THE STRANGE CASE OF TWIN NEUTRON STAR

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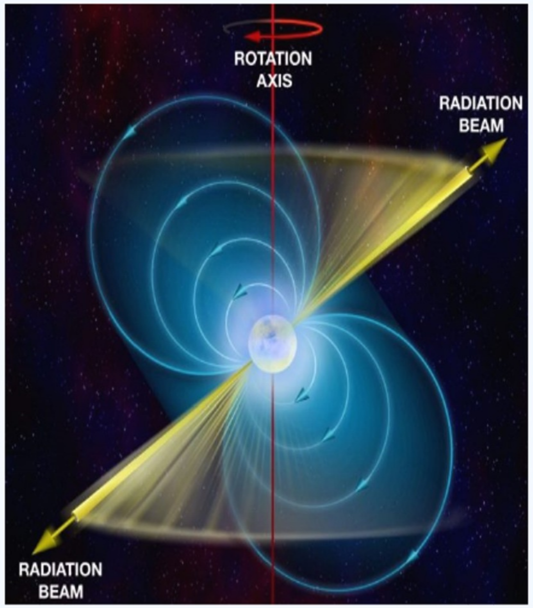
NEUTRON STAR: PROPERTIES

Theorized by Baade and Zwicky in 1934

Discovered around 30 years later as Pulsars in 1967 (Hewish and Bell)



Credits: earthsky.org

Properties of NS

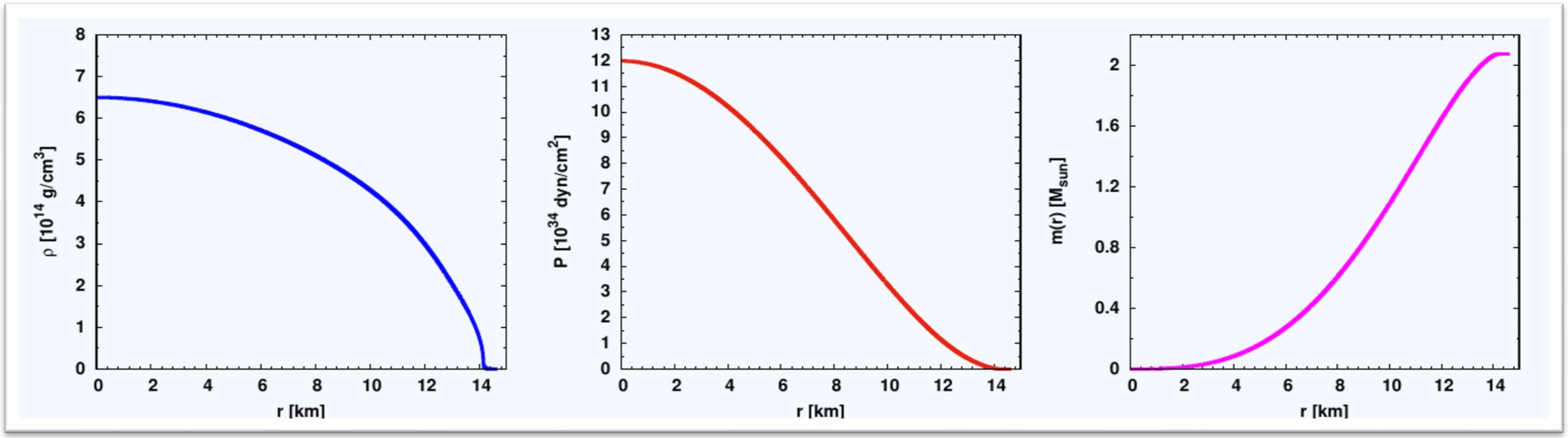
Mass	0.7 - 2.4 solar mass
Radius	10 – 15 km
Period	ms – sec
Density at core	$10^{14} – 10^{15}$ gm/cc
Magnetic field	10^{15} G (max at surface)

NEUTRON STAR: THEORY

$$\underbrace{G_{\mu\nu}}_{\text{Spacetime}} = \frac{8\pi G}{c^2} \underbrace{T_{\mu\nu}}_{\text{Matter}}$$

TOV Equation

$$\frac{dP}{dr} = -G\frac{M(r)\rho(r)}{r^2}\frac{\left[1+\frac{P(r)}{\rho(r)c^2}\right]\left[1+\frac{4\pi r^3 P(r)}{M(r)c^2}\right]}{\left[1-\frac{2GM(r)}{rc^2}\right]}, \qquad M(r) = 4\pi\int_0^r \rho(r)r^2 dr$$



NEUTRON STAR: EOS

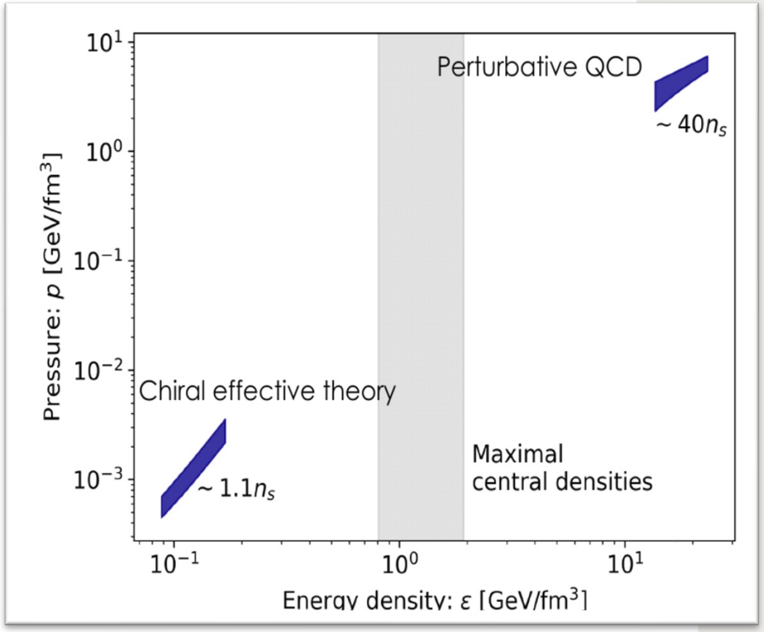
Current Knowledge

Two extremes

We are confident about the matter properties at the two extremes

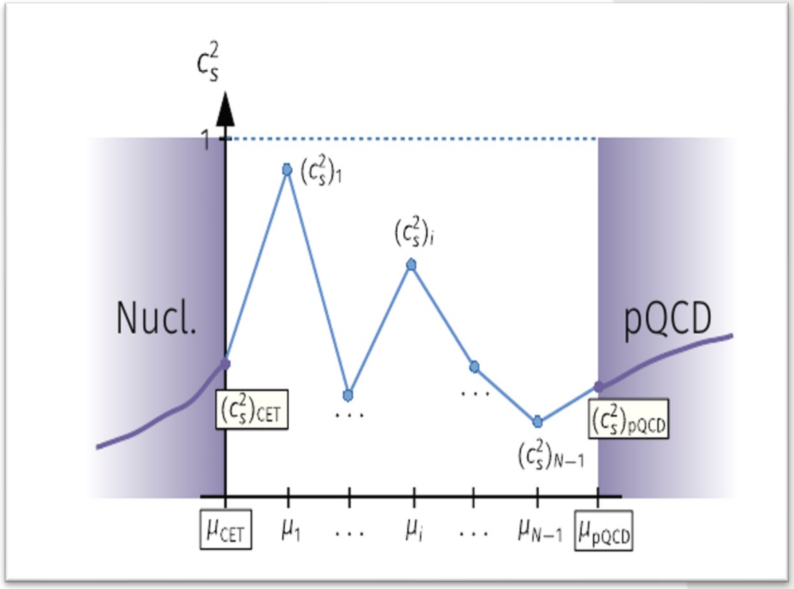
Low density: Chiral effective field theory

High density: perturbative QCD



A Kurkela, Quark Confinement Conf, Stavanger

NEUTRON STAR: EOS



Agnostic EOS

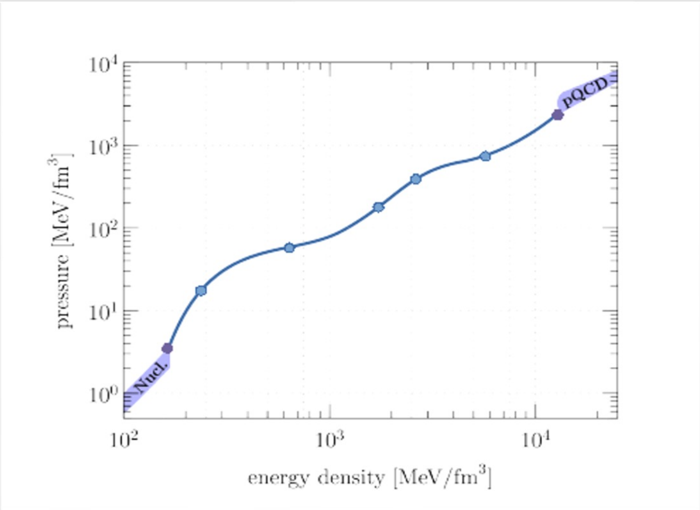
Bounds: Low density ⟹ CMF model

High density ⟹ pQCD

In between randomizing speed of sound as a function of chemical potential (0–1)

Piecewise linear extrapolation to generate continuous curve

Annala et al., PRX 12, 011058, 2022



$$n(\mu) = n_{CET}\, \exp\left(\int_{\mu_1}^{\mu} \frac{d\mu'}{\mu' c_s^2(\mu')}\right)$$

$$p(\mu) = P_o + \int_{\mu_1}^{\mu} d\mu' n(\mu')$$

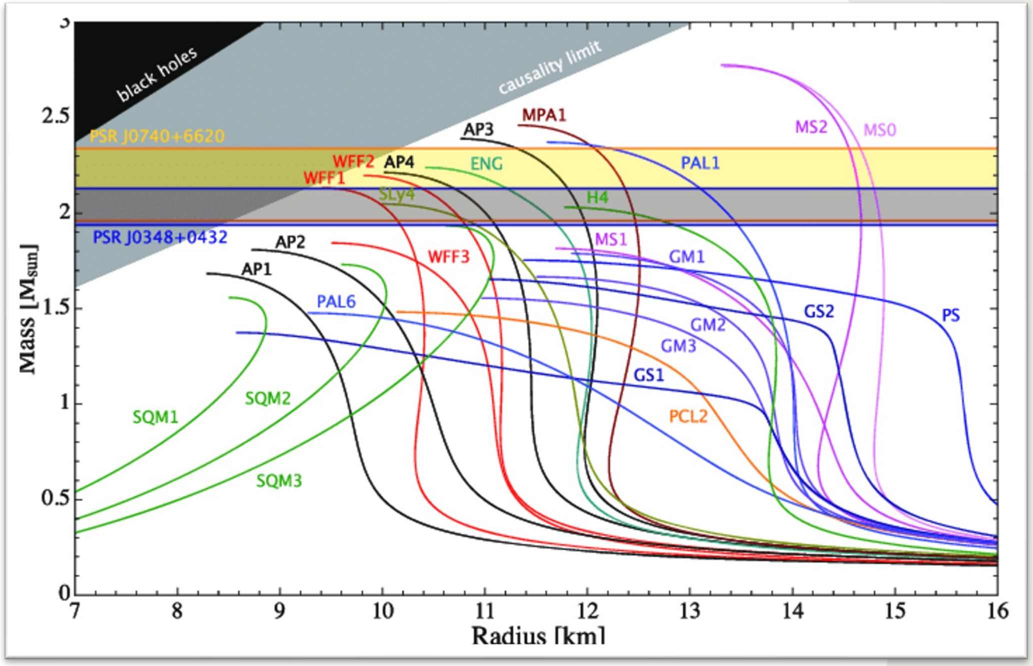
$$c_s^2(\mu) = \frac{(\mu_{i+1}-\mu)\, c_{s,i}^2 + (\mu-\mu_i)\, c_{s,i+1}^2}{\mu_{i+1}-\mu_i}$$

NEUTRON STAR: M-R

Given an ***EOS*** (relationship between thermodynamic variables)

↓

Unique ***Mass-Radius*** curve



Huanchen Hu et al, MNRAS 497, 3118 (2020)

Different curve for different EoS

What is the actual EoS?
The form of matter at the core of NS?

Test the theory of high-density matter with NS observation

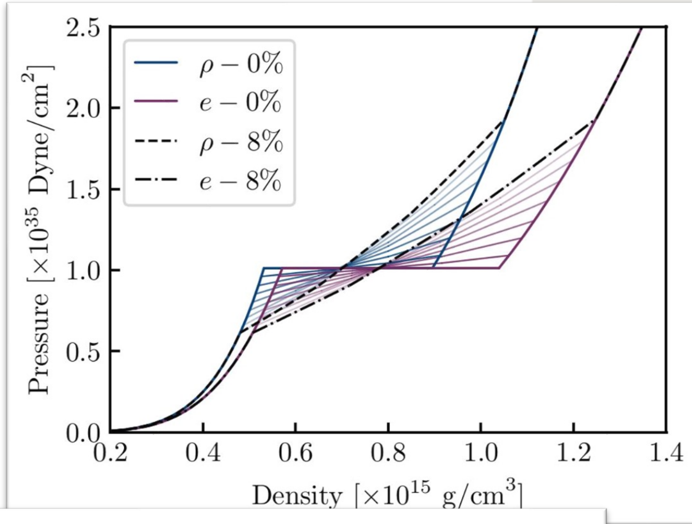
1ST-ORDER PT

1st-order PT

Another Possibility: 1st-order PT

A finite density discontinuity, Pressure is continuous

In mass radius diagram, a new branch emerges

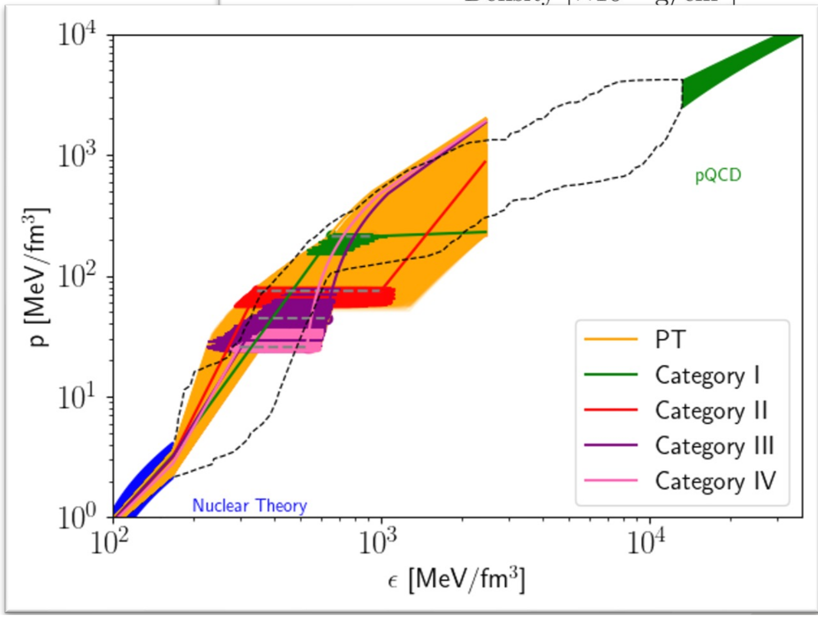


Before PT $\Rightarrow$

$$p(n) = p_{\rm EFT}(n_{\rm EFT})(n/n_{\rm EFT})^{\Gamma_1};$$

After PT $\Rightarrow$

$$p(n) = p(n_{\rm PT})[n/(n_{\rm PT} + \Delta n)]^{\Gamma_2}.$$



The transition density sampled randomly starting from twice the nuclear density

The density discontinuity is also sampled randomly

NEUTRON STAR: TWINS

NEUTRON STAR: TWINS

The speciality about twins

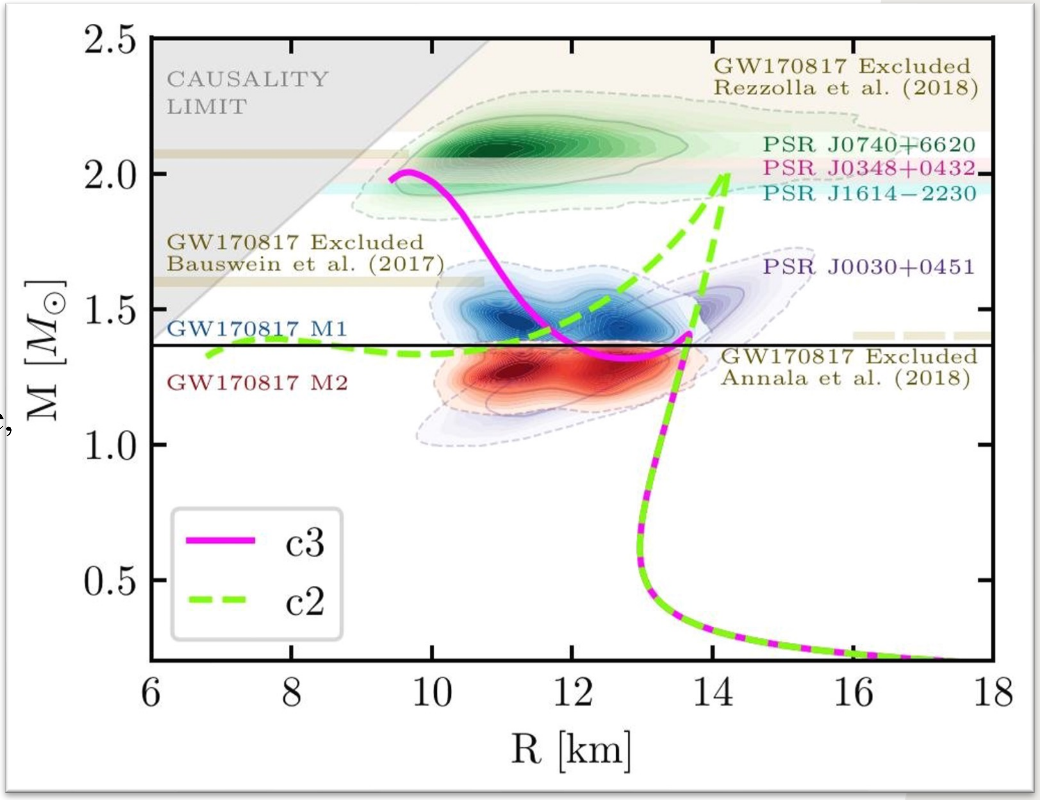
In some cases, the second branch can be unstable for some radius and then again becomes stable

Mass of the star of stable and unstable branch same, but different Radius

→ ***Twins***

Same mass → Different Radius

Can be of various types



NEUTRON STAR: TWINS

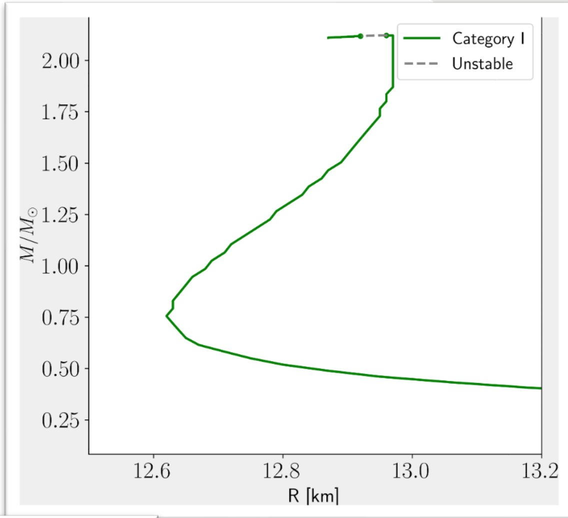
Twin Categories

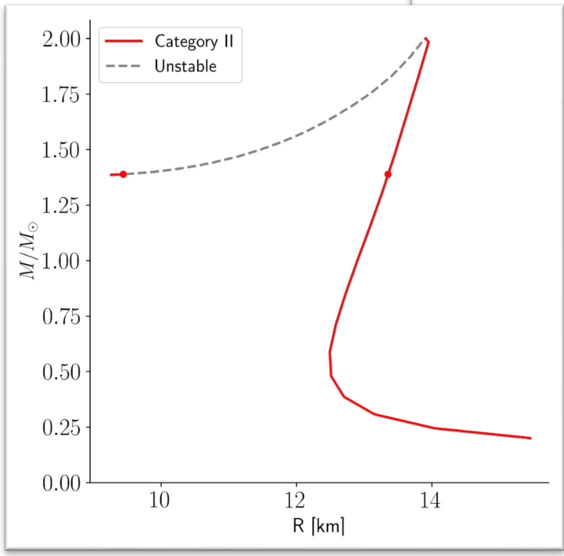
Category I: M $> 2M_\odot$

Category II: M $< 2M_\odot$

Category III: $M_\odot <$ M $< 2M_\odot$

Category IV: M $< M_\odot$





NEUTRON STAR: TWINS

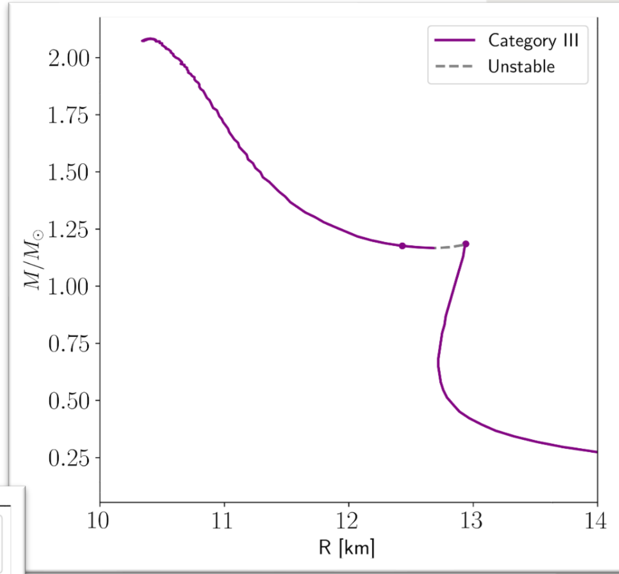
Twin Categories

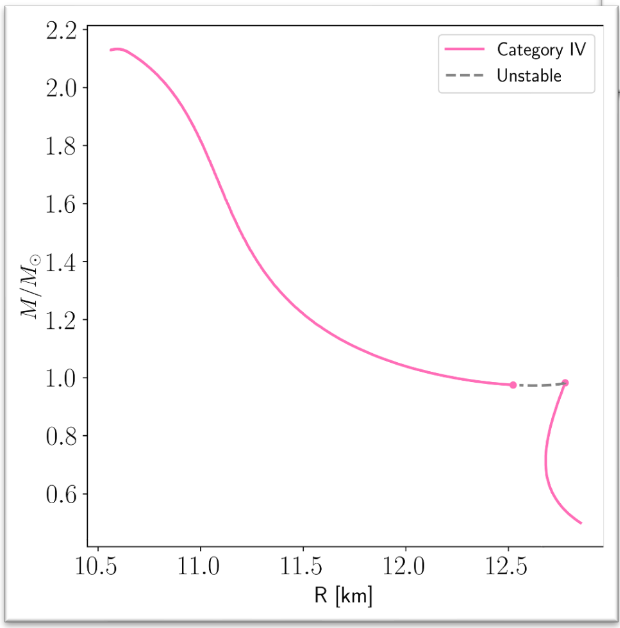
Category I: M > $2M_\odot$

Category II: M < $2M_\odot$

Category III: $M_\odot$ < M < $2M_\odot$

Category IV: M < $M_\odot$





FILTERING TWINS

Universal relations

I – Love – Q (dimensionless)

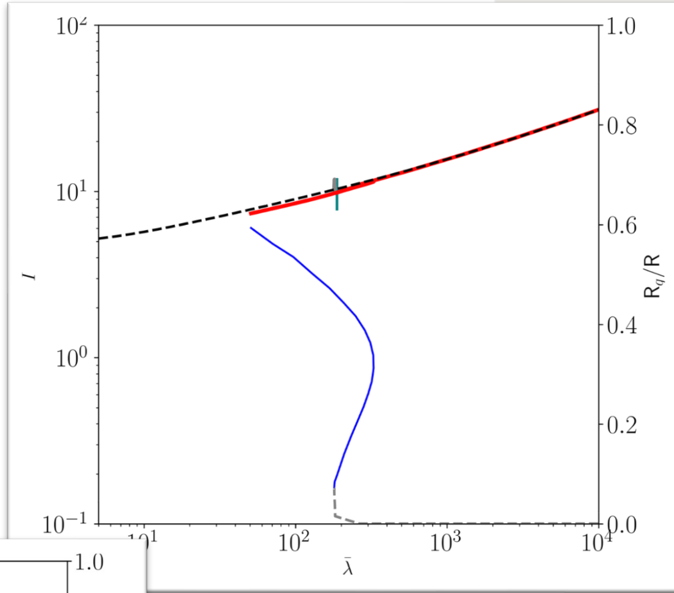
I: Moment of Inertia

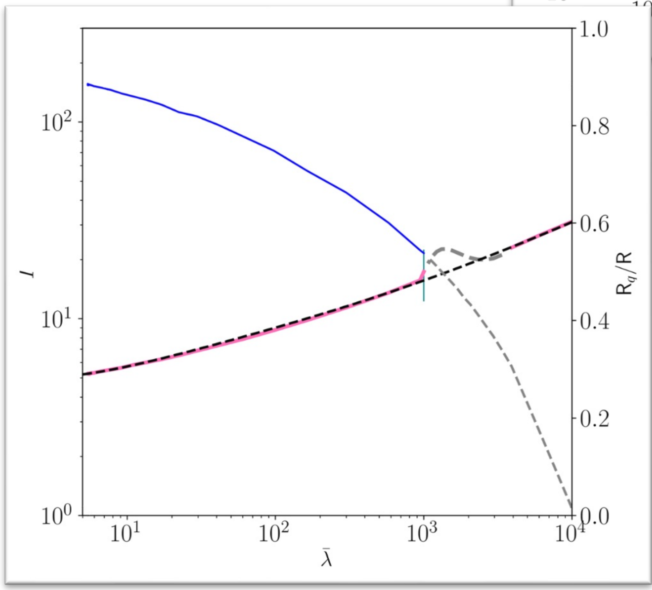
Love: Love Number

Q: Quadrupole moment

Deviates from UR when quark-core appears

Far away ➡ respects the UR





FILTERING TWINS

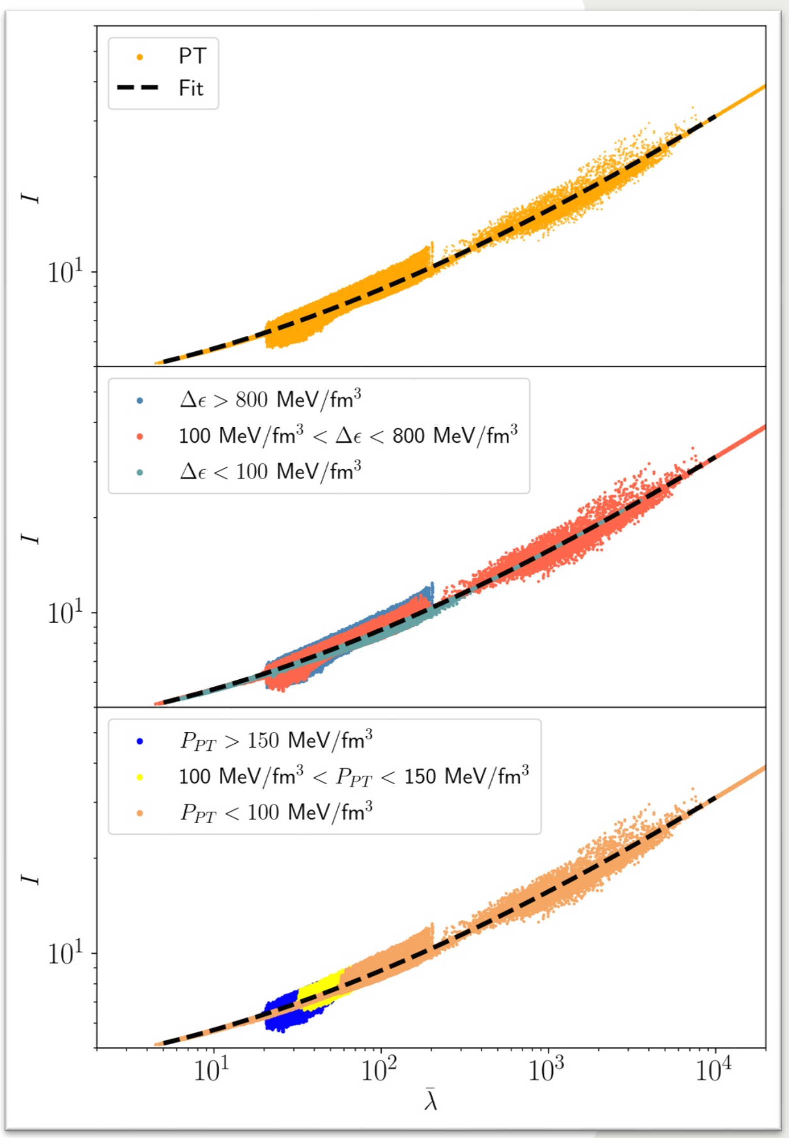
Universal relations

$$y = a_0 + a_1 \log\bar{\lambda} + a_2(\log\bar{\lambda})^2 + a_3(\log\bar{\lambda})^3 + a_4(\log\bar{\lambda})^4$$

	a_0	a_1	a_2	a_3	a_4
$\bar{Q}$	-0.01653	0.16145	0.08245	-0.01876	0.001352
$\bar{I}$	0.64633	0.06582	0.0477	-0.00311	7.5214×10^{-5}

For normal 1st-order PT, deviation depends on density discontinuity

But not much on the point of PT



FILTERING TWINS

Universal relations

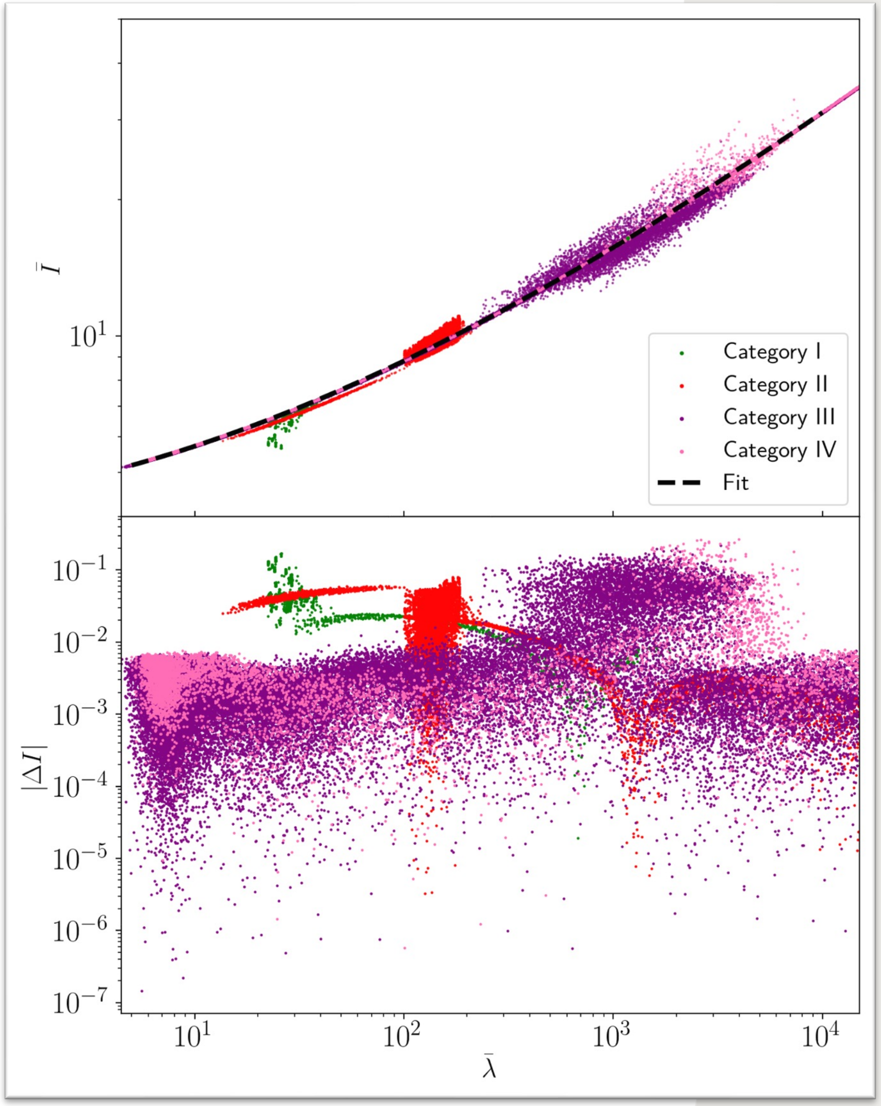
UR for Twin EOS

Fractional percentage error

Category I and shows most deviation for massive stars

Category II shows most deviation for intermediate stars

Category III and IV shows most deviation for low mass stars



FILTERING TWINS

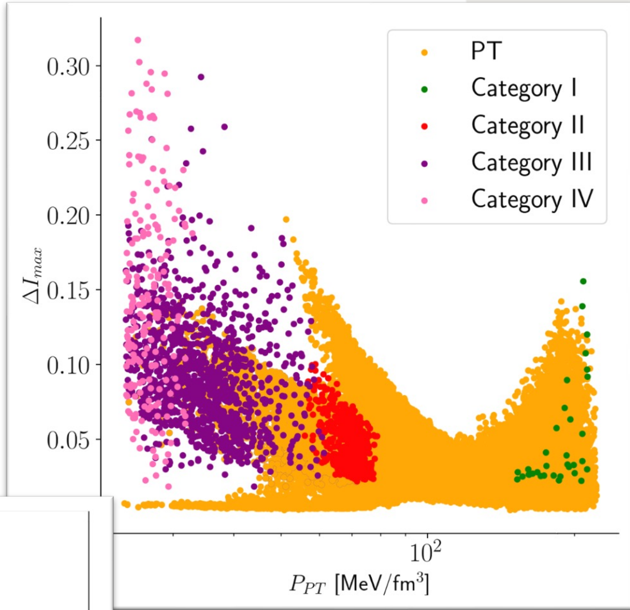
Universal relations

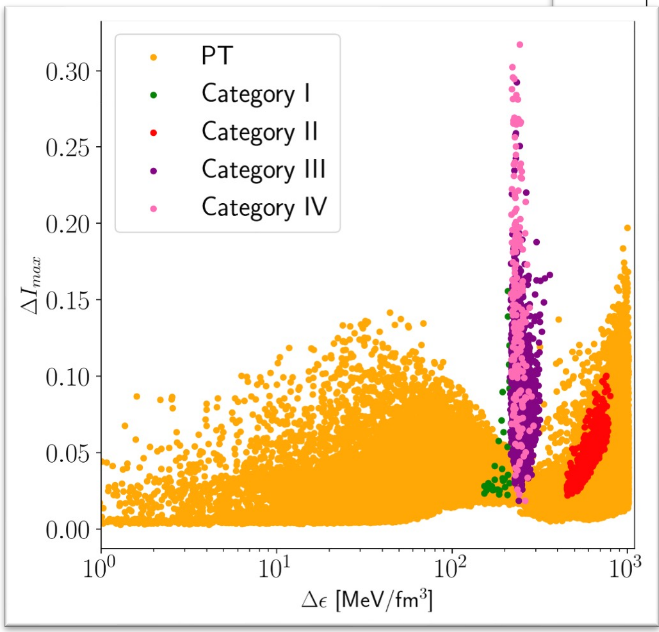
Maximum deviation with P_{PT}

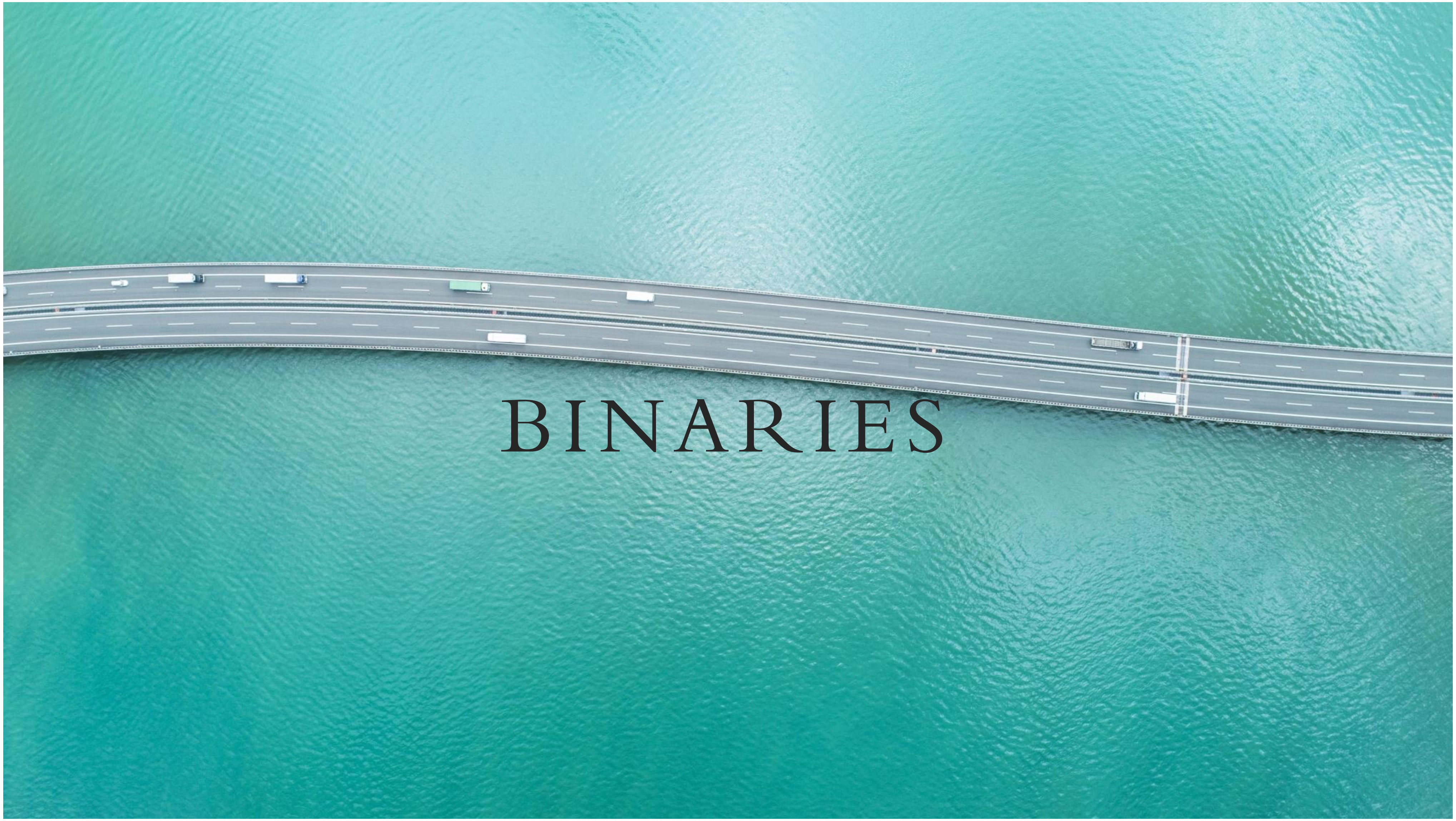
Category III & IV max deviation
Least category I
Breaking of UR depends on P_{PT}

Maximum deviation with discontinuity

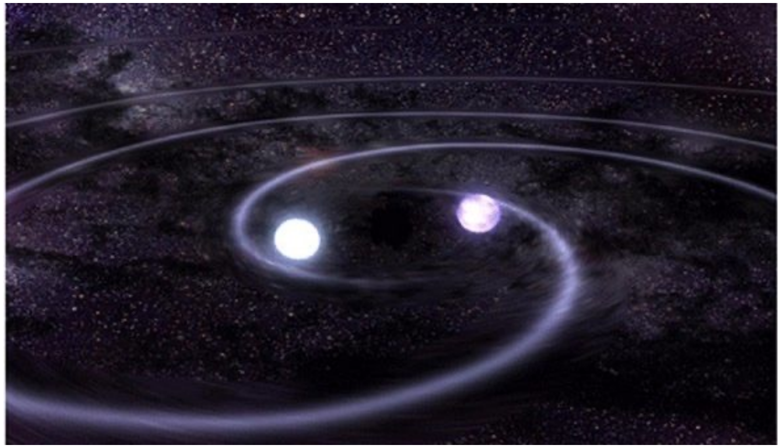
No such dependence



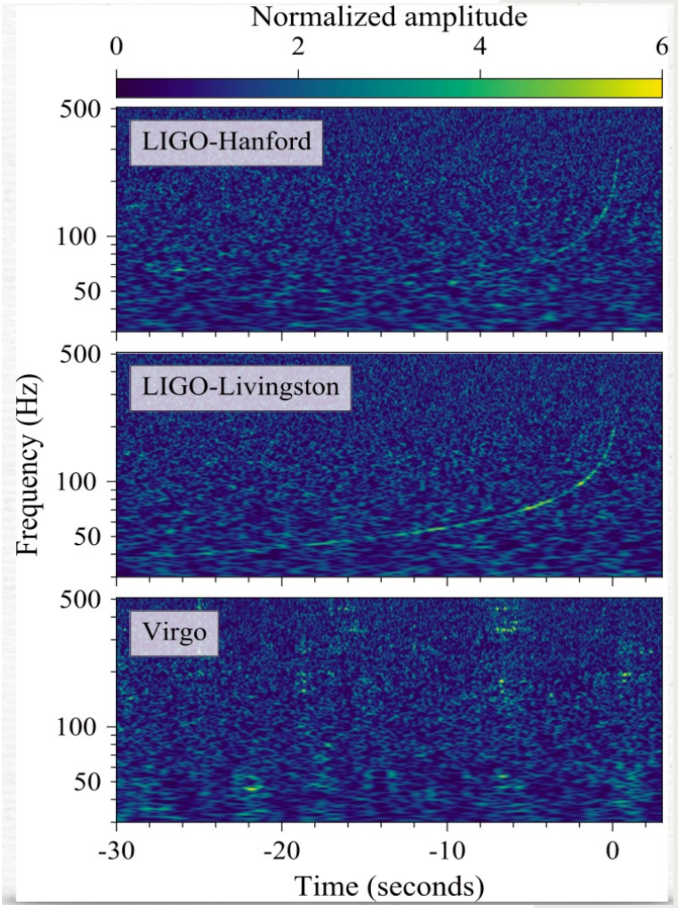



BINARIES

NEUTRON STAR: BINARIES



NASA/Dana Berry, Sky Works Digital



Abbott et al., PRL119, 161101 (2017)

Gravitational Wave

GW170817

NS + NS $\xrightarrow{\text{GW}}$ *HMNS* $\xrightarrow{\text{GW}}$ *BH*

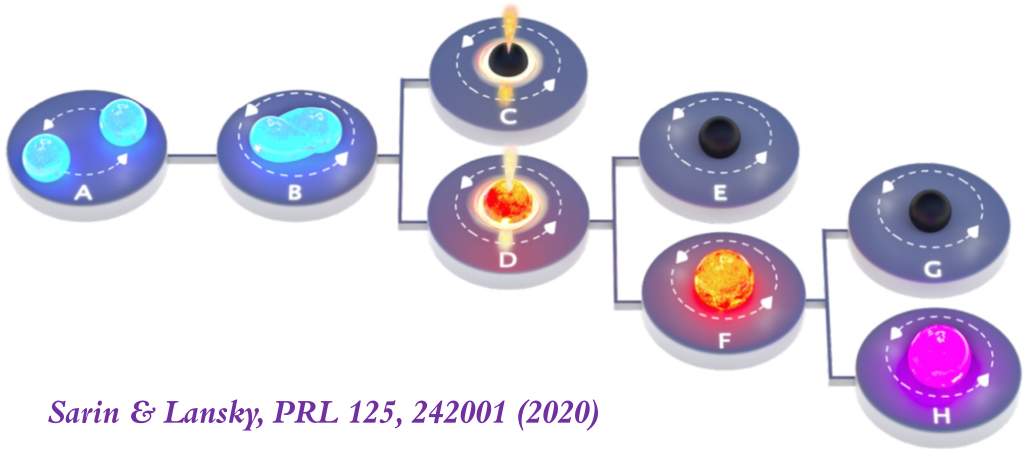
NS BINARIES

Detection of the inspiral part, before the merger

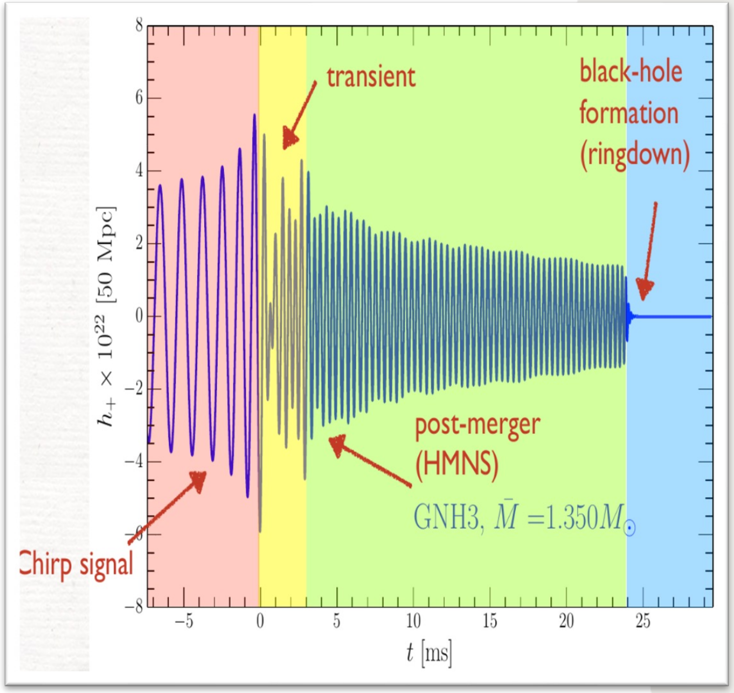
Not only ***GW*** but also ***sGRB*** and ***Electromagnetic Signal***

Multi-messenger signal

Post-merger signal not detected, expected to have more rich physics



Sarin & Lansky, PRL 125, 242001 (2020)

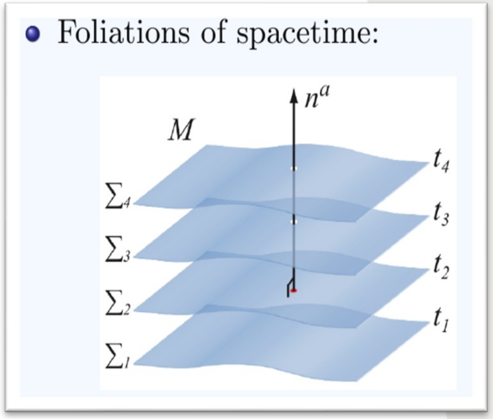


Takami et al., PRL 113, 091104 (2014)

NS: NUMERICAL RELATIVITY

Einstein Equation and Numerical relativity

$$\underbrace{G_{\mu\nu}}_{\text{Spacetime}} = \frac{8\pi G}{c^2} \underbrace{T_{\mu\nu}}_{\text{Matter}}$$



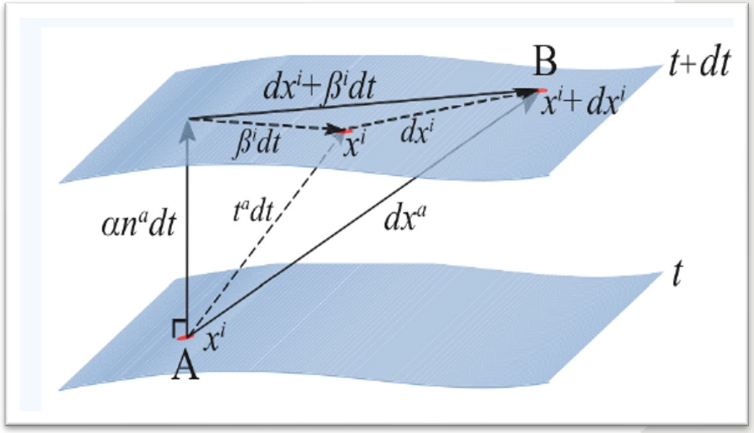
Numerical Relativity: 3+1 Formalism

Foliate 4-d space-time ⟹ 3-d spacelike hypersurface

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}\left(dx^i + \beta^i dt\right)\left(dx^j + \beta^j dt\right)$$

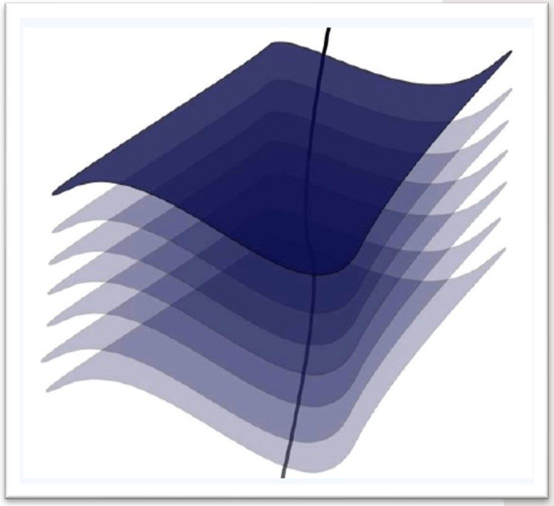
$$g_{ab} = \begin{pmatrix} -\alpha^2 + \beta_l \beta^l & \beta_i \\ \beta_j & \gamma_{ij} \end{pmatrix} = \gamma_{ab} - n_a n_b$$

$$\begin{aligned} &\gamma_{ab} = g_{ab} + n_a n_b \implies \textit{spatial metric} \\ &n^a \implies \textit{normal vector} \\ &\beta^i \implies \textit{shift vector} \\ &\alpha \implies \textit{lapse function} \end{aligned}$$



NS: NUMERICAL RELATIVITY

Einstein Equation and Numerical relativity



Evolution Equation

Spatial metric

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + D_i\beta_j + D_j\beta^i$$

Extrinsic curvature

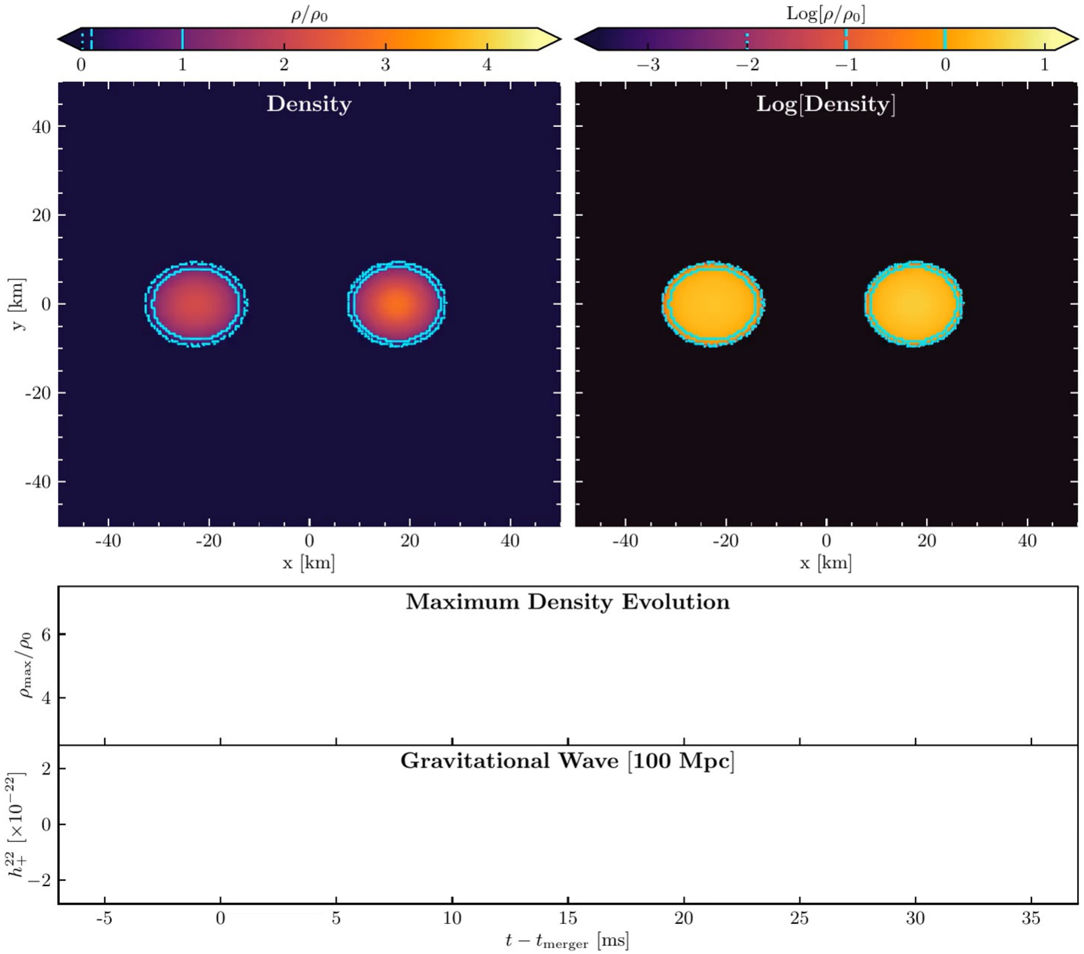
$$\partial_t K_{ij} = -D_i D_j \alpha + \alpha\left(R_{ij} - 2K_{ik}K^k{}_j + KK_{ij}\right) - 8\pi\alpha\left(S_{ij} - \frac{1}{2}\gamma_{ij}(S-\rho)\right) + \beta^k D_k K_{ij} + K_{ik} D_j \beta^k + K_{kj} D_i \beta^k$$

GRHD Equations

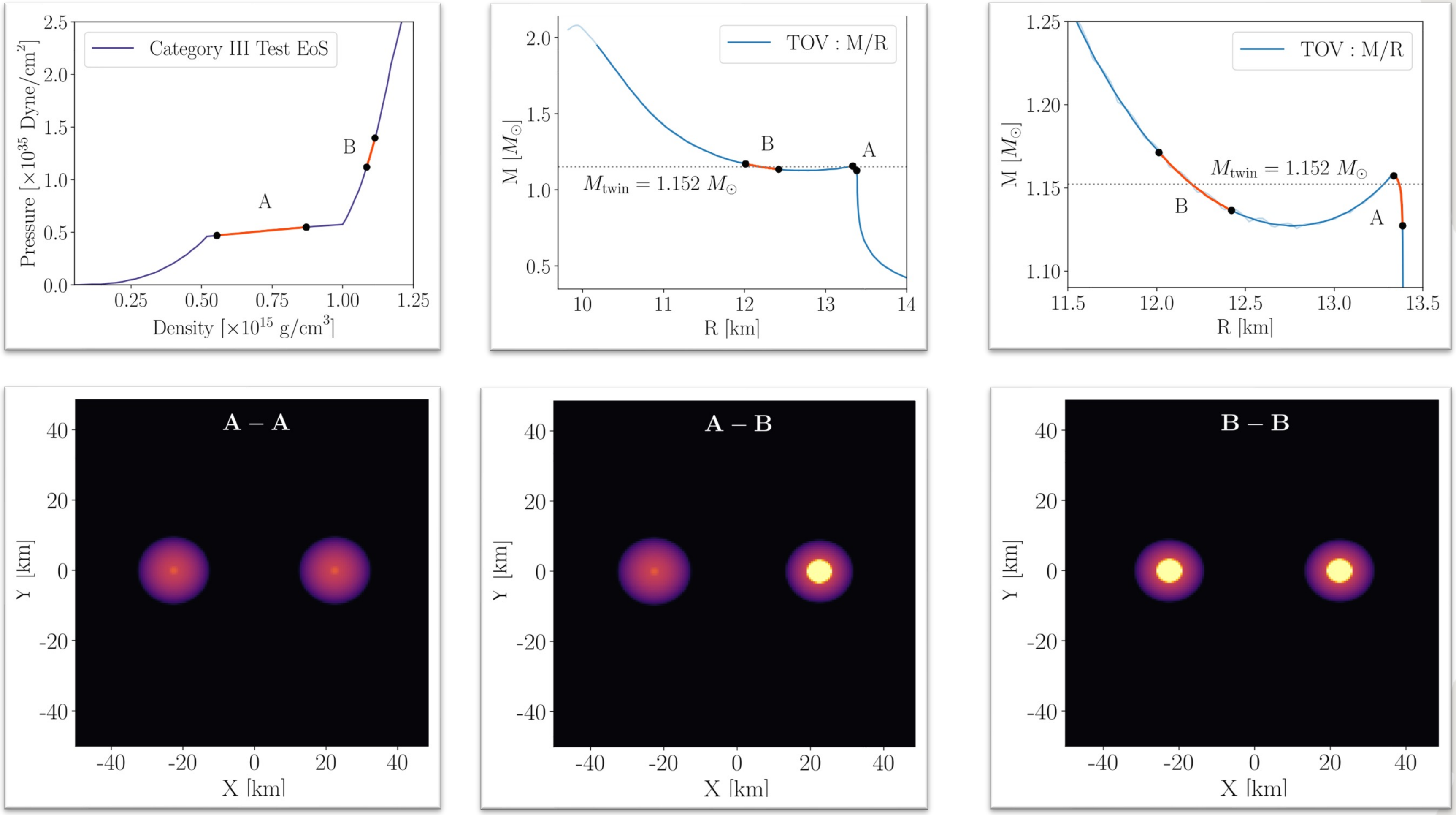
$$\nabla_\nu T^{\mu\nu} = 0 \qquad \nabla_\nu(\bar{\rho}U^\nu) = 0$$

NEUTRON STAR:BINARIES

Binary Merger



TWIN BINARIES



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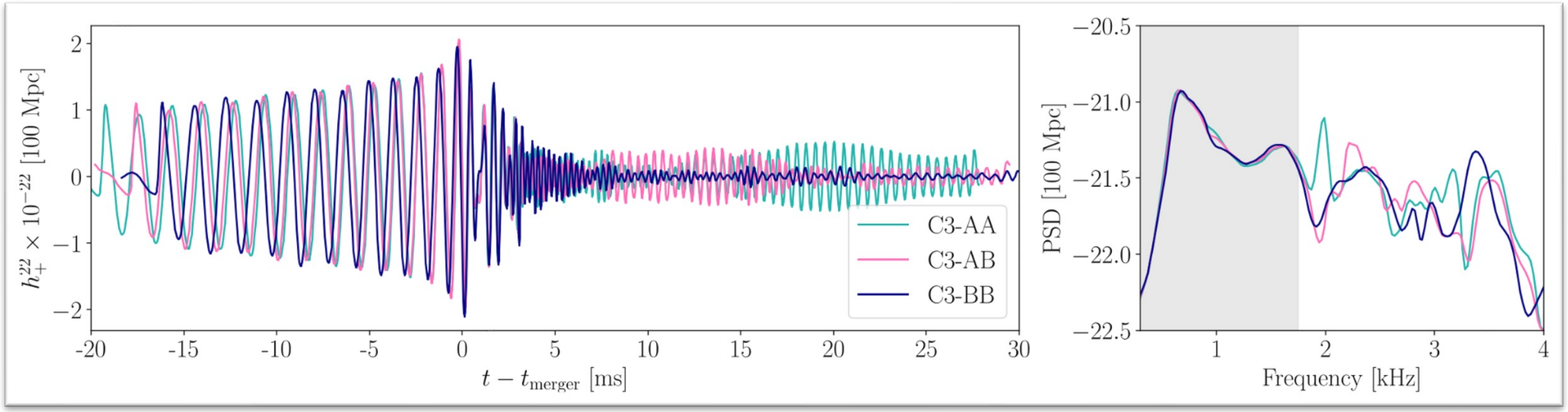

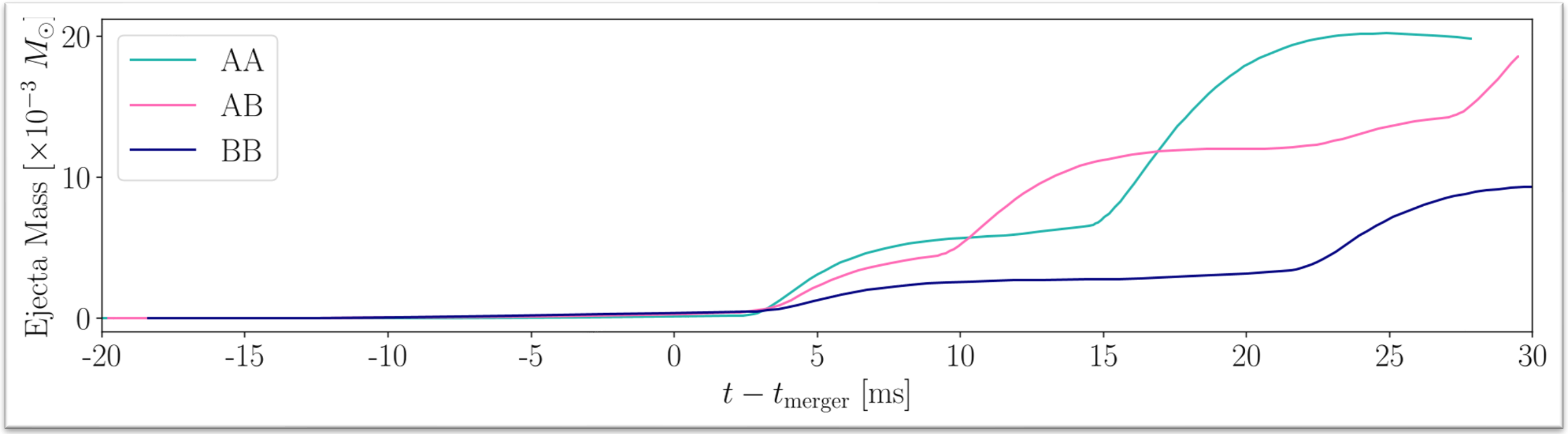

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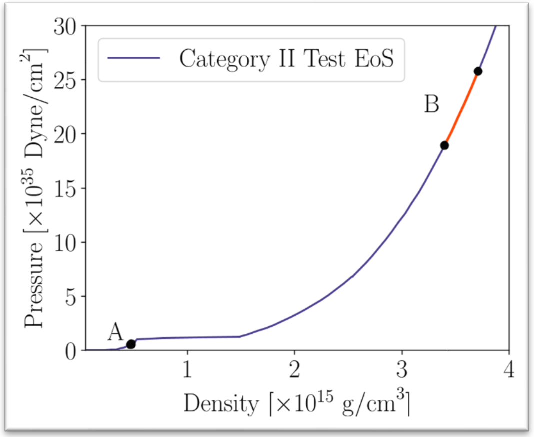

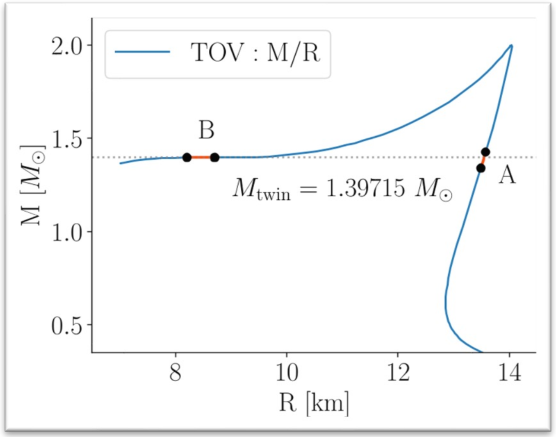

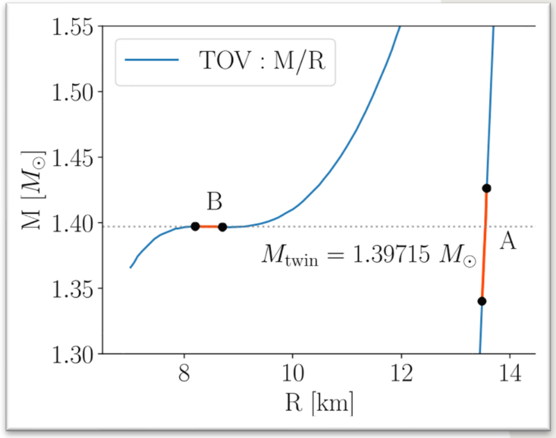

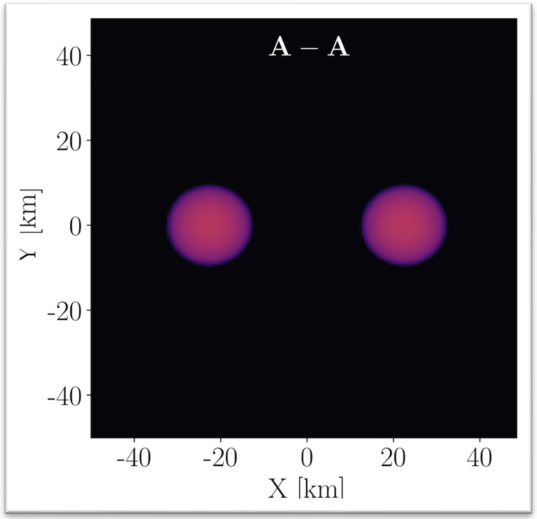

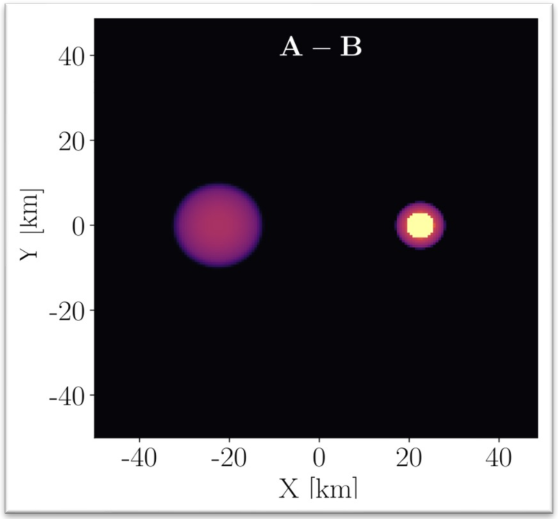

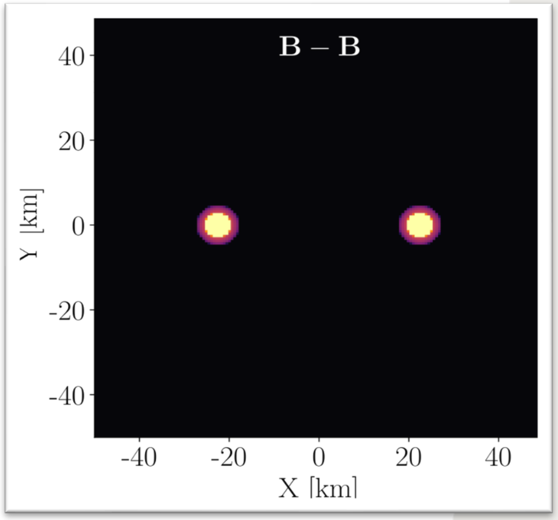

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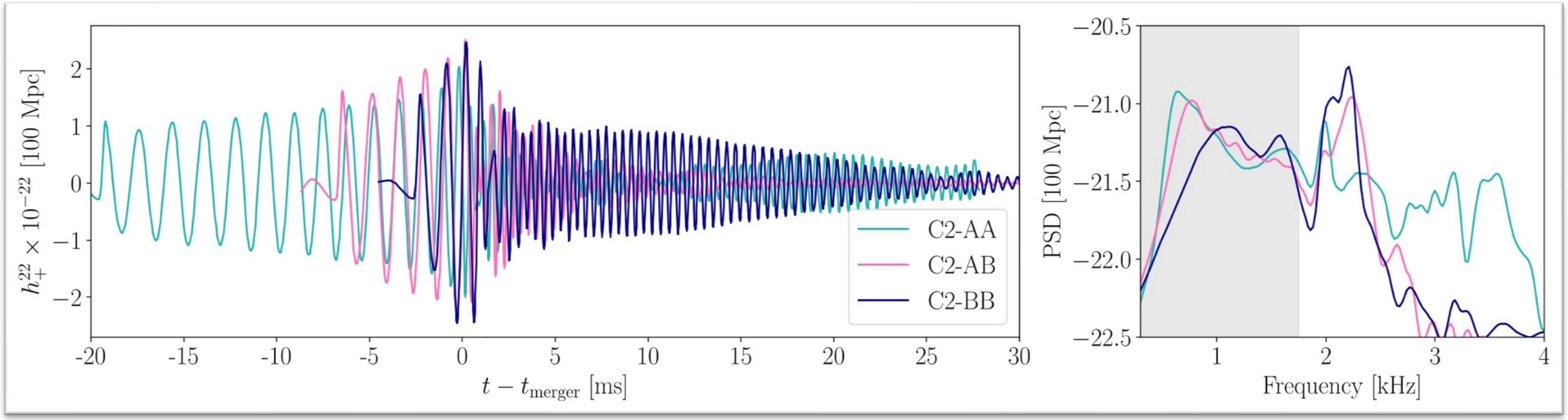
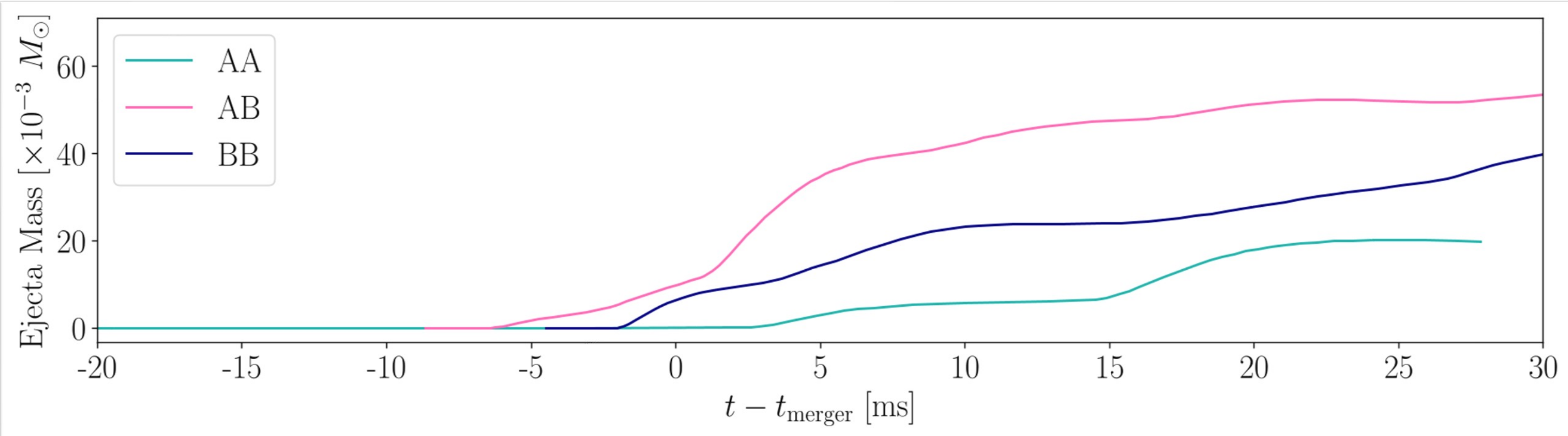




We are the Compact Object ASTrophysics (COAST) Research Group

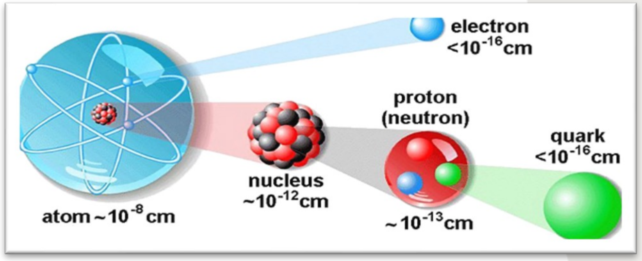
sites.google.com/iiserb.ac.in/coast/home


THANK YOU

ENIGMA OF NEUTRON STARS

High Mass + Small Radius ⟹ Dense matter at core

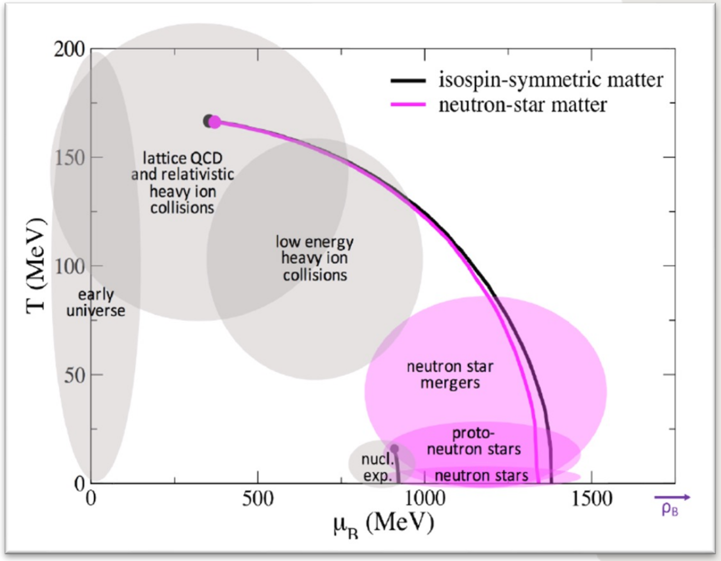
Probe for High density matter



Phase transition in NSs

NS ⟹ *QS/HS*

However, there is still no conclusion



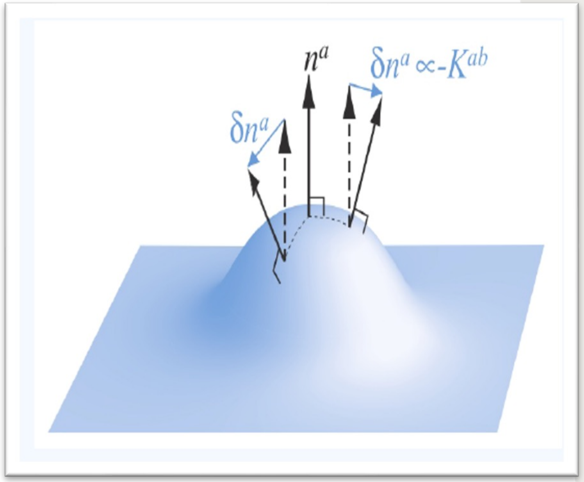
Dexheimer et al., Universe, 2019

NS: NUMERICAL RELATIVITY

Einstein Equation and Numerical relativity

Extrinsic Curvature

$$\begin{aligned} K_{ab} &= -\gamma_a{}^c \gamma_b{}^d \nabla_c n_d \\ &= -\nabla_a n_b - n_a a_b \\ &= -\frac{1}{2}\mathcal{L}_{\mathbf{n}}\gamma_{ab} \end{aligned}$$



Constraints Equations

Hamiltonian

$$R + K^2 - K_{ij}K^{ij} = 16\pi\rho$$

Momentum

$$D_j\left(K^{ij} - \gamma^{ij}K\right) = 8\pi S^i$$

Source Term

$$\begin{aligned} \rho &= n_a n_b T^{ab} \\ S^i &= -\gamma^{ij} n^a T_{aj} \\ S_{ij} &= \gamma_{ia}\gamma_{jb} T^{ab} \\ S &= \gamma^{ij} S_{ij} \end{aligned}$$