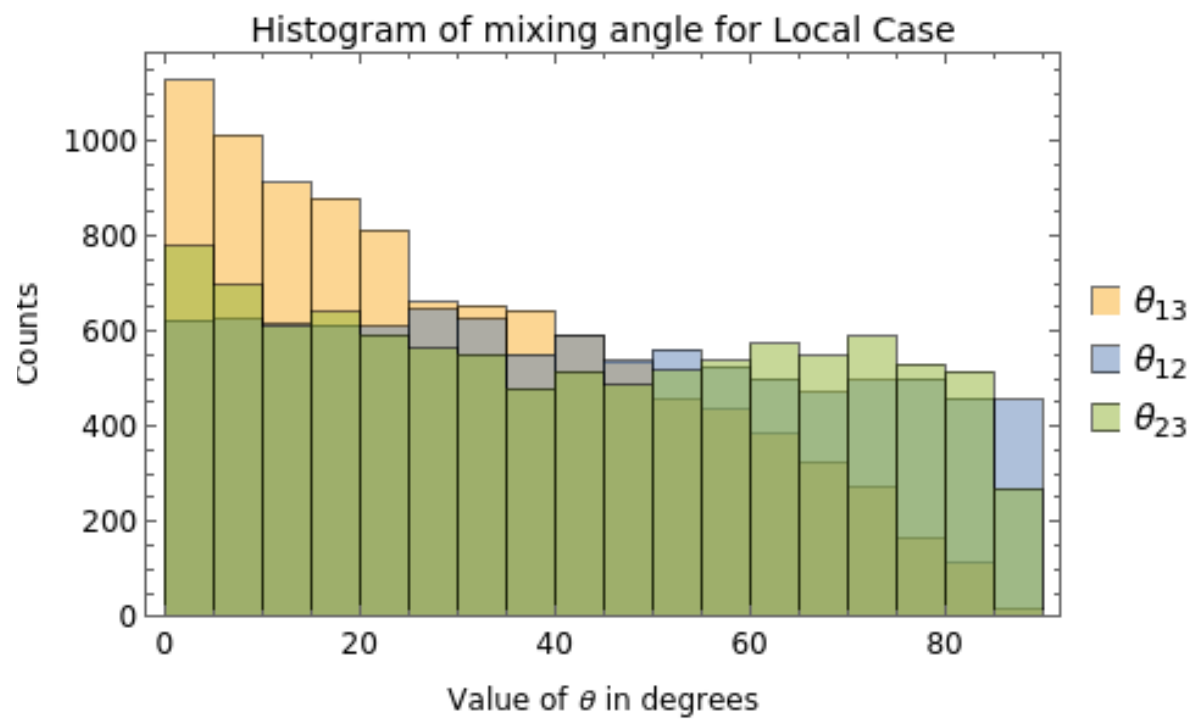
-independent of geometry of the Chain

- Some universal features for neutrino masses and mixing.

Dirac Case

$$\mathcal{L}_{int.} = Y_1 \bar{\nu}_L H R_1 + Y_2 \bar{\nu}_R H L_n + h.c.$$



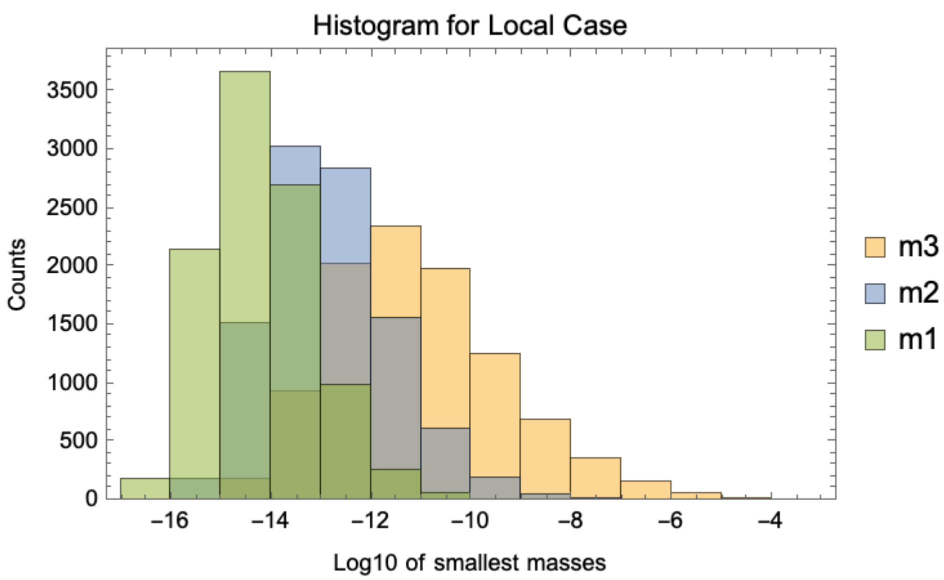


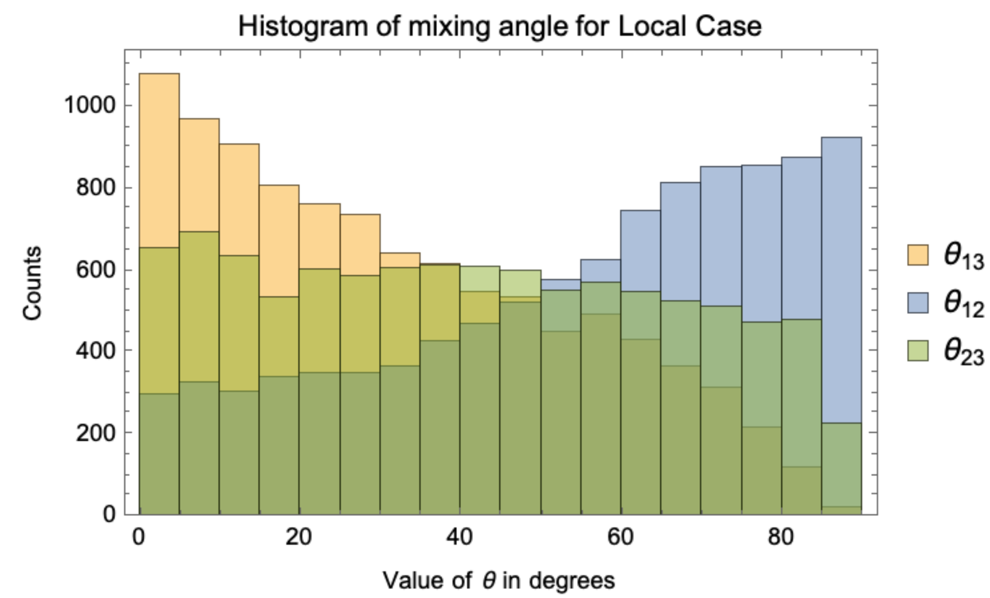
O(1) eV neutrino masses (Demonstration)

Mixing angles are anarchical.

Majorana Case

$$\mathcal{L}_{NP} = L_{kin} - t\bar{L}_1\Psi - \sum_{i,j=1}^{n} \overline{L_i}\mathcal{H}_{i,j}R_j - W\Psi\Psi + h.c.$$





Hierachial neutrino masses with suppression but anarchical mixing angles.

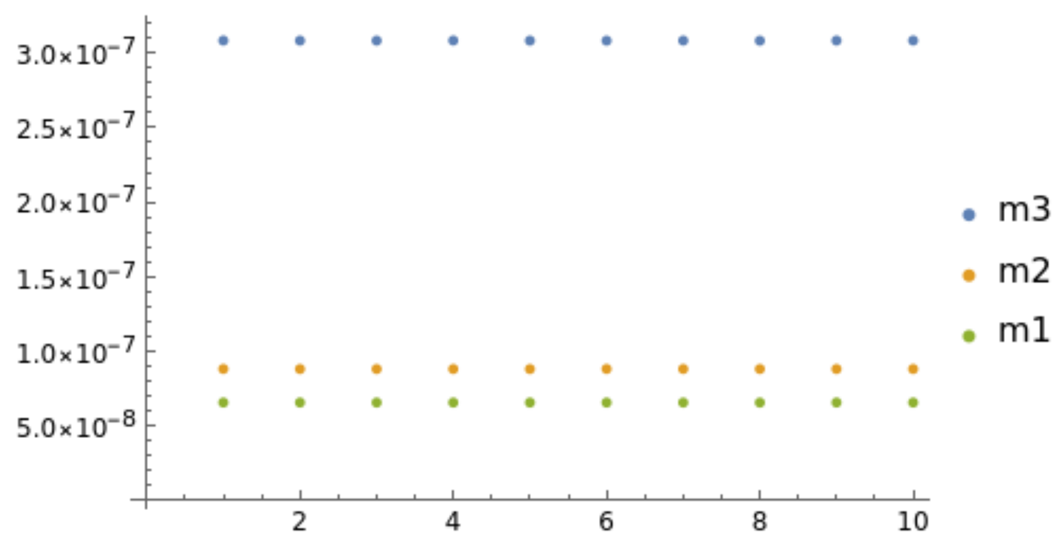
Hierarchial neutrino masses with anarchic mixing angles is a feature of the strong localisation regime independent of the type of geometry, couplings (non-local, partially local etc.)

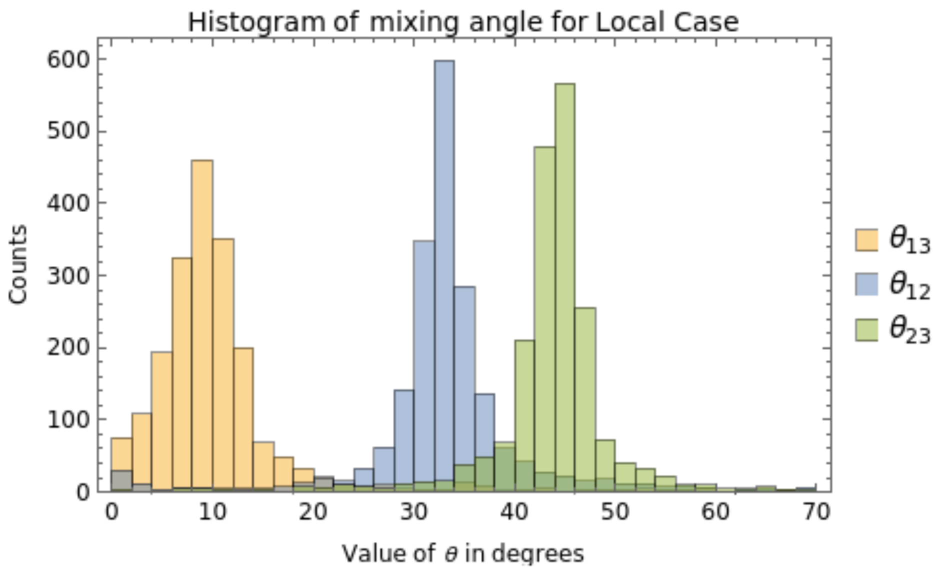
In the case of strong disorder in couplings (t) parameter, $t \gg \epsilon$, geometry does play a mild role, but mixing angles are mostly anarchic, except one !.

Role of Geometry : Weak Disorder

Dirac Scenario : Local Lattice (only nearest neighhour)

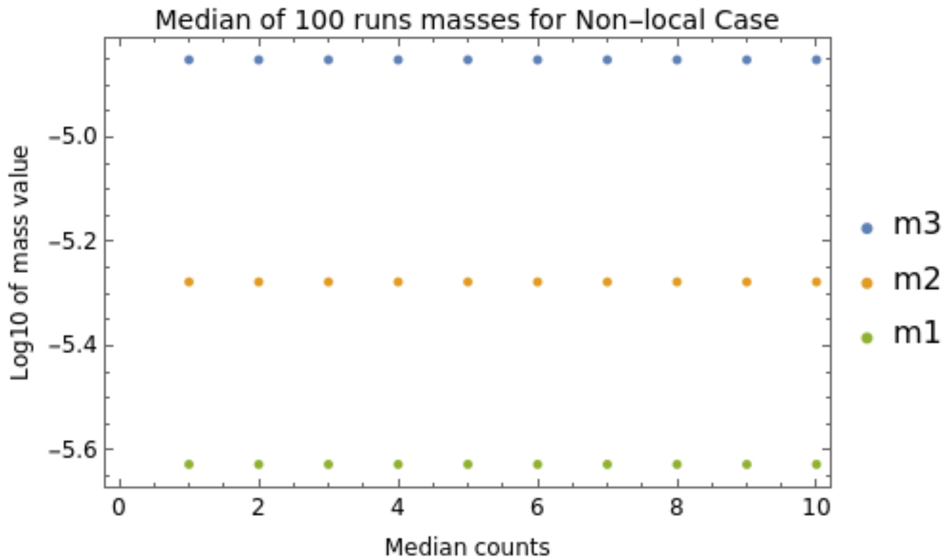
$$\epsilon \lesssim t$$

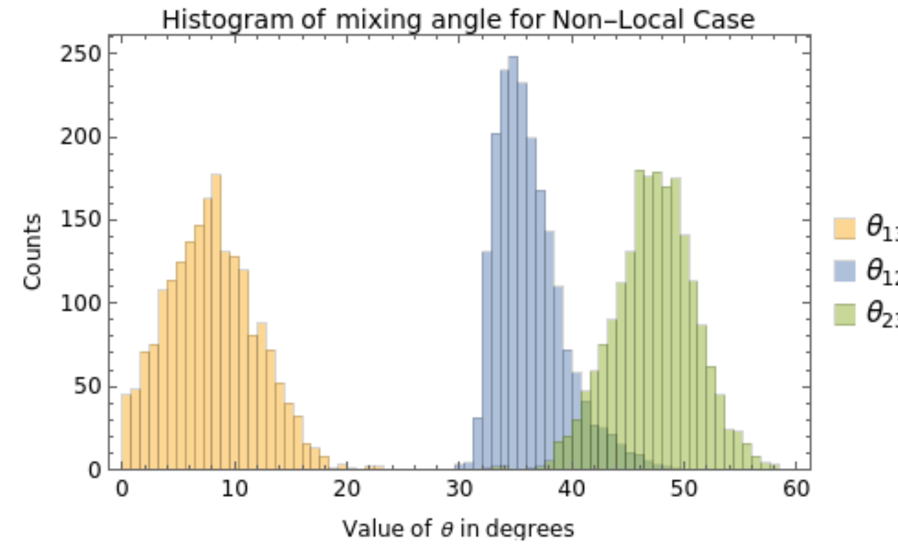




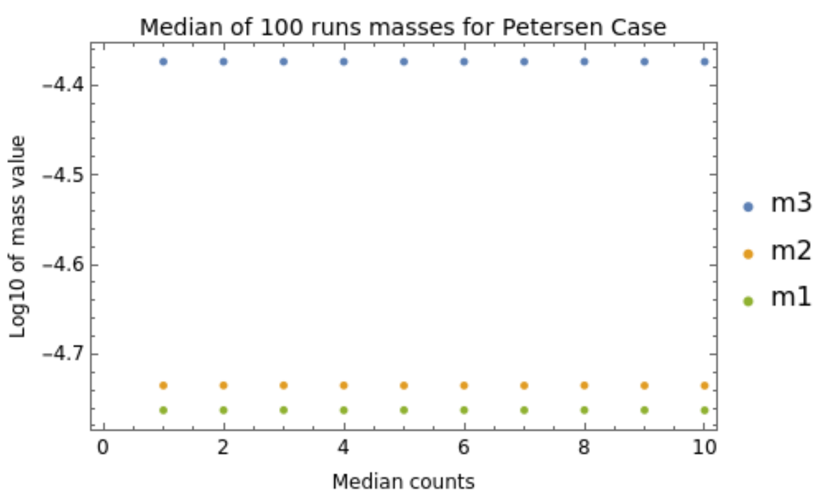
Mixing angles are “localised”.

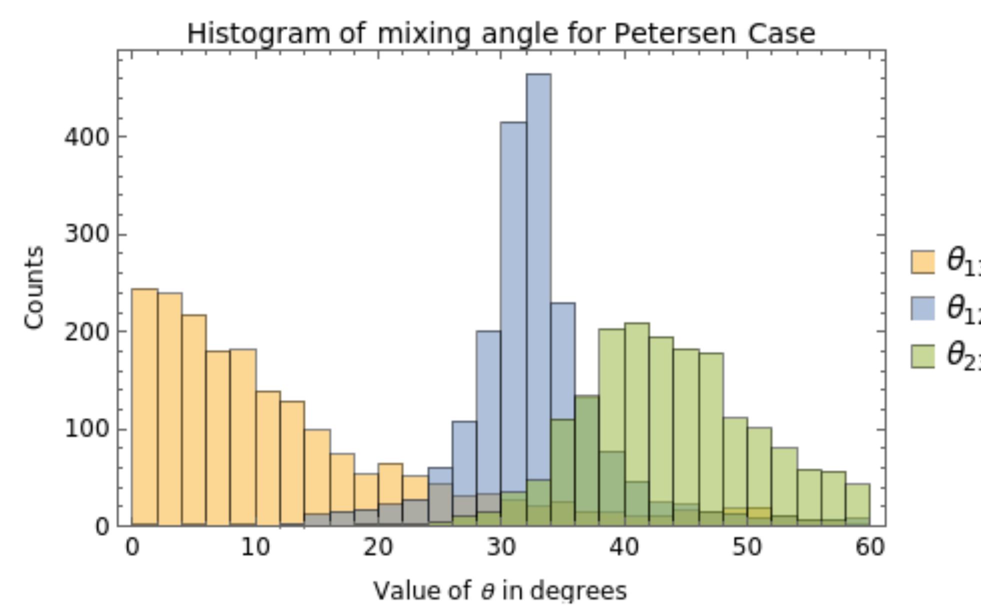
Fully non-local



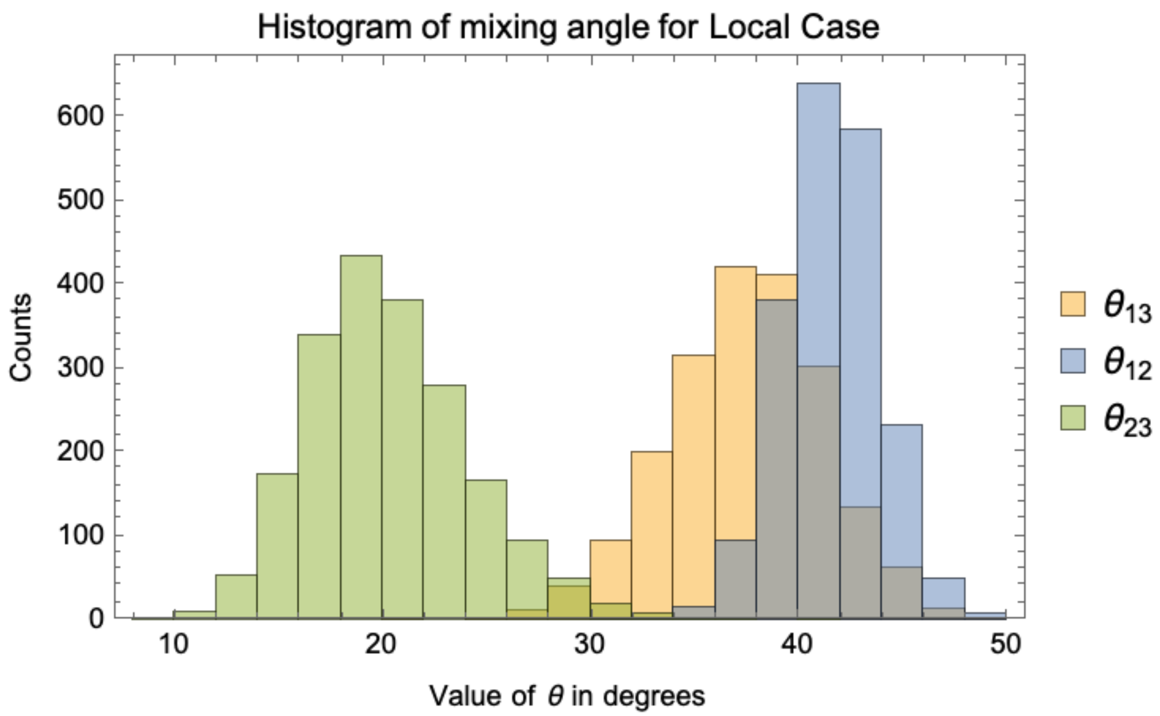


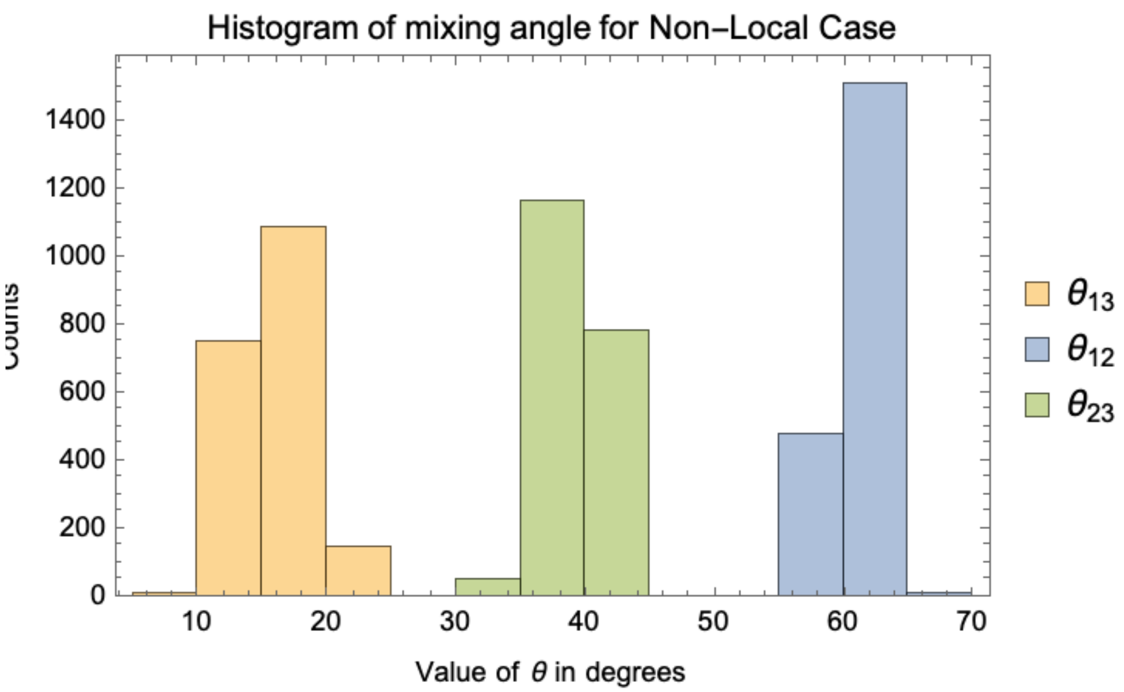
Partially non-local



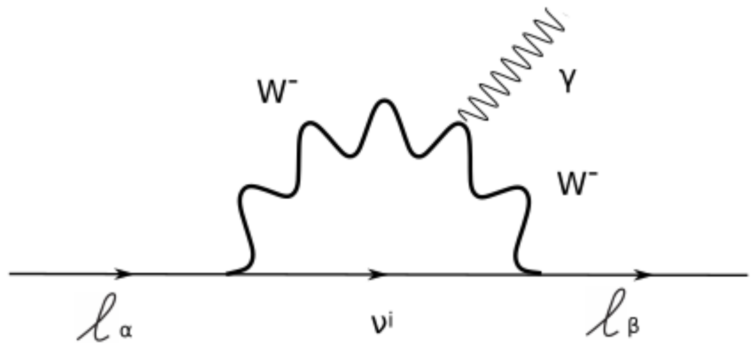


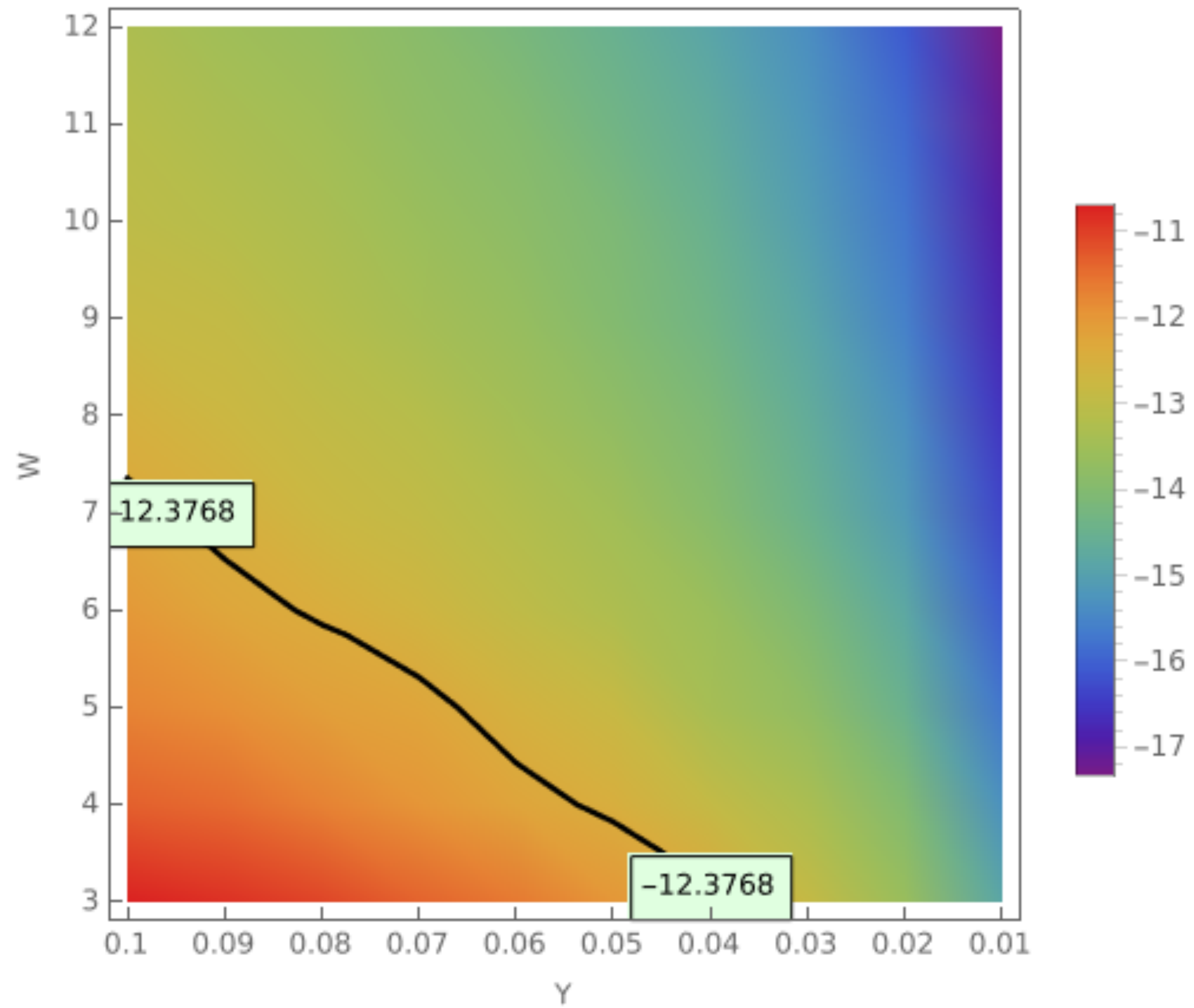
For the Majorana case, we get similar “ localisation”





Phenomenology





Constraints become weaker for non-local and partially local case.

Outlook

Randomness in couplings can lead to exponentially hierarchal couplings.

In the regime of strong coupling, the geometry of the mass chains does not matter significantly. They predict hierarchal neutrino masses and anarchical mixing angles for both Dirac or Majorana scenarios.

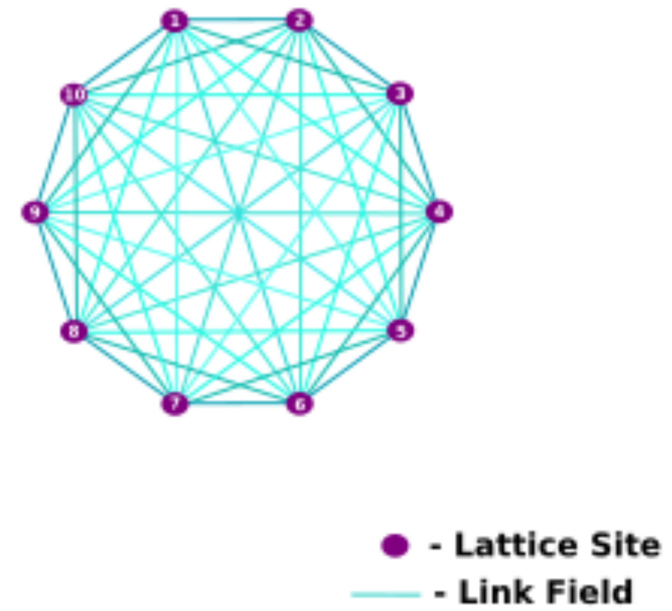
In the weak coupling regime, geometry does play a role and can be chosen carefully to ``localise” the mixing angles.

Experimental signatures become weaker for non-local /partially non-local cases compared to local case.

Back Up

$$M_{fermion} = \begin{bmatrix} 0 & v_1^1 & v_1^2 & v_1^3 & \ldots & v_1^n \\ v_n^1 & \lambda_1 & 0 & 0 & \ldots & 0 \\ v_n^2 & 0 & \lambda_2 & 0 & \ldots & 0 \\ v_n^3 & 0 & 0 & \lambda_3 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ v_n^n & 0 & 0 & 0 & \ldots & \lambda_n \end{bmatrix}$$

$$m_0 \approx \sum_{i=1}^{n} \frac{v_1^i v_n^i}{\lambda_i} \propto \sum_{i=1}^{n} v^2 \frac{e^{-\frac{n}{L_n}}}{\lambda_i}$$



Non Local and Two Dimensional Graphs

Phenomenology